ADAPTIVE CONTROL OF A TWO INPUT – TWO OUTPUT SYSTEM USING DELTA MODELS

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Abstract
This paper presents the design of an adaptive controller for a two input – two output (TITO) system using delta models. This controller has been verified by simulation and real time control of a non-linear laboratory model CE108 - coupled drives apparatus. The recursive least squares method is used in identification part of this controller. The synthesis is based on a polynomial approach. Decoupling, where the compensator is placed ahead of the system, suppresses the interactions between control loops. The results of the simulation and the real-time experiments are also given.

1 Introduction
Many technological processes require that several variables relating to one system are controlled simultaneously. Each input may influence all system outputs. The design of a controller able to cope with such a system must be quite sophisticated. There are many different methods of controlling multivariable systems. Several of these use decentralized PID controllers [10], others apply single input-single-output (SISO) methods extended to cover multiple inputs [6]. Here decoupling methods are used to transform the multivariable system into a series of independent SISO loops [8], [14], [15], [16].

This paper is organized as follows: in Section 2 is defined a $\delta$ - model and it described its identification; Section 3 presents the controlled model; Section 4 describes how feedback control without decoupling is designed; Section 5 describes two decoupling methods; Section 6 describes the system identification method; Section 7 gives the simulation results; Section 8 contains the experimental results; finally, Section 9 concludes the paper.

2 Delta models and their identification
If $G(s)$ is the transfer function of a continuous-time dynamic system ($s$ is a complex variable), then the following expression for the discrete transfer function with the zero - order holder is valid

$$G(z) = \frac{z^{-1}Z \left[ L^{-1} \frac{G(s)}{s} \right]}{z} \quad (1)$$

This step transfer function (1) is a rational polynomial function with variable $z$. The simple model structure, easy recursive identification using measurable data, suitability for the synthesis of the discrete control loop as well as for the description and expression of different types of stochastic process, including disturbance modelling, are all advantages of the $z$ – transform function.

The step $z$ - transfer functions have some disadvantages when the sampling period decreases:
the $Z$-transformation parameters do not converge as the sampling period decreases to the Laplace – transformation continuous parameters from which they were derived,

• very small sampling periods yield very small numbers from the transfer function numerator,

• the poles transfer function approach the unstable domain as the sampling period decreases.

The disadvantages of the discrete models can be avoided by introducing a more suitable discrete model. The $\delta$-model, where operator $\delta$ converges with decreased sampling period $T_0$ to a differential operator $p$,

$$\lim_{T_0 \to 0} \delta = p$$

(2)

is best suited to this purposes.

Middleton and Goodwin [11] or Feuer and Goodwin [5] published one of these approaches to the design of these new discrete $\delta$-models. If the new variable $\gamma$ is introduced then it is possible to prove (Mukhopadhyay et al. [12]), that equality

$$\gamma = \frac{z - 1}{\alpha T_0 z + (1 - \alpha)T_0}$$

(3)

holds for interval $0 \leq \alpha \leq 1$. By substituting $\alpha$ in equation (3) we obtain an infinite number of new $\delta$-models. There are several well-known $\delta$-models in applied use

for $a = 0$ $\gamma = \frac{z - 1}{T_0}$ forward $\delta$-model

(4)

for $a = 1$ $\gamma = \frac{z - 1}{zT_0}$ backward $\delta$-model

(5)

for $a = 0.5$ $\gamma = \frac{2z - 1}{T_0 z + 1}$ Tustin $\delta$-model

(6)

This paper will only be concerned with the forward $\delta$-model (4). The $\delta$-models will be used in process modelling for adaptive control based on the self-tuning controller (STC). The main idea of an STC is based on a recursive identification procedure and selected control synthesis. For this reason it is necessary to apply suitable recursive identification algorithm to this model. To parameters estimates of the $\delta$-model, the recursive least squares method (RLSM) with directional forgetting is applied (Kulhavý [7], Bobál et al. [1]).

A useful model to apply this method of identification is the regression (ARX) model which is often expressed in its compact form

$$y(k) = \Theta^T(k)\Phi(k-1) + n(k)$$

(7)

where $\Theta^T(k)$ is the vector of the parameter and $\Phi^T(k-1)$ is the regression vector ($y(k)$ is the process output variable, $u(k)$ is the controller output variable and $n(k)$ is the non-measurable random component). For the identification a stochastic $\delta$-second order model with following discrete equation is used

$$y_\delta(k) = -\alpha_1 y_\delta(k-1) - \alpha_2 y_\delta(k-2) + \beta_1 u_\delta(k-1) + \beta_2 u_\delta(k-2) + n(k)$$

(8)

where

$$y_\delta(k) = \frac{y(k) - 2y(k-1) + y(k-2)}{T_0^2};$$

$$y_\delta(k-1) = \frac{y(k-1) - y(k-2)}{T_0};$$

$$y_\delta(k-2) = y(k-2);$$

$$u_\delta(k-1) = \frac{u(k-1) - u(k-2)}{T_0};$$

$$u_\delta(k-2) = u(k-2)$$

(9)

From equation (7), (8) and (9) it is obvious that the vector of parameters has the form

$$\Theta_\delta^T(k) = [\alpha_1, \alpha_2, \beta_1, \beta_2]$$

(10)

and the regression vector is

$$\Phi_\delta^T(k-1) = \left[-\frac{y(k-1) - y(k-2)}{T_0}, -y(k-2), \frac{u(k-1) - u(k-2)}{T_0}, u(k-2)\right]$$

(11)

Then in the identification part of the designed controller algorithms the regression (ARX) model of the following form
\[ y_\delta(k) = \Theta_\delta^T(k) \Phi_\delta(k-1) + n(k) \]  

(12)

is used.

The recursive least squares method is utilized for calculating of the parameter estimates \( \hat{\Theta}_\delta(k) \) and adaptation is supported by directional forgetting [4]. The value of the directional forgetting factor \( \varphi(k) \) basically depends on the level of conformity achieved between the model and the real behaviour of the system.

A self-tuning SISO (single input – single output) PID controller based on the recursive identification of the \( \delta \) - model and of a modified Ziegler – Nichols criterion has been designed in [4] and some modifications of the pole placement controllers in [2].

### 3 Description of TITO system

The internal structure of the system is shown in Fig. 1.

Fig. 1. A two input – two output system – the “P” structure

The transfer matrix of the system is

\[
G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}
\]

(13)

It is possible to assume that the system is described by the matrix fraction

\[
G(\gamma) = A^{-1}(\gamma) B(\gamma) = B_1(\gamma) A_1^{-1}(\gamma)
\]

(14)

Where polynomial matrices \( A \in \mathbb{R}_{m\times m}[\gamma] \), \( B \in \mathbb{R}_{m\times n}[\gamma] \) are the left indivisible decomposition of matrix \( G(\gamma) \) and matrices \( A_1 \in \mathbb{R}_{m\times m}[\gamma] \), \( B_1 \in \mathbb{R}_{m\times n}[\gamma] \) are the right indivisible decomposition.

The matrices of the discrete model are

\[
A(\gamma) = \begin{bmatrix} \gamma^2 + \alpha_1 \gamma + \alpha_2 & \alpha_3 \gamma + \alpha_4 \\ \alpha_5 \gamma + \alpha_6 & \gamma^2 + \alpha_7 \gamma + \alpha_8 \end{bmatrix}
\]

and the differential equations of the model are

\[
y_{1,\delta}(k) = -\alpha_1 y_{1,\delta}(k-1) - \alpha_2 y_{1,\delta}(k-2) - \alpha_3 \delta(k) + \beta_1 u_{1,\delta}(k-1) + \beta_2 u_{1,\delta}(k-2) + \beta_3 y_{2,\delta}(k-1) + \alpha_4 \delta(k) - \alpha_5 \delta(k-1) \]

(15)

\[
y_{2,\delta}(k) = -\alpha_1 y_{2,\delta}(k-1) - \alpha_2 y_{2,\delta}(k-2) - \alpha_3 \delta(k) + \beta_1 u_{2,\delta}(k-1) + \beta_2 u_{2,\delta}(k-2) + \beta_3 y_{1,\delta}(k-1) + \alpha_4 \delta(k) - \alpha_5 \delta(k-1) \]

(16)

### 4 Design of feedback MIMO system

In the same way as the controlled system, the transfer matrix of the controller takes the form of matrix fraction

\[
G(\gamma) = P^{-1}(\gamma) Q(\gamma) = Q(\gamma) P^{-1}(\gamma)
\]

(16)

The matrix of an integrator for permanent zero control error is

\[
F(\gamma) = \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix}
\]

(17)

The control law apparent in the block diagram (variable \( \gamma \) will be omitted from some operations for the sake of simplification) has the form

\[
U = F^{-1} Q P^{-1} E
\]

(18)

It is possible to derive the following equation for the system output

\[
Y = A^{-1} B F^{-1} P^{-1} Q (W - Y)
\]

(19)
which can be modified to give

\[ Y = P_1 \left( A F P_1 + B Q_1 \right)^{-1} B Q_1 P_1^{-1} W \]  \hspace{1cm} (20)

The closed loop system is stable when the following diophantine equation is satisfied

\[ A F P_1 + B Q_1 = M \]  \hspace{1cm} (21)

where \( M(\gamma) \in \mathbb{R}^{mm} \) is a stable diagonal polynomial matrix.

\[ M(\gamma) = \begin{bmatrix} \gamma^4 + m_2 \gamma^3 + m_4 \gamma^2 + m_3 \gamma + m_4 & 0 \\ 0 & \gamma^4 + m_2 \gamma^3 + m_4 \gamma^2 + m_3 \gamma + m_4 \end{bmatrix} \]  \hspace{1cm} (22)

The roots of this polynomial matrix are the ruling factor in the behaviour of the closed loop system. They must be inside the circle with the centre at \(-1/T_0\) with the radius \(1/T_0\) if the system is to be stable.

The degree of the controller matrix polynomials depends on the internal properness of the closed loop. The structure of matrices \( P_1 \) and \( Q_1 \) was chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the diophantine equations using the uncertain coefficients method.

\[ P_1(\gamma) = \begin{bmatrix} \gamma + p_{15} & p_{25} \\ p_{35} & \gamma + p_{45} \end{bmatrix} \]

\[ Q_1(\gamma) = \begin{bmatrix} q_{15} \gamma^2 + q_{25} \gamma + q_{35} & q_{45} \gamma^2 + q_{55} \gamma + q_{65} \\ q_{75} \gamma^2 + q_{85} \gamma + q_{95} & q_{105} \gamma^2 + q_{115} \gamma + q_{125} \end{bmatrix} \]  \hspace{1cm} (23)

The solution to the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. The controller parameters are given by solving these equations.

5 Design of decoupling control using compensators

There are several ways to control multivariable systems with internal interactions. Some make use of decentralized PID controllers, whilst others are composed of a string of single input – single output methods.

One possibility is the serial insertion of a compensator ahead of the system. The aim here is to suppress of undesirable interactions between the input and output variables so that each input affects only one controlled variable.

The decoupling conditions are fulfilled when matrix \( H \) is diagonal. Several well – known compensators are given in [8], [9], [15], [16]. Control algorithms were derived for the model above with two compensators. These will be referred to as \( C_1 \) and \( C_2 \). Compensator \( C_1 \) is the inversion of the controlled system. Matrix \( H \) is, therefore, a unit matrix.

\[ H = KG \]  \hspace{1cm} (24)

The solution to the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. The controller parameters are given by solving these equations.

\[ Y = P_1 \left( A F P_1 + Q_1 \right)^{-1} Q_1 P_1^{-1} W \]  \hspace{1cm} (25)

The following equation must be satisfied if the closed loop system is to be stable

\[ F P_1 + Q_1 = M \]  \hspace{1cm} (26)

The structures of the polynomial matrices of the controller were chosen to suit physical demands.

\[ P_1(\gamma) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]
\[
Q_1(\gamma) = \begin{bmatrix} q_{1\delta} & 0 \\ 0 & q_{2\delta} \end{bmatrix}
\]  

(27)

Consequently, matrix \( M \) was chosen to be
\[
M(\gamma) = \begin{bmatrix} \gamma + m_1 & 0 \\ 0 & \gamma + m_2 \end{bmatrix}
\]  

(28)

The controller parameters are the result from the equation (26). The control law can be described by matrix equation
\[
FU = B^{-1}AQ_1P_1^{-1}E
\]  

(29)

Compensator \( C_2 \) is adjugated matrix \( B \). When \( C_2 \) was included in the design of the closed loop the model was simplified by considering matrix \( A \) as diagonal. The multiplication of matrix \( B \) and adjugated matrix \( B \) results in diagonal matrix \( H \). The determinants of matrix \( B \) represent the diagonal elements. When matrix \( A \) is nondiagonal, its inverted form must be placed ahead of the system in order to obtain diagonal matrix \( H \), otherwise it may increase the order of the controller and sophistication of the closed loop system. Although designed for a diagonal matrix, compensator \( C_2 \) also improves the control process for non–diagonal matrix \( A \) in the controlled system. This is demonstrated in the simulation results.

Fig. 5. Closed loop system with compensator \( C_2 \).

The equation for the system output as shown in this block diagram takes the form
\[
Y = P_1( AFP_1 + B, Q_1 )B, Q_1 P_1^{-1}W
\]  

(30)

where
\[
B_v = B \text{adj}(B) = \begin{bmatrix} \text{det}(B) & 0 \\ 0 & \text{det}(B) \end{bmatrix}
\]  

(31)

To achieve stability in the closed loop system the following diophantine equation must be fulfilled
\[
AFP_1 + B, Q_1 = M
\]  

(32)

The controller polynomial matrices are chosen as shown below
\[
P_1(\gamma) = \begin{bmatrix} \gamma^2 + p_{1\delta}\gamma + p_{2\delta} & 0 \\ 0 & \gamma^2 + p_{3\delta}\gamma + p_{4\delta} \end{bmatrix}
\]  

\[
Q_1(\gamma) = \begin{bmatrix} q_{1\delta}\gamma^2 + q_{2\delta}\gamma + q_{3\delta} & 0 \\ 0 & q_{10\delta}\gamma^2 + q_{11\delta}\gamma + q_{12\delta} \end{bmatrix}
\]  

(33)

and matrix \( M \) is
\[
M(\gamma) = \begin{bmatrix} \gamma^2 + m_1\gamma + m_2\gamma + m_5 & 0 \\ +m_7\gamma^2 + m_8\gamma + m_5 & \gamma^2 + m_5\gamma + m_7 + m_8\gamma^2 + m_9\gamma + m_9 \end{bmatrix}
\]  

(34)

Solving the diophantine equation defines a set of algebraic equations, which we subsequently use to obtain the unknown controller parameters.

The control law is given by the block diagram
\[
FU = \text{adj}(B)Q_1P_1^{-1}E
\]  

(35)

6 Recursive identification

The algorithms designed here were incorporated into an adaptive control system with recursive identification. The recursive least squares method proved effective for self-tuning controllers and was used as the basis for this algorithm.

The parameter vector are completed as shown below:
\[
\Theta^r(\delta)(k) = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & \beta_5 & \beta_6 & \beta_7 & \beta_8 \end{bmatrix}
\]  

(36)

The data vector is
\[
\Phi^r(\delta)(k) = [-y_{1\delta}(k-1), -y_{1\delta}(k-2), -y_{2\delta}(k-1), -y_{2\delta}(k-2), u_{1\delta}(k-1), u_{1\delta}(k-2), u_{2\delta}(k-1), u_{2\delta}(k-2)]
\]  

(37)

The parameter estimates are actualized using the recursive least squares method plus directional forgetting. The detailed recursive identification algorithm for TITO system is designed in [3].
7 Simulation examples

The program system MATLAB - SIMULINK was used to create a program and diagrams to simulate and verify the algorithms. Verification by simulation was carried out on a range of systems with varying dynamics. The control of the model below is given here as our example.

\[
A(s) = \begin{bmatrix} s^2 + 2s + 0.7 & 0.2s + 0.4 \\ -0.5s - 0.1 & s^2 + 2s + 0.7 \end{bmatrix}
\]

\[
B(s) = \begin{bmatrix} 0.5s + 0.2 & 0.1s + 0.3 \\ 0.5s + 0.1 & 0.3s + 0.4 \end{bmatrix}
\]

(38)

Fig. 6 shows the system’s step responses

![Step Response Diagram](image)

Fig. 6. The step response of the system

The right side control matrices are denoted as follows: without compensator - \( M_1 \), with compensator \( C_1 - M_2 \), and with compensator \( C_2 - M_3 \).

\[
M_1(\gamma) = \begin{bmatrix} \gamma^2 + 2\gamma + 1.5\gamma^2 + 0.5\gamma + 0.0625 \\ 0 \end{bmatrix}
\]

\[
\begin{bmatrix} \gamma^2 + 2\gamma + 1.5\gamma^2 + 0.5\gamma + 0.0625 \\ 0 \end{bmatrix}
\]

\[
M_2(\gamma) = \begin{bmatrix} \gamma + 0.5 & 0 \\ 0 & \gamma + 0.5 \end{bmatrix}
\]

\[
M_3(\gamma) = \begin{bmatrix} \gamma^2 + 2.5\gamma^4 + 2.5\gamma^2 + 1.25\gamma^2 + 3.125\gamma + 0.0313 \\ 0 \end{bmatrix}
\]

\[
\begin{bmatrix} \gamma^2 + 2.5\gamma^4 + 2.5\gamma^2 + 1.25\gamma^2 + 3.125\gamma + 0.0313 \\ 0 \end{bmatrix}
\]

(39)

The same initial conditions for system identification were used for all the types of adaptive control we tested. The initial parameter estimates were chosen to be

\[
\Theta_\delta^T(0) = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.1 & 0.2 & 0.3 & 0.4 \\ 0.5 & 0.6 & 0.7 & 0.8 & 0.5 & 0.6 & 0.7 & 0.8 \end{bmatrix}
\]

(40)

The results of simulation are shown in Figs. 7 – 12

It is possible to draw several conclusions from the simulation results of the experiments on linear static systems. The basic requirement to ensure permanent zero control error was satisfied in all cases. The criteria on which we judge the quality of the control process are the overshoot on the controlled values and the speed with which zero control error is achieved. According to these criteria the controller incorporating compensator \( C_1 \) performed the best. However, this controller appears to be unsuited to adaptive control due to the size of the overshoot and the large numbers of process and controller outputs. The controller which uses compensator \( C_2 \) seems to work best in adaptive control. With regards to decoupling, it is clear that controllers with compensators greatly reduce interaction.
8 Laboratory experiments

The verification of designed TITO controllers in laboratory conditions operating in real time has been realized using experimental laboratory model CE 108 - couples drives apparatus. This apparatus, based on experience with authentic industrial control applications, was developed in co-operation with the University of Manchester and made by a British company, TecQuipment Ltd. It allows us to investigate the ever-present difficulty of controlling the tension and speed of material in a continuous process. The process may require the material speed and tension to be controlled to within defined limits. Examples of this occur in the paper-making industry, strip metal and wire manufacture and, indeed, any process where the product is manufactured in a continuous strip.

A continuous flexible belt replaces the industrial type material strip. The principle scheme of the model is shown in the Fig. 10. It consists of three pulleys, mounted on a vertical panel so that they form a triangle resting on its base. The two base pulleys are directly mounted on the shafts of two nominally identical servomotors and the apparatus is controlled by manipulating the drive torques to these servomotors. The third pulley, the jockey, is free to rotate and is mounted on a pivoted arm. The jockey pulley assembly, which simulates a material workstation, is equipped with a special sensor and tension measuring equipment. It is the jockey pulley speed and tension which form the principle system outputs. The belt tension is measured indirectly by monitoring the angular deflection of the pivoted tension arm to which the jockey pulley is attached.

The controller output variables are the inputs to the servomotors and the process output variables are the tension and speed at the workstation. There are interactions between the control loops.

The task was to apply the methods we designed for the adaptive control of a model representing a non-linear system with variable parameters which is, therefore, impossible to control deterministically. Adaptive control using recursive identification both with and without use of compensators was performed. As indicated in the simulation,
compensator $C_1$ was shown to be unsuitable and control broke down. The other two methods gave satisfactory results. The time responses of the control for both cases are shown in Fig. 11 and Fig. 12. The figures demonstrate that control with a compensator reduces interaction. Process output variable $y_1$ is the speed and process output variable $y_2$ is the tension. The variables $u_1$ and $u_2$ are the controller outputs – inputs to the servomotor.

9 Conclusions

The adaptive control of a two-variable system based on polynomial theory and using delta models was designed. Decoupling problems were solved by the use of compensators. The designs were simulated and used to control a laboratory model. The simulation results proved that these methods are suitable for the control of linear systems. The control tests on the laboratory model gave satisfactory results despite the fact that the nonlinear dynamics were described by a linear model.
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