Cognitive Underlay Communications with Imperfect CSI: Network Design and Performance Analysis

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Abstract—This paper is concerned with performance analysis of cognitive underlay partial relay networks with imperfect channel state information (CSI). To assure the required quality of service (QoS) at primary receivers, transmit power at secondary nodes is adjusted by using a back-off power scheme under imperfect CSI condition. The back-off power strategy reduces transmit power at secondary nodes and in turn degrades the secondary network performance. Under the performance tradeoff between primary and secondary networks, this paper considers the use of multiple antennas at destination node and partial relay selection to enhance the secondary network performance without sacrificing the performance of the primary network. To justify the benefit of using the network design, several analytical and simulated results of the network performance are provided under various system settings.

Index Terms—Interference probability, outage probability, cognitive radio, imperfect channel state information.

I. INTRODUCTION

Recent explosive growth in the number of wireless broadband services has called for more efficient techniques of accessing radio frequency resource. With the ability to sense the surrounding radio spectrum, cognitive radio (CR) communications has been recognized as the most efficient approach to utilize the limited radio resource. CR networks allow secondary nodes to access the licensed spectrum of primary nodes via underlay, overlay or interweave transmission [1]. In the literature, the underlay transmission strategy has recently received a considerable attention due to its advantage on facilitating concurrent transmission for both secondary and primary nodes. However, interference constrains at primary nodes in an underlay network limit the transmit power at secondary nodes. As a result, cognitive underlay transmission is only able to facilitate short-range communications between secondary nodes.

To extend the communication range in a cognitive underlay network, relay nodes can be employed in transmission between secondary nodes [2]–[7]. Several network designs for combining cognitive underlay and multihop communications have been intensively analyzed in term of outage probability. In particular, the exact form of the system outage probability for underlay amplify-and-forward (AF) relaying networks has been derived for Rayleigh fading channels [2]. Considering Nakagami-m fading channels and the maximum transmit power constraints, the performance of underlay dualhop decode-and-forward (DF) relaying networks has been studied in [3]. In [4], a relay selection scheme for underlay cognitive relay networks has been proposed and its performance over Rayleigh fading channels has been analyzed. By treating a dual-hop link as a virtually equivalent link, Bao et al. [5] has derived an outage probability expression of underlay selective DF relay communications under interference constraints. Besides DF, the best relay selection for AF was considered in [8]. Recently, the problem of secondary relay optimization problem has been studied in [6] and [7] for Rayleigh and Nakagami-m fading channels, respectively.

In the literature of CR underlay relay networks, most of existing studies (e.g., see [2]–[8]) have assumed perfect channel state information (CSI) of interfering channel links is available at secondary nodes in their analyses. Unfortunately, such channel assumption is rarely attainable in practice, whereas imperfect CSI is likely obtained at nodes via existing channel estimation techniques. Recently, the effects of CSI imperfection has been considered in underlay multihop networks [9], partial relay selection [10] and full relay selection [11]. Numerical results in [9]–[11] show that the back-off transmit power strategy is an efficient way to guarantee the quality of service (QoS) of the primary networks but the resulting reduced transmit power also significantly degrades the secondary network performance.

Unlike [2]–[11], this paper considers the use of multiple antennas at the secondary destination node and partial relay selection to improve the secondary network performance without increasing interference power at primary nodes. Simulation results and related analytical ones show that increasing the number of relays and/or antennas at destination node can enhance the secondary network performance while maintaining the QoS of primary networks.

The remaining of this paper is organized as follows. Section II presents the considered cognitive underlay system model. The performance of the system is analyzed in Section III. Section IV provides numerical results to justify the benefits of using multi-antenna destination node and relay selection in the system. Finally, Section V provides some concluding remarks of this paper.

II. SYSTEM MODEL

Consider a cognitive underlay system, where a secondary relay network operates over the same bandwidth with a primary network. The secondary relay network has a source
(s), a destination (d) and N relays denoted by r₁, . . . , r₅ as shown in Fig. 1. The source and relay nodes are equipped with a single antenna while the destination node possesses multiple antennas. The communications between secondary nodes (i.e., s and d) is facilitated via the aid of N relays using partial relay selection [12]. In particular, a relay node receiving the highest signal power in the first hop will be the forwarder in the second hop [13].

Using two orthogonal timeslots, the source node broadcasts its signal to all relays but not to the destination node due to obstacles between s and d. In the second timeslot, the best relay, which is selected by the existing partial relay selection technique [12], forwards the received signal to the destination d. Using multiple antennas, the destination combines received signal components by maximal ratio combiner.

In this paper, it is assumed that the considered system operates over Rayleigh fading channels where hₐb with a ∈ {s, rₖ} and b ∈ {rₖ, dₙ} denote fading channel coefficients between secondary nodes. As a result, the quantity |hₐb|^² follows exponential distribution with parameter λₐb. The background noise at all receivers are assumed to be zero-mean Gaussian random variables with variance N₀. Interference from the primary transmitter is assumed to be additive white Gaussian noise since secondary receivers are usually located far enough from the primary transmitter [14].

Let denote fₛₚ and fᵣₛₚ as the channel coefficients of the s → p and rₖ → p links, respectively, where p denotes the primary receiver. Q stands for interference threshold (i.e., maximum allowable interference level) at the primary receiver. The transmit powers of the source and destination nodes in the secondary network are given by [2], [14]

\[
P_s = \frac{Q}{|f_{sp}|^2}, \quad P_{r_k} = \frac{Q}{|f_{rs_p}|^2},
\]

making no interference on the primary receiver. In practice, fₛₚ and fᵣₛₚ are not perfectly known at secondary corresponding transmitters due to several uncertain factors, e.g., errors in the feedback transmission and/or outdated feedback. In particular, the channel coefficient fₛₚ and its estimated version, ̂fₛₚ, can be expressed as [15]

\[
̂fₛₚ = pfₛₚ + \sqrt{1 - \rho^2}\varepsilon,
\]

where ρ is the correlation coefficient between fₛₚ and fₛₚ, ε is a circular symmetric complex Gaussian random variable with mean zero and variance λₛₚ.

III. PERFORMANCE ANALYSIS

In this section, we analyze the effects of imperfect CSI of interference links on the performance of the primary network and secondary networks. For the primary network, we adopt the concept of the interference probability, which is defined as the probability that the interference power received at the primary receiver exceeds the maximum allowable interference level. For the secondary network, we derive the closed-form expression for outage probability over fading Rayleigh channels.

A. Interference Probability for Primary Networks

In this subsection, our goal is to investigate the interference probability of the primary network over Rayleigh fading channels. Making use the law of the total probability, one can have the interference probability (P_I) given by [16]

\[
P_I = \Pr(P_s|f_{sp}|^2 > Q) + \Pr(P_s|f_{sp}|^2 \leq Q) \Pr(P_{rs_p}|f_{rs_p}|^2 > Q).
\]

Note that the first and second term in (3) are for the first and the second timeslots, respectively. In particular, the first term accounts for the case where s adjusts the transmit power Pₛ incorrectly and causes an interference level higher than the interference threshold. While the second term reflects the case where the selected relay interferes the primary receiver. In (3), kₖ denotes the index of the best relay, i.e., [12]

\[
kₖ = \arg \max_{k=1, \ldots, K} |h_{sk}|^2.
\]

Since Pr(Pₛ|fₛₚ|^2 > Q) + Pr(Pₛ|fₛₚ|^2 ≤ Q) = 1, in order to derive (3), we need to calculate Pr(Pₛ|fₛₚ|^2 > Q) and Pr(P_{rs_p}|f_{rs_p}|^2 > Q). For Pr(Pₛ|fₛₚ|^2 > Q), we have

\[
\Pr(P_s|f_{sp}|^2 > Q) = \Pr\left(\frac{|f_{sp}|^2}{|f_{sp}|^2} > \frac{|f_{sp}|^2}{|f_{sp}|^2}\right) = \int_{y=0}^{\infty} \int_{x=0}^{\infty} f_{|f_{sp}|^2,|f_{sp}|^2}(x,y)dxdy,
\]

where f_{|f_{sp}|^2,|f_{sp}|^2}(x,y) is the joint probability density function (PDF) of fₛₚ and ̂fₛₚ given by [15]

\[
f_{|f_{sp}|^2,|f_{sp}|^2}(x,y) = e^{-\frac{x+y}{(1-\rho^2)\lambda_{sp}}} I_0\left(\frac{2\rho\sqrt{xy}}{(1-\rho^2)\lambda_{sp}}\right),
\]

where I₀(.) is the zeroth-order modified Bessel function of the first kind [17]. Substituting (6) into (5) and then taking the resulting integral, we have

\[
\Pr(P_s|f_{sp}|^2 > Q) = 1/2.
\]
For $\Pr(P_{tx,p}|f_{tx,p}|^2 > Q)$, using the same approach as for (5), we have

$$\Pr(P_{tx,p}|h_{tx,p}|^2 > Q) = \sum_{k=1}^{N} \Pr \left( \left| \frac{|h_{tx,p}|^2}{|h_{tx,p}|^2} > 1 \right| \right) \times \Pr \left( \left| h_{sr_k} \right|^2 > \max_{i=1,\ldots,N;i \neq k} \left| h_{sr_i} \right|^2 \right).$$

Making use of the fact that all links from the secondary source to secondary relays are independent and identically distributed (i.i.d.), we have

$$\Pr(P_{tx,p}|h_{tx,p}|^2 > Q) = \sum_{k=1}^{N} \Pr \left( \left| \frac{|h_{tx,p}|^2}{|h_{tx,p}|^2} > 1 \right| \right) \times \sum_{n=0}^{N-1} \left( \frac{N-1}{n} \right) \frac{(-1)^n}{n+1}.$$  \hspace{1cm} (9)

Based on the result in [18, 0.159.4], i.e.,

$$\sum_{n=0}^{N-1} \left( \frac{N-1}{n} \right) \frac{(-1)^n}{n+1} = \frac{1}{N},$$

it is straightforward to arrive at

$$\Pr(P_{tx,p}|h_{tx,p}|^2 > Q) = \Pr \left( \left| \frac{|h_{tx,p}|^2}{|h_{tx,p}|^2} > 1 \right| \right) = 1/2$$  \hspace{1cm} (10)

leading to $P_I = 1/2 + 1/2^2 = 0.75$. It is noted that $P_I$ is independent of the settings of the secondary networks, e.g., average channel gains of interference links and the number of antennas sited at the secondary destination. In addition, the value of $P_I$ is too high and unacceptable in practice. To reduce $P_I$ or to assure the QoS of the primary networks, the possible solution is the back-off technique, where the transmit powers of secondary transmitters are adjusted with a back-off power control coefficient, $\eta$. In particular, we have $P'_s = \eta P_s$ and $P'_r = \eta P_r$, with $0 \leq \eta \leq 1$. Similarly, for a given $\eta$, the interference probability is of the form

$$P_I = \Pr(P'_s|h_{sp}|^2 > Q) + \Pr(P'_s|h_{sp}|^2 \leq Q) \Pr(P'_r|h_{tx,p}|^2 > Q) = 1 - \left( 1 + \frac{1}{2} \frac{\eta - 1}{\sqrt{\eta + 1/2 - 4\rho^2\eta}} \right)^2 \times \left( 1 + \frac{3}{4} \frac{\eta - 1}{\sqrt{\eta + 1/2 - 4\rho^2\eta}} \right).$$

Based on (11), one can determine the value of $\eta$ for a given interference probability ($P_I$) as follows:

$$\eta = \frac{-1 - \psi^2 + 2\psi^2 \rho^2 + 2\sqrt{\psi^2 - \psi^2 \rho^2 + \psi^4 \rho^2} + \psi^4 \rho^4}{1 - \psi^2},$$

where $\psi = 2\sqrt{1 - P_I} - 1$.  \hspace{1cm} (12)

### B. Outage Probability for Secondary Networks

In this subsection, we investigate the outage probability of secondary networks for a given $\eta$. By making use of the fact that the weaker hop in dual hop DF networks will dominate the overall network performance, we have the end-to-end SNR of secondary networks given by [19, Prop. 1]

$$\Gamma_\Sigma \approx \min(\Gamma_1, \Gamma_2),$$

where $\Gamma_1$ and $\Gamma_2$ are respectively the first and second hop SNR of secondary networks. For the first hop, due to partial relay selection, $\Gamma_1$ is of the form [12]

$$\Gamma_1 = \frac{\xi}{|f_{sp}|^2} \max_{r_k \in \mathbb{R}} |h_{sr_k}|^2.$$  \hspace{1cm} (14)

where $\xi = \eta Q$ and $\mathbb{R} = \{r_1, \ldots, r_N\}$. Since MRC is used at the destination, we can write $\Gamma_2$ as

$$\Gamma_2 = \frac{\xi}{|f_{tx}|^2} \sum_{j=1}^{M} |h_{tx,d_j}|^2.$$  \hspace{1cm} (15)

Denoting $X_1 = \max_{r_k \in \mathbb{R}} |h_{sr_k}|^2$, $Y_1 = |f_{sp}|^2$, $X_2 = \sum_{i=1}^{M} |h_{tx,d_i}|^2$ and $Y_2 = |f_{tx}|^2$, $\Gamma_1$ and $\Gamma_2$ can be rewritten as

$$\Gamma_1 = \xi X_1 Y_1^{\frac{1}{2}}, \quad \Gamma_2 = \xi X_2 Y_2^{\frac{1}{2}}.$$  \hspace{1cm} (16)

From (13) and under the assumption of independent fading channels, the CDF of $\Gamma$ can be written as [16]

$$F_{\Gamma}(\gamma) = F_{\Gamma_1}(\gamma) + F_{\Gamma_2}(\gamma) - F_{\Gamma_1}(\gamma)F_{\Gamma_2}(\gamma),$$

where $F_{\Gamma_k}(\gamma)$ with $k = 1, 2$ denotes the CDF of $\Gamma_k$.

Outage probability is an important performance metrics defined as the probability that the end-to-end SNR is below the predetermined SNR threshold, $\gamma_{th}$. Mathematically, we can write $OP = F_{\Gamma}(\gamma_{th})$. The next step is to calculate $F_{\Gamma_k}(\gamma)$. We start with $F_{\Gamma_1}(\gamma)$, which is explicitly expressed as

$$F_{\Gamma_1}(\gamma_{th}) = \int_{0}^{\infty} F_{X_1} \left( \frac{\gamma_{th}}{\xi^2} \right) f_{Y_1}(y_1) dy_1 = \int_{0}^{\infty} \left[ 1 - \exp \left( -\frac{y_1}{\xi \lambda_{sr}} \right) \right] N \frac{1}{\lambda_{sp}} e^{-\frac{y_1}{\lambda_{sp}}} dy_1.$$  \hspace{1cm} (18)

Using binomial expansion and then taking the integral, we have

$$F_{\Gamma_1}(\gamma_{th}) = \frac{1}{\lambda_{sp}} \sum_{k=0}^{N} \binom{N}{k} (-1)^k \int_{0}^{\infty} \exp \left( -\frac{k y_1}{\xi \lambda_{sr}} - \frac{y_1}{\lambda_{sp}} \right) dy_1 = \sum_{k=0}^{N} \binom{N}{k} (-1)^k \frac{1}{1 + \frac{k \gamma_{th} \lambda_{sr}}{\xi \lambda_{sr}}}.$$  \hspace{1cm} (19)

For $F_{\Gamma_2}(\gamma_{th})$, using the same approach as for (18) and noting that $F_{X_2}(x_2) = \frac{\gamma_{th} x_2}{\xi x_2^{1/\xi}}$, where $\Gamma(.)$ and $\gamma(.)$ respectively
denote the Gamma and incomplete Gamma function [17], we can write

\[ F_{\Gamma_2}(\gamma_{th}) = -\frac{1}{\Gamma(M)} \int_0^\infty \gamma(M, \frac{\gamma_{th}y^2}{\xi \lambda_{rd}}) d\exp\left(-\frac{y^2}{\xi \lambda_{rp}}\right) \]

\[ = \frac{\gamma(M, 0)}{\Gamma(M)} + \frac{1}{\Gamma(M)} \int_0^\infty e^{-\frac{y^2}{\xi \lambda_{rp}}} e^{-\Gamma(\frac{M}{2}, \frac{\gamma_{th}y^2}{\xi \lambda_{rd}})} \frac{\Gamma(M-1, \frac{\gamma_{th}y^2}{\xi \lambda_{rd}})}{\Gamma(M)} dy^2. \]  

(20)

After some manipulations and with the help of [18], we have

\[ F_{\Gamma_2}(\gamma_{th}) = \frac{\gamma(M, 0)}{\Gamma(M)} + \left(\frac{\lambda_{rp} \gamma_{th}}{\xi \lambda_{rd} + \lambda_{rp} \gamma_{th}}\right)^M. \]  

(21)

Having the CDF of \( \Gamma_k \) with \( k = 1, 2 \) at hands allows us to obtain the closed-form expression for OP by plugging (19) and (21) to (17) and then evaluating the resultant at \( \gamma = \gamma_{th} \).

IV. NUMERICAL RESULTS

In this section, we present some typical simulation results to verify the correctness of the analysis. For illustrative purpose, the network and channel parameters are chosen as follows: \( \lambda_{sr} = 10 \) dB, \( \lambda_{rd} = 10 \) dB and \( \lambda_{sp} = 2 \) dB, and \( \lambda_{rp} = 3 \) dB.

In Fig. 2, we study the interference probability against the correlation coefficient for different power control coefficients. It is observed that the theoretical results are in good agreement with the simulation ones. For \( \rho = 1 \), e.g., the CSI of the interference links is perfect, the interference probability is always equal to zero. But for \( \rho < 1 \) and \( \eta = 1 \), the interference probability is always of 0.75 irrespective of \( \rho \) and average channel gains. Fig. 2 also show that \( P_I \) can be significantly decreased if the value of \( \eta \) is decreased.

Fig. 3 presents the outage probability for secondary networks versus \( Q \). As expected, the outage probability decreases as the interference threshold \( Q \) grows. In addition, an increase of the number of relays and/or number of antennas at the destination can reduce the outage probability.

To illustrate the performance tradeoff between primary and secondary networks, Fig. 4 provides the numerical results of outage probability versus interference probability under different numbers of receive antennas at the destination node in the secondary network. In addition, Fig. 4 helps to determine the secondary network performance gain when one increases the number of receive antennas at the destination node. With the performance tradeoff, the efforts to improve the secondary network performance will degrade the primary network performance. The results show that higher secondary network performance gain comes at the price of sacrificing the primary network performance and vice versa.
V. Conclusion

This paper analyzed the performance of a cognitive underlay relay system where secondary nodes have only imperfect CSI of interference links. The CSI imperfection causes non-zero interference probability at primary receiver. This in turn leads to the performance tradeoff between primary and secondary networks. To improve the secondary network performance without sacrificing the QoS of the primary network, this paper introduced the use of multi-antenna destination node and partial relay selection for the communications between secondary nodes.

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References


