Blind Multi-user Detection of a Chaos-based CDMA System using Support Vector Machine

Johnny W. H. Kao¹, Stevan M. Berber¹, and Vojislav Kecman²
¹Department of Electrical and Computer Engineering
²Department of Mechanical Engineering
University of Auckland
Auckland, New Zealand
j.kao@ece.auckland.ac.nz, s.berber@auckland.ac.nz, v.kecman@auckland.ac.nz

Abstract—This paper presents the algorithms and the results of multi-user detectors (MUD) on a synchronous chaos-based code division multiple access system (CDMA), which uses chaotic sequences as the spreading codes. Popular linear and non-linear MUD algorithms such as the decorrelator detector, minimum mean square error (MMSE) detector and parallel interference cancellation (PIC) detector are all considered in this paper. These conventional detectors are used to compare the BER performance with a novel blind-MUD receiver. The blind-MUD is achieved by a recently emerged learning technique called support vector machines (SVM). This method can be used to replace the conventional matched filter of the receiver and can be implemented on the forward link. All the MUD schemes are simulated over an AWGN channel and result shows that the blind-MUD compare favorably with other techniques.

Keywords- chaos, code division multiple access, multi-user detection, support vector machine

I. INTRODUCTION

Code division multiple access (CDMA) is one of the most well-studied systems for a multi-user spread spectrum communication network [1]. In a typical direct-sequence CDMA (DS-CDMA) system, each user is distinguished by a unique spreading code. During transmission, the information from all users are modulated by the spreading codes and then mixed together. Therefore in order to separate the information for each user at the receiver’s side, those spreading codes should ideally be orthogonal with each other.

Over the past decade, an intensive research on non-linear dynamic system modeling leads to the generation of chaotic signals which are aperiodic, deterministic and most importantly uncorrelated. These are desirable characteristics which make chaotic signal suitable for multiple-access spread spectrum systems and secure communication. The chaotic signals are highly sensitive to their initial conditions hence it is much easier to generate uncorrelated sequences. Therefore it can increase the system’s overall capacity by allowing more users to share the channel simultaneously and improve the security issue because it becomes much more difficult to replicate the chaotic signal without knowing its initial state. Over the last few years, chaotic communication emerges as a new field in multiple-access spread spectrum systems [2]. It is now commonly agreed that the chaotic sequences can be used as the spreading codes for a typical DS-CDMA system [3, 4].

A major concern for a DS-CDMA system is the non-orthogonality of the users’ spreading code would create interference with each other. This problem of multi-access interference (MAI) is worsened when more users are transmitting or when the interferers are sufficiently powerful at the receiver’s side to cause performance degradation, which is known as the near-far problem [1]. With the increasing use of cellular mobile devices nowadays, this problem becomes the major limiting factor on the system’s performance.

Multi-user detectors are specifically designed to combat this problem. Verdú proposed an optimal MUD in 1986 based on maximum sequence likelihood estimation (MSLE) [5]. The optimum detector provides huge performance improvement. Unfortunately, the algorithm has an exponential computational complexity with the number of users [5]. As a result, it is impossible to implement on a practical system. Later on, a variety of sub-optimal MUDs have been proposed that approaches the performance of the optimum detector with a much reduced complexity. These suboptimum receivers can be broken down into two general categories: linear and non-linear. These MUDs all create data estimate based upon some transformations of the sufficient statistics (i.e., the outputs at the bank of matched filters) [1, 5-7]. The linear detectors considered in this paper are the decorrelator detector and the minimum mean square error (MMSE) detector. The non-linear detector is based on parallel interference cancellation (PIC) detection.

It is worthy to note that in general, MUDs would require knowing the spreading sequences of all users in the system in order to perform the estimation, hence it is commonly agreed that MUD is mostly designed for the reverse link scenario (user to base-station) [7]. So far, very few literatures are found on investigating MUD for chaotic communication systems. In 2005, He and Leung [8] proposed a MUD of chaotic systems using optimal chaos synchronization, which is still only feasible for the reverse link. Blind-MUD which can be realized
for the forward link (base-station to user) possesses a huge challenge for MUD design. Lau et al. [9] shows a blind adaptive MUD of chaotic CDMA system using adaptive filters. In that investigation, it does not have clear comparisons of the bit error rates with other conventional MUDs hence it is difficult to determine the algorithm’s performance.

A machine learning technique called support vector machine (SVM), which originated from statistical learning theory, became immensely popular and highly recognized in the fields of data-mining and pattern classification [10]. Recently, there have some preliminary investigations on applying SVM on CDMA systems as a blind multi-user detector [11-13]. All results have shown that MUD based on SVM performs closely to an optimum receiver. However, those findings were usually investigated under short PN codes with only a very few users.

In this paper, the design approach is completely different from the previous works, where SVM is applied directly to replace the original matched filter. All results are simulated on a chaos-based CDMA system on a larger scale for a more practical measure of the system performance. Therefore the main focus of this paper is comparing the proposed SVM approach to reduce the level of MAI. The main principle of this system model.

II. CHAOS-BASED CDMA WITH MULTI-USER DETECTION

A. System Model

The chaos-based CDMA model is very similar to a traditional direct sequence code division multiple access (DS-CDMA) system except the spreading codes used are replaced by chaotic sequences.

Assuming a synchronous DS-CDMA system, the transmitter output is,

\[ s(t) = \sum_{i=0}^{M} \sum_{k=1}^{K} b_k(i) c_k(t) \]  

(1)

For the \( k^{th} \) user in the \( i^{th} \) time interval, \( b_k(i) \in \{+1, -1\} \) is the transmitted symbol in that period, \( M \) is the total number of symbols transmitted, \( K \) is the total number of users, and \( c_k(t) \) is the spreading sequence of that user, which is zero outside the interval \([0, T]\), so there is no inter-symbol interference. Assume there are \( 2\beta \) chips per bit, which is the spreading factor of the system. The chaotic chips are generated from cubic mapping [4]. Each user has a unique chaotic sequence which only differs from the initial condition of the mapping.

The received input signal from additive white Gaussian noise (AWGN) channel is,

\[ r(t) = s(t) + n(t) \]  

(2)

where \( n(t) \) is the white Gaussian noise, which is a zero mean Gaussian random variable with a variance of \( N_0/2 \).

The receiver estimates the transmitted symbol by making a hard-decision from the output of a matched filter, which is given by,

\[ z_k(i) = \sum_{r=2\beta(i-1)+1}^{2\beta(i)} r(t)c_k(t) \]  

(3)

This receiver operation is known as the matched filter detection. This receiver is not optimal because it treats the multi-access interference (MAI), which is inherent in CDMA, as if it were additive noise with Gaussian distribution after despreading. Multi-user detector, attached after the bank of matched filters, is designed to specifically suppress MAI and improve the system performance. Figure 1 shows the overall system model.

B. Decorrelator Detector

The output of the matched filter receiver \( y \) of a \( K \)-user synchronous CDMA system can be expressed in a matrix form,

\[ y = RAb + n \]  

(4)

where \( R \) is a \( K \times K \) normalized cross-correlation matrix, \( b \) represents the transmitted symbol vector, and \( n \) is the AWG noise samples. \( A \) is a diagonal matrix to denote the square root of users’ signal energy [1].

The decorrelator detector completely removes the MAI introduced from the spreading waveforms and makes the estimate by,

\[ \hat{b}_k = \text{sign}(R^{-1}y)_k \]  

(5)

The complexity only linearly increases with \( K \) [1]. The only drawback is that it would generally increase the level of background noise [5].

C. MMSE Detector

In contrast to the decorrelator detector, the minimum mean square error (MMSE) detector gives the best compromise solution that takes both the interfering spreading codes and the background noise into account by finding a function \( b(y) \) that minimize the sum of square errors [5]. Comparing with (5), the MMSE detector makes an estimate of the transmitted symbol for user \( k \) by [5],

\[ \hat{b}_k = \text{sign}(([R + \sigma^2A^{-2}]^{-1}y)_k) \]  

(6)

The complexity is also linear. To let the MMSE detector to perform well however, there needs to be a good channel estimator to evaluate the noise variance \( \sigma^2 \).

D. PIC Detector

The interference cancellation (IC) technique is a non-linear approach to reduce the level of MAI. The main principle of this
technique is to estimate, regenerate the original transmitting signal and subtract the estimated user interference [1, 5]. This cancellation procedure can be achieved in series (SIC) or parallel (PIC). In general, the SIC detector performs better when users have distinctly different power; otherwise PIC is a better solution [1]. The IC detectors are usually connected in multi-stage in order to improve reliability of the estimation. The estimated symbol for user \( k \) in a single stage can be expressed as,

\[
\hat{b}_k(i) = \text{sign} \left( \sum_{i=2}^{2^l} \left[ (r(t) - \sum_{k=1}^{K} \hat{b}_k(i)c_k(t))c_k(t) \right] \right)
\]  

(7)

III. BLIND-MUD BASED ON SVM RECEIVERS

Apart from the basic matched filter detection, all the MUD techniques mentioned above requires the knowledge of all users’ spreading codes. Therefore, as mentioned before, these methods are difficult to implement on the forward link. A blind-MUD based on SVM receiver, does not have this requirement, hence can be implemented on both the forward and the reverse link. For the sake of simplicity, the notation for each user is omitted from here on because each user will have a unique SVM receiver which is trained according to its spreading sequence, and the variables used are not directly related to the previous section.

SVM classification is achieved in two stages: the initial training or learning stage, which only needs to perform once unless the channel condition has varied significantly, and the actual testing stage. At the first stage, some training examples are given to the machine to create certain decision functions in order to differentiate the different types of objects, or so-called classes. During the second stage, an unforeseen object, which is a new noisy data stream, is then classified by those decision rules.

A. The Training Phase

The input of the SVM receiver in the training stage is a set of received data streams from \( l \) number of message bits. The \( i \)th training data stream with \( n \) number of sample points can be represented by \( x_i = \{x_{i1}, x_{i2}, \ldots, x_{in}\} \), which is the received signal for one message bit (i.e. one bit has \( n \) sample points) that has been multiplied by the chaotic spreading signal. The multiplication in this stage is applied in order to decorrelate the information from each other user.

Each data stream has an associate binary message of what was sent. Therefore the whole dataset can be represented as \( (x_i, y_i), (x_2, y_2), \ldots, (x_n, y_n) \), where \( x_i \in \mathbb{R}^l \), \( i = 1, \ldots, l \) and \( y_i \in \{+1, -1\} \), which represents the desired output result.

During the initial training stage, a decision function for a non-linear SVM is constructed via,

\[
d(x) = \sum_{i=1}^{l} y_i \alpha_i K(x, x_i) + b
\]  

(8)

where \( \alpha_i (\alpha \geq 0) \) is a Lagrangian constant, \( K(x, x_i) = \phi(x) \cdot \phi(x_i) \) is a kernel function, where \( \phi(x) \) maps the training data vector \( x \) into the high-dimensional feature space, and \( b \) is a bias term.

In this case, the coefficient vector \( w \) is defined as,

\[
w = \sum_{i=1}^{l} \alpha_i y_i \phi(x_i)
\]  

(9)

Then the training is completed by solving the following optimization problem [14],

\[
\text{minimize} \frac{1}{2} w^T w + C \sum_{i=1}^{l} \xi_i
\]  

(10)

Subject to the following constraints,

\[
y_i[w^T \phi(x_i) + b] \geq 1 - \xi_i, i = 1, \ldots, l
\]  

\[
\xi_i \geq 0, C > 0
\]  

(11)

where \( C \) is the tradeoff parameter between the training error and the margin of the decision function, and \( \xi \) is a slack variable to compensate for any non-linearly separable training points.

The output is a reduced set of those training data, because most training data \( x_i \) would have \( \alpha_i \) equal to 0. Those training examples which have non-zero \( \alpha_i \) are used as the final decision variables, also called the support vectors (SV).

B. The Testing Phase

After completing the training phase, the SVM receiver is ready for estimating the source bit based on classifying a test object, \( z = \{z_1, z_2, \ldots, z_n\} \), which is a newly received data stream that has been multiplied with the spreading sequence. The task of the receiver then becomes a pattern classification problem. The transmitted message symbol is estimated by making a hard-decision based on the decision function formed earlier in (8),

\[
\hat{b} = \text{sign}(d(z)) = \text{sign} \left( \sum_{i=1}^{l} y_i \alpha_i K(z, x_i) + b \right)
\]  

(12)

IV. SIMULATION RESULTS AND DISCUSSION

A simulator of a synchronous chaos-based CDMA system that uses SVM receiver was designed in order to compare the bit error rate (BER) performance with other conventional multi-user detectors. The system was tested under various cases with different number of users and spreading factor. These scenarios were all on much larger scale than the previous works on SVM-MUD [11-13].

The training of the SVM receiver was carried out by iterative single data algorithm (ISDA), which is suitable for working with a large dataset [15]. The complexity of the SVM on the testing stage is independent on the number of users, but rather on the number of features per SV, i.e. \( O(n) \).
### A. Kernel Selection

Two types of kernel to functions were used to compare the performance with each other. The first is the simplest linear kernel, given as

$$K(x, y) = x^T y$$  \hspace{1cm} (13)

The second is a more popular Gaussian radial basis function (RBF) kernel, shown as

$$K(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right)$$  \hspace{1cm} (14)

where $\sigma$ controls the width of the function.

The system was simulated on 4 users of equal power with a spreading factor of 20. The SVM receiver was trained at $E_b/N_0$ of 0 dB for 6000 random bits. For the Gaussian RBF kernel, a 10-fold cross-validation sweep from the training samples was used to find the optimum parameters of $C$ and $\sigma$. Figure 2 shows that the width $\sigma$ has a more dominating effect on the error rate than the penalty parameter $C$. When $\sigma$ is below a certain threshold (in this case is 30), the SVM receiver has the best performance, regardless of the $C$ parameter.

A similar search was conducted for the linear kernel, but it only has the $C$ parameter to adjust. Hence, there was not much performance variation. Table 1 summaries the optimum SVM model obtained after the parameter search.

#### Table I. Comparison of SVM Models

<table>
<thead>
<tr>
<th>Selected Kernel</th>
<th>Gaussian RBF</th>
<th>Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>#SVs</td>
<td>2063</td>
<td>1692</td>
</tr>
</tbody>
</table>

The linear kernel has less support vectors than the RBF one; therefore it has a less computational complexity and thus would perform faster. In order to compare the bit error rate (BER) performance fairly, both kernels used by the SVM receiver were classifying exactly the same received signals.

### B. MAI Resistance

The ability of the SVM receiver to combat MAI was tested by putting the system on a heavy load (i.e. let $K$ be relatively close to $2\beta$). Apart from continuing with the previous example, a larger scale simulation that consisted of 50 synchronous users of equal power with a spreading factor of 100 was carried out. All the conventional multi-user detectors mentioned in the previous section was also simulated under the same environment. This also serves as an original validation of applying the conventional multi-user detectors on a chaos-based communication system.

Figure 4 shows the BER performance of various MUDs under a system which has been heavily degraded by the presence of MAI. In both cases, the MMSE detector performs the best, approaching the closest to the single user bound. In the first case, the PIC detector seems to be suffering from an error floor where the BER would not drop below $2 \times 10^{-3}$. This may be due to its limitation on MAI reduction as this is the main limiting factor in the high SNR regions.

For the latter case, the SVM receiver performs better overall than the PIC detector, and it is very close to the decorrelator detector. This demonstrates that the performance as a blind-MUD of the SVM receiver does not suffer much degradation comparing with other well-known MUD techniques, which would all require extra information from the system. Moreover, the SVM receiver performs better than most MUDs in the low SNR regions (i.e. $E_b/N_0 < 3$ dB), where the system is corrupted by both MAI and strong additive noise.

Figure 3 (above) shows the bit error rate performance of the SVM receiver when employing different kernels. Evidently, the linear kernel, though much simpler, has slightly better performance than the Gaussian RBF kernel. This finding has not been cited on any previous work on multi-user detection related to SVM. Training on a ‘worst-case’ scenario (i.e. $E_b/N_0$ of 0 dB in this case) works well, proving that the SVM receiver does not need to be frequently re-trained in different $E_b/N_0$ ratios.
C. System Tolerance

One common uncertainty or criticism about an adaptive learning technique is the system’s tolerance to the adversity of the testing environment. For example, if the SVM receiver is only trained for 10 users, it will definitely have some performance difference when the actual number of users in the system varies. To test the SVM receiver’s tolerance in this case, let \( K_{\text{trained}} \) denote the number of users used for training and \( K_{\text{actual}} \) be the actual number of users. Both the linear and Gaussian RBF kernels are used for training the SVM receiver based on \( K_{\text{trained}} \) users and tested on different \( K_{\text{actual}} \) users.

Figure 6 shows that the two kernels used by SVM receiver produce a similar result. At a low SNR, it has a better BER than the conventional receiver when \( K_{\text{actual}} / K_{\text{trained}} \) is around \( 0.5 \sim 1.5 \). Beyond that limit, the SVM receiver would have a similar bit error rate performance as the matched filter. On the hand, when the system is on a much lower SNR, the SVM receiver tolerance is increased to 3. However, more analysis is required on a finer resolution to have a more conclusive view on the system’s tolerance. Nevertheless, this test indicates that the worst performance of the SVM receiver is similar to the matched filter, when the actual number of users is severely different from the trained model.

V. Conclusions

This paper describes and investigates the application of multi-user detectors on a chaos-based code division multiple access communication system. More importantly, a novel blind multi-user detector based on support vector machine is introduced and analyzed. From the fundamental kernel property selection, it is found that a linear kernel can achieve a similar performance to a Gaussian radial basis one. Hence using a linear kernel can reduce the model’s complexity and increase the computational speed.

Moreover, the proposed SVM receiver can directly replace the conventional matched filter and thus it can be implemented on the forward link transmission. Simulation results show that regardless of the amount of noise in the channel and the number of users in the system.
the SVM receiver can effectively reduce the impact of multi-access interference. When 100 spreading chips are used, the SVM receiver can achieve a similar BER performance as a decorrelator detector. Furthermore, the system's performance is still acceptable when the number of users remains within three times away from its training condition.

Some potential future work for this investigation would take Rayleigh's fading into account in order to model a multi-path channel hence making the system more realistic and look for the possibility of combining multi-user detection with channel coding in order to increase the reliability of the system even further.

REFERENCES