Abstract

In this paper a new method for real time estimation of vehicular flows and densities on motorways is proposed. This method is based on fusing traffic counts with mobile phone counts. The procedure used for the estimation of traffic flow parameters is based on the hypothesis that “instrumented” vehicles can be counted on specific motorway sections and traffic flow can be measured on entrance and exit ramps. The motorway is subdivided into cells, assuming that mobile phones entering and exiting every cell can be counted during the observation period. An estimate of “instrumented” vehicle concentration is obtained and propagated on the network in time and space. This allows one to estimate traffic flow parameters by sampling “instrumented” traffic flow parameters using a "concentration" (the ratio of the densities of instrumented vehicles to the density of overall traffic) propagation mechanism.

Keywords: Transportation, Traffic flow, Simulation.

1. Introduction

The widespread use of mobile phone communications in the majority of developed countries, and the increasing numbers of customers using mobile phones allows for the exchange of large quantities of data and continuously updated information (Nilsson 1999).

There are many possibilities that stem from the use of these technologies; among them, fundamental for the development of improved transport system management and control, is the estimation of traffic flow parameters. Intelligent transportation systems (ITS) include applications of new technologies to traffic management and control. ITS strategies are designed to supply services that increase the safety, efficiency and reliability of the transportation system. In order to accomplish this, traffic control systems must be able to estimate the traffic state on the system, and this is usually accomplished by the application of many different technologies: from automatic sensors
to direct traveler reports. Experience has shown that it is difficult to accurately estimate the true system state and it remains a fundamental objective in the development of new ITS.

Vehicular traffic flow is a complicated random process, described with parameters that depend on space and time. The deployment of widespread information and communication networks may now facilitate the extraction of detailed, accurate information on traveler position from the localization of mobile phones. Given the hypothesis that the mobile phones on some “instrumented” vehicles can be localized and counted this can improve the accuracy of the estimation of the parameters that characterize vehicular flow. This hypothesis is enhanced by the new mobile phone localization systems that is being introduced in the USA as a consequence of enhanced 911 service (Docket 94-102 October 1996 of the Federal Communications Commission FCC) and in Europe for commercial reasons.

Recent literature has described the potential for the estimation of traffic flow parameters by using mobile phone localization information (Astarita, 2002; Smith et al. 2001; Lovell, 2002; Astarita-Guido, 2002). Further Bolla et al. (2000) studied the problem for some types of roads including motorways with controlled access.

In this paper a new method for real time estimation of vehicular flows and densities on motorways is proposed. This method is based on fusing traffic counts with mobile phone counts. It is important to note that the procedure used for the estimation of traffic flow parameters is based on the hypothesis that “instrumented” vehicles can be counted on specific road sections and traffic flow can be measured on entrance and exit ramps. It is not necessary to assume that “instrumented” vehicles are vehicles with a mobile phone on board. In fact any electronic or non electronic device, that can be used on only a fraction of the total flow, and that can be counted at specific road sections, introduces the same estimation problem.

The motorway is subdivided into cells, assuming that “instrumented” vehicles entering and exiting every cell can be counted during the observation period. Moreover, the number of vehicles that enter the first cell of the network and the number that enter and exit on ramps is also known. The “instrumented” vehicle concentration is obtained and propagated over the network in time and space. This allows one to estimate traffic flow
parameters by sampling “instrumented” traffic flow parameters using a concentration propagation mechanism.

In the next section, two numeric calculation techniques are introduced for the estimation of traffic flow parameters within the theoretical framework introduced. The first is an implicit calculation technique for the solution of partial differential equations and the second technique is based on explicit numerical calculations. Though subject to some numerical instabilities, the second technique has exhibited better results forcing the use of a well studied time-space grid. Results are compared and discussed using a real-life test scenario.

2. Analytical framework

It can be useful to estimate the flow, density and speed, parameters that clearly define traffic conditions. In this paper "instrumented" vehicles are vehicles that have on-board any kind of technical apparatus that can be localized and counted on some given road section. Values relative only to "instrumented" vehicles are indicated with the sub index \( c \).

\[
\begin{align*}
Flow: (Q; Q_c) & \approx f(x, t) \left[ \frac{\text{vehicles}}{h} \right], \\
Density: (\rho; \rho_c) & \approx f(x, t) \left[ \frac{\text{vehicles}}{km} \right], \\
Speed: (v; v_c) & \approx f(x, t) \left[ \frac{km}{h} \right].
\end{align*}
\]

These parameters are functions of time and space. \( Q_c, \rho_c, v_c \) are the values of flow, density and speed relative only to "instrumented" vehicles. In this paper, flow is considered as a fluid composed of two continuously mixed fluids: instrumented and not instrumented vehicles.

This assumption is based on the decomposition of an overall traffic streams into two classes as frequently done in literature decomposing traffic stream into trucks and automobiles.

Based on Edie (1965), we have the following two relationships for the overall traffic flow (1) and for only instrumented vehicles (2):
Two conservation equations can be used for the overall traffic flow (3) and for the instrumented vehicles (4):

\[ \frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0, \]  
\[ \frac{\partial \rho_c}{\partial t} + \frac{\partial Q_c}{\partial x} = 0. \]

These relationships (3) and (4) are valid for general road sections with no entrance and exit ramps.

We can imagine that an instrumented vehicle has a mobile phone on board that can be localized. If we hypothesize that mobile phone position can be estimated even when in stand-by condition (when no call is being made) there is no reason for the average speed of instrumented vehicles to be different from that of non-instrumented vehicles. We can assume that the overall speed is equal to the speed of instrumented vehicles and also equal to the speed of non instrumented vehicles \( v_{nw} \):

\[ v(x,t) = v_c(x,t) = v_{nw}(x,t). \]

Assuming (5) simplifies the calculations leading to an elegant analytical formulation. As shown in the following, some analytical results can be obtained even with a less restrictive assumption.

We can define as "concentration" \( \varphi \) the ratio of the densities of instrumented vehicles to the overall traffic density and \( \psi \) the ratio of instrumented vehicles flow to the total traffic flow:

\[ \varphi(x,t) = \frac{\rho_c(x,t)}{\rho(x,t)}, \]  
\[ \psi(x,t) = \frac{Q_c(x,t)}{Q(x,t)}. \]

Applying (5) we obtain in (6):

\[ \varphi(x,t) = \frac{\rho_c(x,t) \cdot v_c(x,t)}{\rho(x,t) \cdot v(x,t)}, \]

and applying (1) and (2) we obtain:
The continuity equation (4) relative to instrumented vehicles can be written:
\[
\frac{\partial \rho_{i}(x,t)}{\partial t} + \frac{\partial Q_{i}(x,t)}{\partial x} = \frac{\partial(\rho(x,t)\varphi(x,t))}{\partial t} + \frac{\partial(Q(x,t)\varphi(x,t))}{\partial x} = 0,
\]
we obtain:
\[
\varphi(x,t)\cdot \frac{\partial \rho(x,t)}{\partial t} + \rho(x,t)\cdot \frac{\partial \varphi(x,t)}{\partial t} + \varphi(x,t)\cdot \frac{\partial Q(x,t)}{\partial x} + Q(x,t)\cdot \frac{\partial \varphi(x,t)}{\partial x} = 0.
\]
Then by regrouping :
\[
\varphi(x,t)\cdot \left(\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x}\right) + \rho(x,t)\cdot \frac{\partial \varphi(x,t)}{\partial t} + Q(x,t)\cdot \frac{\partial \varphi(x,t)}{\partial x} = 0.
\]
The first addend is equal to zero:
\[
\rho(x,t)\cdot \frac{\partial \varphi(x,t)}{\partial t} + Q(x,t)\cdot \frac{\partial \varphi(x,t)}{\partial x} = 0.
\]
by rewriting and regrouping we obtain:
\[
\rho(x,t)\cdot \left(\frac{\partial \varphi(x,t)}{\partial t} + v(x,t)\cdot \frac{\partial \varphi(x,t)}{\partial x}\right) = 0.
\]
This means that the total derivative of \(\varphi\) with respect to time \(t\) is equal to zero:
\[
\frac{d \varphi(x,t)}{dt} = 0. \tag{9}
\]
The ratio \(\varphi\) (and \(\psi\)) is constant along a trajectory of a traffic flow particle.
If the average speed of instrumented vehicles is not equal to the average speed of all traffic then another alternative functional relationship can be assumed between the speeds of instrumented and non-instrumented vehicles, for example, we could assume:
\[
v_{e}(x,t) = v_{nc}(x,t) + c. \tag{10}
\]
or:
\[
v_{e}(x,t) = v_{nc}(x,t) \cdot c.
\]
Where \(c\) is a constant value.
For the sake of brevity, we present only some consequences of assuming the relationship depicted by (10). In this case we can assume that the density is the sum of the two densities relative to instrumented and non-instrumented vehicles (Edie,1965; Daganzo,1997):
\[ \rho = \rho_c + \rho_{nc}, \quad (11) \]

we also have that the total flow is the sum of the two flow components:

\[ Q = Q_c + Q_{nc}. \quad (12) \]

We can write this as:

\[ \rho \cdot v = \rho_c \cdot v_c + \rho_{nc} \cdot v_{nc}, \]

obtaining for the speed of all flow:

\[ v = \frac{\rho_c \cdot v_c + \rho_{nc} \cdot v_{nc}}{\rho}. \quad (13) \]

From (13) by applying (11) and (10) we obtain:

\[ v = \frac{\rho_c \cdot v_c + (\rho - \rho_c)(v_c - c)}{\rho} = \frac{\rho_c \cdot v_c}{\rho} + \frac{\rho \cdot v_c}{\rho} - \frac{\rho \cdot c}{\rho} - \frac{\rho_c \cdot v_c}{\rho} + \frac{\rho_c \cdot c}{\rho}, \]

after simplification:

\[ v = v_c - c \left( 1 - \frac{\rho_c}{\rho} \right). \]

Applying (6), we can write

\[ v = v_c - c (1 - \varphi). \quad (14) \]

This last formula represents the relationship between the speed of all flow and the speed of only instrumented vehicles assuming (10).

By combining (2) and (14) we obtain:

\[ Q_c = \rho_c \cdot (v + c (1 - \varphi)). \quad (15) \]

By substituting into the continuity equation (4) we have:

\[ \frac{\partial \rho_c \cdot (v + c (1 - \varphi))}{\partial x} + \frac{\partial \rho_c}{\partial t} = 0, \quad (16) \]

after some simplification and using (1) and (6) we have:

\[ \frac{\partial Q}{\partial x} \varphi + \frac{\partial \varphi}{\partial x} Q + c \cdot \rho \frac{\partial (1 - \varphi)}{\partial x} + (1 - \varphi) \cdot c \cdot \varphi \frac{\partial \rho}{\partial x} + \varphi \frac{\partial \rho}{\partial t} + \rho \frac{\partial \varphi}{\partial t} = 0. \]
By simplification and by extracting the total derivative of $\varphi$ with respect to time we have:

$$\frac{d\varphi}{dt} = -c \cdot \frac{\partial (1-\varphi)}{\partial x} \frac{\varphi}{\rho} \frac{\partial \rho}{\partial x}.$$  \hspace{1cm} (17)

In this case the total derivative of $\varphi$ is not null and the density ratio (concentration) $\varphi$ varies along a trajectory of flow, but still it is possible to solve the problem numerically once the relationship between $\varphi$ and $\psi$ is established.

We can rewrite (7) as:

$$\psi = \frac{v \cdot \varphi}{v},$$ \hspace{1cm} (18)

and applying (14) we can rewrite it as:

$$\psi = \frac{v + c(1-\varphi)}{v} \varphi = \varphi + \frac{c(1-\varphi)}{v} \varphi.$$ \hspace{1cm} (19)

This is a relationship between $\varphi$ and $\psi$ that can be applied to solve the analytical problem in the case that the constant $c$ in equation (10) is not null. In this paper, for the sake of brevity, this case is not considered further and the proposed numerical method is applied assuming that relationship (5) is true.

3. A methodology for traffic parameter estimation

The methodology presented in this paper aims at traffic parameter estimation on motorway networks and is based on the hypothesis that "instrumented" vehicles can be counted when moving from one cell to another on the network and traffic flow can be measured on ramps.

By using the fundamental equations (1) and (2), continuity equations (3) and (4) and assuming that "instrumented" vehicles move on the network with the same average speed of the overall flow (equation 5), we obtain a system of partial differential equations. The problem may seem similar to the Lighthill and Whitham model (L-W model, 1955), but here, no relationship is introduced for speed dynamics. This methodology can be used without assuming that any particular analytical traffic model is valid, including the L-W model, the Payne model or any other traffic model.
This kind of partial differential equation problem can be solved by using time and space discretization. Using space discretization that is based on cells where we have traffic counts of "instrumented vehicles" simplifies the solution, for this reason in the following we assume that "instrumented" vehicles can be counted at the entrance and the exit of the discrete space cells used for space discretization.

By discretizing equations (3) and (4) we obtain:

\[
\Delta \rho = -\frac{\Delta Q}{\Delta x} \cdot \Delta t 
\]

(20)

\[
\Delta \rho_i = -\frac{\Delta Q}{\Delta x} \cdot \Delta t
\]

(21)

Here follows the notation used to describe each entity considered (Fig. 1):

- **T** = the instant in time of the estimation time period;
- **i** = \{1, 2, ..., M\}, the index of cell i, M is the total number of cells and the index i grows in the direction of traffic flow;
- **t_j** = the beginning instant of time step j;
- **\Delta t** = the time step used in time discretization (\(T = t_{j+1} - t_j\); with \(j = 1..T/\Delta t\));
- **j** = \{1, 2, ..., \(T/\Delta t\)\}, the index of time step j;
- **L_i** = the spatial length of cell i;
- **\(N_{v(i,j)}\)** = the number of vehicles in cell i at time \(t_{j+1}\);
- **\(N_{e(i,j)}\)** = the number of "instrumented" vehicles in cell i at time \(t_{j+1}\);
- **\(N_{v0(i,j)}\)** = the number of vehicles exiting cell i in time interval \((t_j, t_{j+1})\) and entering cell \(i+1\);
- **\(N_{e0(i,j)}\)** = the number of "instrumented" vehicles exiting cell i in time interval \((t_j, t_{j+1})\) and entering cell \(i+1\);
- **\(E_{n(i,j)}\)** = the number of vehicles entering cell i by a ramp in time interval \((t_j, t_{j+1})\);
- **\(Out_{(i,j)}\)** = the number of vehicles exiting cell i by a ramp in time interval \((t_j, t_{j+1})\);
• \( \varphi_{i,j} \) = the ratio of "instrumented vehicles" to the total number of vehicles in cell \( i \) at time instant \( t_{j+1} \);

• \( Q_{i,j} \) = traffic flow exiting cell \( i \) during time interval \([t_j, t_{j+1})\);

• \( \rho_{i,j} \) = density of cell \( i \) at the end of time interval \( j \);

• \( Q_{e(i,j)} \) = traffic flow relative to “instrumented vehicles” exiting cell \( i \) during time step \([t_j, t_{j+1})\);

• \( \rho_{e(i,j)} \) = traffic density relative to “instrumented vehicles” in cell \( i \) at the end of time step \( j \);

• \( v_{i,j} \) = average velocity in cell \( i \) at the end of time step \( j \).

![Diagram](http://example.com/diagram.png)

Fig. 1: Schema of flow (cells and vehicles) entering and exiting the cells.

Using this notation in (20) and (21) it is possible to write:

\[
\frac{N_{v(i,j)} - N_{v(i,j-1)}}{L_e} = \frac{N_{vu(i,j)} - N_{vu(i-1,j)}}{L_e}
\]

Obtaining for every cell without ramps

\[
N_{v(i,j)} = N_{v(i,j-1)} - N_{vu(i,j)} + N_{vu(i-1,j)}
\] (22)

In general:

\[
N_{v(i,j)} = En_{(i,j)} - Out_{(i,j)} + N_{v(i,j-1)} - N_{vu(i,j)} + N_{vu(i-1,j)}
\] (23)

where \( N_{vu(i,j)} \) and \( N_{vu(i-1,j)} \) are undefined.

Assuming (5) we obtain

\[
\varphi(x,t) = \psi(x,t) = \frac{Q_e(x,t)}{Q(x,t)}
\]

which allows us to find the values of \( N_{v(i,j)} \):
\[ N_{x(i,j)} = En_{(i,j)} + Out_{(i,j)} + N_{v(i,j-1)} - \frac{N_{ca(i,j)}}{\phi(i, j-1)} + \frac{N_{ca(i-1,j)}}{\phi(i-1, j-1)} \] (24)

in which we assume
\[ \phi_{(i,j)} = \frac{N_{c(i,j)}}{N_{v(i,j)}} \] (25)

The "forward propagation" of ratio \( \phi \) happens on the basis of one space-time discretization.

A bidimensional schematic representation of the model is observed in Fig. 2, in which on the abscissas we have single cells, and on the ordinates, we have time intervals, considering the step(\( j \)) for example time intervals that elapse between the instant \( j \) and the instant \( j+1 \).

Fig.2: Space-time discretization

In order to estimate the values of capacity and density it is possible to use the following expressions:

\[ Q_{(i,j)} = \frac{N_{v(i,j)}}{\phi_{(i,j)} \cdot \Delta T} \] (16)
\[ \rho_{(i,j)} = \frac{N_{v(i,j)}}{L_{v(i,j)}} \]  

Finally, it is possible to estimate the value of the speed from the ratio of the \( Q \) and \( \rho \):

\[ v_{(i,j)} = \frac{Q_{(i,j)}}{\rho_{(i,j)}} \]

This new formulation is slightly different from another formulation presented in Astarita and Guido (2002), that provided the resolution of the problem using an implicit method of calculation. The explicit formulation here presented provides reasonable results given that the extension of the step applied for the time discretization is not excessive, in fact, while the method appears more precise and reliable than the implicit scheme, the misuse of space-time discretization could lead to numerical instabilities.

The application of this method of calculation on a test network offers optimal results, considering that the information on the concentration of instrumented vehicles does not propagate only following the direction of flow. In fact, with particular conditions applied to the ramps, the method is applicable to the propagation of the concentration of the instrumented vehicles in both directions.

4. Numerical application on a test network

In order to verify the efficiency of the proposed algorithm, the latter has been re-run using Integration, a traffic microsimulation model (Van Aerde 1995), that enabled simulation of driver behavior.

This model has been used in order to simulate traffic on a freeway equipped with on- and off- ramps, toll stations where it is possible to record the number of passing vehicles and radio base stations tracking the mobile phones.

The test network is a three-lane motorway that extends for 20 km in length with on- and off- ramps (see Fig. 3). In the entering (A) and exiting (B) sections it is supposed that complete traffic counts are obtained continuously in time from toll stations. The network is subdivided into 20 cells each with a length of 1 km. It is also supposed that “instrumented” vehicles can be counted when moving from one cell to another,
assuming that the link (cell) coincides with the cell of the network in which it is possible to measure signals emitted from the mobile phone.

On this network seven scenarios have been simulated representing situations that often are found in reality, relating the behavior of the model to variations in the percentage of instrumented vehicles in relation to the total number of vehicles on the network.

The estimation methodology is applied to information obtained from traffic counts recorded by the simulated scenarios (*Integration*). This way it is possible to compare the estimated value obtained with the presented methodology and the “known” values produced by the microsimulation. A desired value of $\phi$ can be reproduced generating randomly “instrumented” vehicles following the chosen preset ratio for $\phi$.

Different scenarios have been analyzed (in a single Integration run) in order to estimate the effectiveness of the new method. Results relative to a single scenario (scenario A) are presented as an example: entering traffic flow is equal to 3600 (veh/h) and, in the last cell, there is a bottleneck, reducing the capacity to 3200 (veh/h). In this scenario a queue forms and propagates upstream from the last cell. Results shown in Fig. 4 and 5 are relative to the percent estimation error of density and velocity for an average value of $\phi$ equal to 0.2, 0.4, 0.6, 0.8.
As shown, results obtained in all scenarios indicate that an increase in the average value of $\phi$ improves the quality of estimates. A more complete appraisal of the quality of the proposed method can be performed through the introduction of the variable time in the parameter estimation.

Therefore, we can examine the magnitude of the percentage error in density and flow...
estimation in one cell of particular interest for every scenario analyzed and observe their
trends over time.

For simplicity, only the density observations in the 19th cell of the network are shown,
in order to examine the effects produced from the bottleneck (Fig. 6).

![Graphs showing percent error in traffic density estimation over time for different flow values](image)

It has been observed that for lower flow values the method produces larger errors, since
in some sections of the network the algorithm amplifies the difference between
observed values and measured values, in a way inversely proportional to the proportion
of instrumented vehicles.

All the results exposed are relative to the application of the new methodology, adopting
a time-step of observation (step) equal to 10 seconds. Further, it has been observed than
the instability problem is solved by using a time step smaller than a critical value,
beyond which the model becomes less reliable. The following equation is applied to
obtain the critical value of the time step:

\[
\nu < \frac{\Delta x}{\Delta t}
\]
Having adopted the value of $\Delta x$ equal to 1 kilometer and value of $v$ equal to 110 km/h, the critical value of $\Delta t$ is 32.4 seconds.

The model becomes reliable for values of $\Delta t$ equal to 30 seconds, obtaining, however, better results for 10 second steps.

For thoroughness, results from the model proposed in this article are compared, for the following five scenarios (Fig.7), with results from a previous implicit method (Astarita and Guido, 2002):

Scenario A: lane drop in the last cell of the simulated motorway;
Scenario B: reduced capacity caused by an incident in the middle of the simulated road.
Scenario C: bad visibility conditions in 3 cells with reduced free speed.
Scenario D: lane drop and reduced speed in the middle of the simulated motorway due to a work zone.
Scenario E: increase in traffic demand for an exceptional event.

Fig.7: Comparison of results produced from the various methodologies of estimation (density)
5. Conclusion

This paper presents a new methodology for traffic parameter estimation based on new technologies, the result of the interaction between the classical theory of traffic flow and the application of data transmissions technologies to the transportation systems.

The results produced from the proposed model result from the combination of traditional traffic counts with the counting of instrumented vehicles on specific sections of the network.

The results of the new methodology appear promising with minimal differences between estimated density values and observed density values.

The precision of the estimate improves with an increase in the ratio between instrumented vehicles (mobile phone on board) and total vehicles on the network.

The scenarios analyzed represent only some of the possible traffic conditions that can occur, but the proposed model can be applied to any type of road configuration.

The methodology proposed in this paper assumes that propagation of the concentration of instrumented vehicles only happens in the same direction of the flow, but considering particular conditions at the boundaries of the final cell of the network, it is possible to take advantage of information to use mixed propagations.

The mobile communication trend, in Italy as in nearly all the industrialized countries, is clearly rising and the directives are supplied, also in Italy, from a recent “Piano Nazionale dei Trasporti” which proposes data transmission to be increasingly present in applications for the study and management of the transport system.

Models such as the one proposed in this article can be useful for analysis and simulation of multiple situations on our roads. Customers and managers of the transportion system would find, in the application of these new methodologies, a valid contribution to the resolution of problems relative to traffic.

Further research efforts should be devoted to an application of the methodology to real traffic data on an Italian toll motorway, when these data will be available and to address other possible numerical schemes and different boundary conditions.

In this paper only the main aspects of the problem have been presented opening the road for additional research.

Motorway traffic parameter estimation from mobile phone counts is a combination of traditional traffic flow theory with new technologies and it is one of the new techniques
that can be obtained by telematics applied to transportation systems. Many ITS have been applied and designed in recent years, but the great upcoming challenge is to combine different sources of information, owned by different entities, and to extract what may be relevant to various control systems, with the aim of supplying better, more efficient transportation services to road travelers.

6. References


