Discussion on: “Passification of Non-square Linear Systems and Feedback Yakubovich–Kalman–Popov Lemma”

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The main goal of the paper by A. Fradkov is to obtain (necessary and) sufficient conditions for the existence of an output feedback control law

\[ u = K y + L v, \quad K \in \mathbb{R}^{m \times l}, \quad L \in \mathbb{R}^{m \times m} \]

which makes a linear, non-square \((m \neq l)\) system

\[ \dot{x} = A x + B u, \]
\[ y = C x \]

strictly \(G\)-passive, where \(G \in \mathbb{R}^{m \times l}\) is given. Here \(A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}\) and \(C \in \mathbb{R}^{p \times n}\).

The (strict) \(G\)-passivity concept as such is equivalent to the hyperstability in the sense of Popov. If we consider system (1)–(2) of Section 16 in [3] with \(J = 0, K = 0, M = -\mu I, L^* = \frac{1}{2} G, L = \frac{1}{2} G^*\) we obtain the Popov system

\[ \dot{x} = A x + B u(t), \quad x(0) = x_0, \]
\[ \eta(0, t) = \int_0^t (\text{Re}(u^*(\tau)Gx(\tau)) - \mu \|x(\tau)\|^2) \, d\tau. \]

Theorem 1 in Section 16 says (among other claims) that hyperstability in the sense of Popov is equivalent to the representation of \(\eta(0, t)\) as

\[ \eta(0, t) = x^*(t) U x(t) - x_0^* U x_0 + \int_0^t \|V u(\tau)\|^2 \, d\tau \]

with \(U \geq 0\), and also to \(\eta(0, t) \geq 0\). Therefore the system is strictly \(G\)-passive. Of course, hyperstability is also equivalent to the positive realness of the so-called Popov function. Other connections between dissipativity/passivity and hyperstability can be found in [4].

It is also worth mentioning that among the ideas of Popov’s theory is the invariance of various properties with respect to some transformations that are a combination of coordinate change plus a state-feedback. The aim of the paper in discussion is somehow converse: to find a state-feedback to ensure the required properties.

The strict version of \(G\)-passivity for linear systems, with storage functions of quadratic form, \(V(x) = x^* H x\), is equivalent – via the KYP Lemma – to the following matrix inequalities:

\[ A^* H + H A < 0 \quad \text{and} \quad H B = C^* G^*. \]

Thus, the problem reduces to the following system of bilinear matrix inequalities (in the unknowns \(H > 0\) and \(K\)):

\[ (A^* + C^* K^* B^*) H + H (A + B K C) < 0 \]
\[ \text{and} \quad H B = C^* G^*. \]

Such problems are in general difficult to solve, but it has been shown in [2] that some inequalities of similar form can be solved via LMI, obtaining a complete description of matrices \(H\) and \(K\).

The paper has the merit to extend the concept of passivity to non-square systems (where the number of inputs differs from the number of outputs), motivated by applications in adaptive control, where preserving the number of adjustable parameters is important. However, from technical point of view, there are not many differences with respect to the (standard) square
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case. To be more specific, solvability conditions for (1) are given in terms of the “squared” system $GW(s)$. It is the place here to notice the strongness of Theorem 4, which gives checkable conditions for the solvability of the constrained inequality (1), which is known to be an NP hard problem. This is not surprising, since Theorem 4 can be regarded as the Feedback KYP Lemma; it just illustrates once more how powerful the Kalman–Yakubovich–Popov Lemma is.

Going further into technical details, one might ask if the method proposed by Huang et al. [2] for solving (in $P > 0$ and $\tilde{K}$) the bilinear matrix inequalities

$$(A^t + \tilde{C}^t \tilde{K} B^*) P + P (A + B \tilde{K} \tilde{C}) < 0 \quad \text{and} \quad PB = \tilde{C}^t$$

(2)

can be easily adapted to deal with (1). In (2), $\tilde{K}$ is a square matrix, while the number of columns of $B$ and the number of rows of $\tilde{C}$ are the same — the “square” case is treated. At a first glance, one could apply their method and get the matrix $\tilde{K} = G^* K \in \mathbb{R}^{l \times l}$. But such a $\tilde{K}$ has rank at most $m$, hence it is of low rank. Can one cope with this supplementary restriction? It seems that such a “squaring up” brings too much information into the problem. Probably one can adapt the techniques in [2] in a more direct way, obtaining successful results.

Although not mentioned explicitly, throughout the paper the author adopted tacitly the assumption $m \leq l$ ($G$ and $K$ must be “fat”, not “tall”). Otherwise, the minimum-phase conditions in Definition 3 become degenerate, since $\det GW(s) = 0$, a.e. on $\mathbb{C}$. Moreover, for $GW(s)$ to be strictly or hyper minimum phase, $G$ must be chosen such that $GCB$ is non-singular, respectively symmetric and positive definite. Thus, when passifying a given non-square system by static output feedback, the choice of $G$ plays a decisive role.

As emphasized in the last part of the paper, it would be of great interest to identify subclasses of matrices $K$ which make system (1) easier to solve. One of these classes is described by Corollary 3, which looks like a direct application of the Small Gain Theorem.

The paper is in our opinion a good illustration of various difficulties that are to be overcome when static output feedback is used. From the classical “structural conditions for non-void stability domain in the parameter space” of Aizerman and Gantmacher [1], up to the minimal stability property introduced by Popov [3], different approaches to this challenge have been taken more or less successful.

References

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