Rapid transit network design for optimal cost and origin–destination demand capture

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ABSTRACT

This paper proposes a tractable model for the design of a rapid transit system. Travel cost is minimized and traffic capture is maximized. The problem is modeled on an undirected graph and cast as an integer linear program. The idea is to build segments within broad corridors to connect some vertex sets. These segments can then be assembled into lines, at a later stage. The model is solved by branch-and-cut within the CPLEX framework. Tests conducted on data from Concepción, Chile, confirm the effectiveness of the proposed methodology.

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1. Introduction

Many cities throughout the world have invested in recent years in the construction, expansion or modernization of rapid transit systems, such as metros, light trains, pre-metros, commuter trains and monorails. This is partly in response to increased traffic congestion and to the need to reduce carbon emissions. There is no clear definition of a rapid transit system, but these are commonly defined as networks that do not interfere with road or pedestrian traffic and are primarily designed to serve city needs. This definition therefore excludes bus lines and inter-city railway networks. There are around 400 light rail systems worldwide and 116 cities with metro systems [13], the most important of which, measured in terms of length or number of stations, are located in New York, Paris, Shanghai, Madrid and Seoul.

The design and construction of rapid transit systems are major endeavours requiring long-term planning as well as the involvement of several players such as urban planners, geologists, engineers, politicians and various interest groups. This process involves a large amount of uncertainty, particularly in the case of underground metros where unexpected delays are frequent. From a methodological point of view, the design problem is a large-scale often non-linear multi-objective problem which cannot be solved optimally unless major simplifications are made. As argued by Vuchic [35] and Laporte et al. [18], operational research tools can be put to use to suggest alternative designs among which the decision makers can choose, or to solve some well-defined sub-problems. For surveys on rapid transit network design, see [20,22]. A survey of related problems can be found in [6]. Note that this paper is uniquely concerned with the design of rapid transit systems from a strategic point of view, whereas the Transit Network Design Problem (TNDP) [15], also related to these systems, is more concerned with operational issues such as frequencies on individual lines, rolling stock, and so on.

A frequently used objective in rapid transit network design is to maximize the covered population, i.e., population located close to the stations, rather than maximizing covered trips. This is operationalized by estimating the population living close to a potential station, irrespective of travel demand. Under this approach, important locations such as city centres, hospitals, universities, shopping malls, etc., must be assigned large populations, such as the number of people frequenting them during the day. Maximizing population coverage requires relatively few data and usually yields tractable models (see e.g., [28,10,26,27,9,18]). This is also the case with the literature on stop location [29,32,36]. In particular, Wu and Murray [36] propose a model for reducing the travel time in an existing system by decreasing the number of stops. The travel time is not weighted by the traffic. A second objective maximizes covered population, rather than origin–destination traffic, which is not taken into account. The population coverage objective also makes sense from a planning perspective. Since rapid transit systems are designed for the long term, they are located to cover dense population areas rather than focus on present day travel patterns. Once a system is in place, one can argue that people will tend to relocate over time in order to satisfy
their travel requirements, no matter what the shape of the system is. However, if the optimal layout of the lines is to be found, seeking a better traffic coverage is a must. Origin/destination (O/D) demand forecasts can be used, and these are typically available in the transportation sector.

In fact, whereas population coverage constitutes a sensible and easy to operationalize objective, it does not adequately reflect the aim of a rapid transit system which is to improve population’s mobility. As a result, several authors have proposed models that explicitly maximize traffic coverage. One way to operationalize this concept is to first compute an O/D matrix and to design a system that will cover as much traffic as possible, subject to budgetary and operational constraints. This is what was done in Laporte et al. [24,23]. The first of these two papers proposes a heuristic for the design of a single-line in the absence of a competing mode and classifies the single line problem as NP-Hard, whereas the second paper models and solves a similar problem in the presence of competition, that is, users associated with a given O/D pair will opt for the fastest mode. Guan [11], Schöbel and Scholl [33], and Borndörfer et al. [2] have described algorithms to select a set of lines from a pool to connect several O/D pairs under a budget constraint. Gutiérrez-Jarpa et al. [12] proposed an integer programming formulation for the single-line traffic capture problem, extended here to multiple lines. The candidate lines under consideration take directness of travel into account.

Since the single-line problem is NP-Hard, so is the multiple-line problem. The scientific contribution of this paper is the introduction of a tractable model for the multiple-line problem, in which travel cost is minimized and traffic capture maximized, and a simple constraint relaxation mechanism is used, capable of solving it efficiently on a realistic data set. This is the first model that finds the exact shape of a set of lines (not one but several), minimizes cost and captures maximum origin–destination traffic (as opposed to capturing population living close to stops), optimally locates stops, and solves the problem exactly on instances of reasonable size (real data are used). Other researchers have not addressed this problem in such detail. We remark that knowing how many users will choose a transportation system is not possible without considering origin–destination traffic, as opposed to considering origins and destinations separately, as do most researchers in the field of metro location. Naturally, an exact method like the one proposed in the paper, if applicable, is better than a heuristic, as in all previous research on traffic capture.

As for most formulations for rapid transit network design, our mathematical model works on a graph in which some of the nodes are potential locations for stations. From practical and computational points of view, it makes sense to restrict the set of these locations. For example, broad corridors corresponding to heavy traffic flows can be predefined for some of the lines, leaving the determination of the specific station locations to the optimization process. These lines can be combined in several ways to yield various network topologies such as a star, a cartwheel or a triangle (Fig. 1). Laporte et al. [19] have analyzed these and other configurations from a generic standpoint and have developed a number of measures to assess them. One of their conclusions is that cartwheels and triangles are preferable to stars and grids in terms of directness and effectiveness when a uniform O/D distribution is assumed; if this is not the case, then their conclusions may not hold.

Bruno and Laporte [4] and Bruno et al. [3] have designed a heuristic and a decision support system enabling planners to design a rapid transit network possessing a given topological structure. The user first defines a configuration using a menu, and a tabu search heuristic is then applied to locate an alignment within each of the corridors specified by the user. Tests were successfully performed on data from the City of Milan. It is of course possible to replace the heuristic part of the system with an exact optimizer, provided the instance size is not too large. This is essentially the approach taken in this paper except that we simultaneously minimize cost and maximize travel demand coverage. We also include a median objective to the problem, i.e., we consider the cost for users of reaching their closest station.

The remainder of this paper is organized as follows. The problem just described is formally defined and modeled in Section 2. Computational results on data from the city of Concepción, Chile, are presented in Section 3, followed by conclusions in Section 4.

2. Formal problem definition

The rapid transit network design problem (RTNDP) is defined on an undirected graph \( G(N,E) \), where \( N = \{1, \ldots, n\} \) is a node set and \( E = \{(i,j) : i,j \in N, i < j\} \) is an edge set. Let \( d_{ij} \) be the length of edge \((ij)\). The transit network consists of one or several lines with stations in \( N \). In the latter case, the lines intersect at intersection points so that the transit network is connected and passengers can transfer from one line to another. Each line is made up of segments which are chains of edges connecting two intersection points of a line, or an intersection point with an end-point. The star configuration of Fig. 1, for example, contains six segments, one intersection point and six end-points. The cartwheel configuration is made up of twelve segments, five intersection points and four end-points. There are typically several ways of combining segments into lines [35,18]. For example, the star configuration of Fig. 1 (a) could operate as three intersecting lines or as six lines meeting at a common intersection point, among several possibilities. Similarly, the cartwheel configuration of Fig. 1 (c) could have three lines (one of them circular), or more. The output of the model is a set of segments. Their combination into lines is beyond the scope of this study and is contingent upon considerations related to train scheduling, capacity, and passenger ride times, with or without transfers.

Fig. 1. Three basic configurations for a rapid transit system. (a) star, (b) triangle and (c) cartwheel.
Let $S$ be the set of segments and let $N_t \subseteq N$ be the set of nodes that are candidate locations for the stations of segment $s \in S$. In practice, $N_t$ is often defined as a corridor associated with an important traffic flow in a city. Also define $N_s = \bigcup_{s \in S} N_t$. The nodes at which each segment starts or ends must belong to a predefined extreme set, the set of nodes that are candidates to the location of end nodes. Intersecting segments of course cross in the same extreme set. Let $T_k$ denote the $k$th extreme set. Let $O_k \subseteq S$ be the set of all segments having one of their end-nodes in $T_k$. Fig. 2 depicts the candidate nodes and extreme sets for a simple star configuration. We denote by $c_{ij}$ the (construction) cost of edge $(ij)$ and by $t_{kl}$ the traffic O/D demand (in both directions) between nodes $k$ and $l$, both originating or having a destination at these nodes, and attracted from their neighborhood. This is explained below.

Computing traffic demands on an existing network is generally achieved by means of O/D surveys which can be conducted through the use of questionnaires. When designing a new network, one can use attraction functions $\beta_{ki}$ which depend on the distance between a user located at node $k$ and a potential station located at node $i$. It is customary to assume that a pedestrian user is attracted to only one station because the acceptable walking distance is usually much less than the inter-station distance [35]. Most models developed to compute attraction functions are discrete. Thus Bruno et al. [3] use iso-distance curves which are circles if walking distances are Euclidean, and diamonds if they are Manhattan. These curves are defined for a limited number of distances from a potential station, with attraction levels that decrease with distance. Thus $\beta_{ki}$ could be defined as

$$\beta_{ki} = \begin{cases} 
1 & (d_{ki} \leq 50) \\
0.75 & (50 < d_{ki} \leq 100) \\
0.5 & (100 < d_{ki} \leq 150) \\
0.25 & (150 < d_{ki} \leq 200) \\
0 & (d_{ki} > 200),
\end{cases}$$

where $d_{ki}$ is expressed in meters. Laporte et al. [21] have combined this idea with population counts obtained from census tracts to compute catchment areas of potential stations. We explore...
another way of defining $\beta_{k}$, which is to use a truncated decay function of the type

$$
\beta_{k} = \begin{cases} 
e^{-d_{ki} / r} & (d_{ki} \leq r) \\ 0 & (d_{ki} > r), 
\end{cases}
$$

where $r = 200$ (meters) for example. Traffic demands $t_{ij}$ can be computed as a function of attractions to the origin and the destination. For example, a user located at $k$ and traveling to $k'$ may be attracted to the O/D pair $(ij)$ with probability $\beta_{kij} = min(\beta_{ki}, \beta_{kj})$ or $(\beta_{ki} + \beta_{kj}) / 2$, for example. Current et al. [7] have used a similar concept within their shortest path covering model. They assume that a traffic demand is captured if both its origin $k$ and its destination $k'$ lie within $r$ units of their respective stations $i$ and $j$. This amounts to defining $\beta_{k}$ as in Eq. (1) with only two discretization intervals and setting $\beta_{k} = min(\beta_{ki}, \beta_{kj})$. Then, the total traffic between stations $i$ and $j$ will be $\sum_{k} \beta_{kij} x_{ij}$, where $k$ and $k'$ include nodes $i$ and $j$.

We model the RTNDP with binary variables $x_{ij}$ equal to 1 if and only if edge $(ij)$ belongs to segment $s$, binary variables $y_{i}$ equal to 1 if and only if node $i$ is an extreme node, and binary variables $w_{ij}$ equal to 1 if and only if node $i$ belongs to segment $s$. In addition, let $v_{ij}$ be a variable equal to 1 if and only if the O/D traffic between nodes $i$ and $j$ is captured by the network. As is common in such models (see e.g., [30]), the distance between two consecutive stations must lie within an interval $[l_{\text{min}}, l_{\text{max}}]$, typically $[0.5 \text{ km}, 2 \text{ km}]$. This is easily handled in our problem by removing from $E$ the edges whose length does not satisfy this condition, and adding an explicit constraint to avoid forbidden interstation distances.

The basic model for the RTNDP can be stated as follows:

$$
(P) \text{Min } z_{c} = \sum_{s \in S,S' \in N_{t}} c_{ps} x_{ij}^{S}
$$

$$
(P) \text{Max } z_{c} = \sum_{i,j \in E,i \neq j} (t_{ij} + l_{ij})w_{ij}
$$

$$
\sum_{i \in S} \sum_{j \in S} (x_{ij}^{S} + x_{ij}^{S'}) \geq 1 \text{ } \forall k \in K, \ s \in O_{k}
$$

$$
\sum_{j \in S} (x_{ij}^{S} + x_{ij}^{S'}) = 2w_{ij} - y_{i} \text{ } \forall s \in S, \ i \in N_{t}
$$

$$
\sum_{i \in S} y_{i} = 1 \text{ } \forall k \in K
$$

$$
w_{i} + w_{j} \leq 1 \text{ } \forall s \in S \text{ and } i,j \in N_{t} : i > j \text{ and } d_{ij} < l_{\text{min}}
$$

$$
v_{ij} \leq \sum_{s \in S} w_{ij} \text{ } \forall i,j \in N_{t} : i < j
$$

$$
\sum_{s \in S} w_{ij} \leq \sum_{i \in S} w_{ij} \text{ } \forall i \in N_{t}
$$

$$
x_{ij}^{S} \in \{0,1\} \text{ } \forall (i,j) \in E, \ s \in S
$$

$$
v_{ij} \in \{0,1\} \text{ } \forall i,j \in N_{t} : i \neq j
$$

$$
y_{i} \in \{0,1\} \text{ } \forall i \in N_{t}
$$

$$
w_{ij} \in \{0,1\} \text{ } \forall s \in S, \ i \in N_{t}
$$

Objectives (3) and (4) represent construction cost and traffic capture, respectively. Constraints (5) require each segment with an extreme in site $k$ to have an edge connecting a node in the site with a node in the corridor. Eq. (6) is continuity constraints for each segment in its corridor and extreme set, which state that if a node is the extreme of a segment, it must have one incident edge (because $y_{i}=1$ when $i$ is in the extreme set); otherwise, it must have two incident edges (because $y_{i}=0$ when $i$ is not the extreme set). Constraints (7) state that there exists a single node acting as a transfer node in an extreme set, which is visited by all segments starting or ending in that set. Constraints (8) force two stations on the same segment to be farther away than the minimum allowed distance, even if both stations are not adjacent. Constraints (9) and (10) prevent traffic between two nodes from being considered as captured unless both belong to some of the segments that are opened. Note that connectivity between segments is guaranteed by construction of the corridors and extreme sets, and because there is exactly one extreme node in each extreme set. Constraints (11) eliminate subrouts from the solution. The remaining constraints define the range of the variables.

The model just described can easily be extended to a median-traffic capture model. This is essentially a bi-objective model

![Fig. 4. Three network configurations for the city of Concepción. (a) star, (b) triangle and (c) cartwheel.](http://dx.doi.org/10.1016/j.cor.2013.06.013)
which jointly minimizes the network design cost and the walking distance to and from the stations, and maximizes the captured traffic. This problem has already been studied under different names in the context of single path notation. It was introduced by Current et al. [8] and was later studied by Labbé et al. [16,17], Nepal and Park [31], Avella et al. [1], and Lari et al. [25], for example. This extension can easily be formulated within the context of our problem. Let \( z_{ik} \) be defined as 1 if node \( i \) is served by station \( k \) (the closest station), and 0 otherwise. The objective (3) is replaced by a linear combination of the following expressions:

\[
Z_c' = \sum_{s \in S} \sum_{i,j \in N_s} c_{ij} x_{ij}
\]

where \( N(i) \) is the set of stations within a preset distance from \( i \). These objectives incorporate both the cost and the total walking distance of passengers to their closest station. In addition, constraints (9) and (10) are replaced with

\[
v_{ij} \leq \sum_{k \in N(i)} z_{ik} \quad \forall i,j \in N : i < j \quad (9')
\]

\[
v_{ij} \leq \sum_{k \in N(j)} z_{jk} \quad \forall i,j \in N : i < j \quad (10')
\]

requiring the traffic \( i \) to \( j \) being captured only if both nodes are within preset distances of a station. Finally, the formulation must include the constraints

\[
z_{ik} \leq \sum_{s \in S} w_s^k \quad \forall i \in N, k \in N(i),
\]

i.e., a node \( i \) can be served by station \( k \) only if there is such a station.

Finally, the attraction function (2) can be incorporated in the model using the following equations:

\[
v_{ij} \leq \sum_{k \in N(i)} \beta_{ik} z_{ik} \quad \forall i,j \in N : i < j \quad (9'')
\]

\[
v_{ij} \leq \sum_{k \in N(j)} \beta_{kj} z_{jk} \quad \forall i,j \in N : i < j \quad (10'')
\]

together with (16) and

\[
\sum_{k \in N(i)} z_{ik} \leq 1 \quad \forall i \in N.
\]

Constraints (17) preclude customer site \( i \) from being assigned to more than one station \( k \).

Fig. 5. Trade-off curves for the star configuration.

Fig. 6. Structure of the resulting network found using different models. (a) \( r = 0 \), (b) \( r = 500 \) attraction, (c) \( r = 500 \) median and (d) \( r = 1,500 \) median.

Fig. 7. Trade-off curve for the triangle configuration.

3. The case of Concepción, Chile

We have generated a test case using data for the city of Concepción, Chile. Traffic data between different zones of the city can be found in SECTRA [34]. Unfortunately, the latest available data are for 1999, and some new zones of the city are not covered by this data set. For these zones, we have estimated traffic using figures from adjacent zones having similar areas. Fig. 3 depicts a map of the city of Concepción and the demand zones, represented by 108 nodes. The total population of the city is 800,000 and its area is 222 km². However, the city has several peripheral zones, some of them rather disconnected from its center, which would significantly increase the construction cost of the system if considered. Consequently, we consider only the remaining central, dense zones, and concentrate on a total population of 330,000. The approximate number of daily trips is 360,000.

The choice of the shape of the transportation system, as well as the selection of the nodes that belong to the extreme sets and the
corridors, can be made according to multiple criteria: the direction of the traffic on different streets, road intersections that must be excluded or those whose inclusion increases the probability of providing a good service, O/D surveys, expert knowledge on passenger flows, and so on. The edges in the tests satisfy the upper and lower bounds limits on the distances between stations, and their cost was considered to be proportional to their length.

We have compared three possible network configurations for this city: the star with four segments and five extreme sets; the triangle, with seven segments and seven extreme sets; and the cartwheel, with twelve segments and nine extreme sets. The shapes of the configurations, as well as their corridors and extreme sets are depicted in Fig. 4.

We have solved the problem for all configurations using CPLEX 12.0 with AMPL 9.0, on a PC Intel Core i7 running at 2.93 GHz, with 4 GB RAM. Whereas the model discussed in Section 2 works with undirected edge variables, we have found it is more efficient, from a computational point of view, to replace each such variable with two opposite directed arc variables and modify the constraints accordingly. Since the formulation requires an exponential number of subtour elimination constraints (11), its size can become unmanageable. We therefore solve it in a branch-and-cut fashion.

To deal with both objectives and to identify a non-dominated set of solutions, we have used the NISE method of [5]. The method starts by solving the problem for each objective separately. The two solutions found in this way are non-dominated, and become the extreme points of the trade-off curve. A new linear combination of the two objectives is minimized, corresponding to the straight line passing through the two extreme solutions just found. With a high likelihood, if there exists an intermediate non-dominated solution, it will be found. The process continues by using as new objectives the straight lines passing through adjacent non-dominated solutions, in the hope of finding intermediate solutions, and finishes when no new solutions are found, or any other stopping criterion is met. Each time a new pair of solutions is considered, the new model inherits the set of constraints (11) that were added to its “parents”. The process ends when all pairs of adjacent solutions have been analyzed and no new intermediate solutions have been found. We remark, however, that since this method uses weights on the objectives to construct a single objective, it can conceivably happen that, given the integer (non-continuous) nature of the variables, there exist some non-dominated solutions that will not be found because they fall within an integrality gap. If all non-dominated or Pareto-optimal solutions must be found, a constraint method can be applied [14,5].

We first solved the models for each configuration. The trade-off curves for the star configuration are depicted in Fig. 5. For these problems, we used $l_{min}=500$ m and $l_{max}=4000$ m. The trade-off curve denoted as $r=0$ corresponds to the formulation (3)–(15), i.e., the only passengers using the system are those who start or end their trips at one of the stations. The curve “$r=500$ attraction” shows the efficient frontier for median-traffic capture with the attraction function (2), when the longest distance walked by users is 500 m. The curves “$r=500$ median” and “$r=1500$ median” show the results for the median-traffic capture formulation, for two capture radii, without the attraction function. Objectives $z^*$, and $z^{c}$, in the median-traffic model were added together, with equal costs.

As Fig. 5 illustrates, when not only nodes on the path are served, the trade-off curves naturally move up, i.e., more traffic is captured for the same construction cost. The inclusion of attraction reflects the decrease in capture due to distance, and the corresponding curve moves to the left as compared to the basic model, and it lies between the basic and the median curves. When the capture radius is higher, more capture can be achieved at the expense of a higher construction and walking cost.

Fig. 6 shows the network topologies for the cases shown in the trade-off curve of Fig. 5. Figs. 7 and 8 show results for $r=0$; $r=500$ attraction and $r=500$ median, for the triangle configuration, while Figs. 9 and 10 display the results for the cartwheel configuration.

For an easier comparison of the configurations, Fig. 11(a)–(c) show, on the same graph, the trade-off curves for star, triangle and cartwheel, for different objectives. As Fig. 11(a), (b) and (c) show, among the three tested, the best configuration is the star, since it captures the most traffic for the same cost. As opposed to the triangle and cartwheel, it also allows low cost configurations.

The remaining tests were conducted only for the star configuration. Fig. 12 shows the trade-off curve (Pareto-optimal solutions) for the star configuration, with \( l_{\text{min}} = 500 \) m and \( l_{\text{max}} = 4,000 \) m. Some of the solutions are identified with a letter \((a, b, c \text{ or } d)\) and are shown in Fig. 5(a), (b), (c) and (d), respectively. In Fig. 13, the grid highlighting corresponds to the extreme sets, while the grey highlighting corresponds to the corridors. New segments are indicated by dark lines. Node 20 belongs to two corridors. The four cases illustrate how, as the segments become longer (and more expensive), the capture level increases, but the shape of the segments becomes more crooked at the same time.

Table 1 provides the values of \( l_{\text{max}} \) and \( l_{\text{min}} \). The first and second columns of Table 1 provide the value of the inter-station bounds. The third column gives the number of non-dominated solutions found; the next two columns give the number of subtours contained in the solution and the number of constraints required during the whole NISE process. The total time represents the time in seconds necessary to find all non-dominated solutions. When the difference between \( l_{\text{max}} \) and \( l_{\text{min}} \) increases, the number of non-dominated solutions also increases, as does the frequency of occurrence of subtours in the solutions. These two effects combined significantly increase the solution time. This is basically due to the fact that as the difference between \( l_{\text{max}} \) and \( l_{\text{min}} \) increases, more arcs become feasible. Fig. 14 depicts the trade-off curve for the instances of Table 1. Fig. 15 shows four of the same cases, for four instances with a weight \( \alpha = 0.89 \) on the cost objective.

4. Conclusions

We have modeled and solved the problem of locating a rapid transit network to minimize construction cost and maximize origin–destination traffic capture. A mixed integer formulation was presented and Pareto-optimal solutions were found by means of the NISE multi-objective method. Data from the city of Concepción in Chile were used for the experiments. Our model yields realistic solutions within relatively short computing times. This paper contributes to the modeling and algorithmic literature on metro location. It proposes, for the first time, a methodology capable of designing a metro network having a prespecified topology. It does so by locating stations within corridors associated with the various segments of the proposed topological configuration. The algorithm also simultaneously optimizes two objectives.

![Fig. 13. Solutions for four corridors in the city of Concepción.](image)

![Fig. 14. Trade-off curves for different values of \( l_{\text{min}} \) and \( l_{\text{max}} \).](image)

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<th>( l_{\text{max}} ) (m)</th>
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This type of methodology thus allows decision makers to construct and compare several families of solutions. We remark that knowing how many users will choose a transportation system is not possible without considering O/D traffic, as opposed to considering origins and destinations separately, as this is done in most of the literature on metro location. To our knowledge, other researchers in the field have not addressed this problem in such detail. Furthermore, using an exact model and solution method like those proposed in the paper, results in better solutions than a heuristic, which is the approach used in all previous research on traffic capture. The modeling framework’s complexity (in size and scale) has resulted in a formulation size which allows finding the optimal solution for the combined bi-objective function in the model. We are aware that the analysis presented in the paper is limited to the results obtained from the NISE-based solution approach, which finds several but not necessarily all of the non-dominated solutions. Future efforts may be focused on finding all solutions for much larger networks. Additionally, new solution methods (such as the constraint relaxation idea presented by the authors or a multiple single-line RTNDP-type approach) may be developed.

We have compared three well-known network configurations for the city of Concepción: the star, the triangle and the cartwheel. The star is the best configuration, in terms of cost per captured traffic. For this configuration, we have considered several demand capture functions, and have also studied the case in which average or total walking distance is minimized. The resulting trade-off curves were depicted. In general, as the cost increases, the lines become more crooked.

We intend to study two possible extensions to this work: the consideration of user choice and competition with other transportation systems or modes, in terms of travel time, and developing a solution strategy based on solving a series of single line RTNDPs, as was done by Gutiérrez-Jarpa et al. [12].

Acknowledgments

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