Adaptive Tracking Control Using Synthesized Velocity from Attitude Measurements*

H. Wong,† M.S. de Queiroz,‡‡† and V. Kapila†

†Department of Mechanical Engineering Polytechnic University
6 Metrotech Center, Brooklyn, NY 11201
hwong01@utopia.poly.edu, vkapila@poly.edu

‡ Mechanical Engineering Department Louisiana State University
Baton Rouge, LA 70803-6413

†Corresponding author.

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Abstract

The attitude tracking control problem of uncertain rigid spacecraft without angular velocity measurements is addressed in this paper. The adaptive control law, which incorporates a velocity-generating filter from attitude measurements, is shown to ensure the asymptotic convergence of the attitude and angular velocity tracking errors despite unknown spacecraft inertia. Simulation results are presented to illustrate the theoretical results.

1 Introduction

The attitude control of rigid bodies has important applications ranging from rigid aircraft and spacecraft systems to coordinated robot manipulators (see [19] for a literature review of such applications). Rigid spacecraft applications in particular often require highly accurate slewing and pointing maneuvers of large angle amplitudes. As noted in [1], these requirements necessitate the use of the nonlinear dynamic spacecraft model for control system synthesis. Further complications arise from uncertain spacecraft mass and inertia properties due to fuel consumption, payload variation, appendage deployment, etc.

The attitude motion of a rigid body is represented by a set of two vector equations [7, 8, 19] – Euler’s dynamic equation, which describes the time evolution of the angular velocity vector, and the kinematic equation, which relates the time derivatives of the orientation angles to the angular velocity vector. Several kinematic parametrizations exist to represent the orientation angles, including singular three-parameter representations (e.g., Euler angles, Gibbs vector, Cayley-Rodrigues parameters, and Modified Rodrigues parameters) and the nonsingular four-parameter representation given by the unit quaternion (Euler parameters). Whereas the three-parameter representations

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†Corresponding author.
always exhibit singular orientations (i.e., the Jacobian matrix in the kinematic equation is singular for some orientations), the unit quaternion globally represents the spacecraft attitude without singularities; however, an additional constraint equation is introduced with the four parameters.

Solutions to the attitude control problem have been presented in the literature since the early 1970’s [12]. See [19] for a comprehensive literature review of earlier work. In [19], the authors designed a class of attitude tracking control laws which include PD, model-based, and adaptive controllers. Adaptive tracking control schemes were presented in [15, 17] to compensate for the unknown spacecraft inertia matrix. In [20], an adaptive tracking controller was developed which guaranteed the inertia matrix identification under certain richness conditions. Recently, [1] proposed an adaptive attitude tracking controller that identified the inertia matrix via periodic command signals. The problem solved in [1] was later extended to the angular velocity tracking problem in [2]. An $H_\infty$-suboptimal state feedback controller was developed in [4]. In [9], the authors designed an inverse optimal control law for attitude regulation using the backstepping method.

A typical feature in all the above-mentioned attitude control schemes is the requirement of angular velocity measurements. Unfortunately, this requirement is not always satisfied in reality. Thus, a common practice is to approximate the angular velocity signal through an ad-hoc numerical differentiation of the attitude angles. With this in mind, an angular velocity observer was developed in [14]; however, the observer was based on an unproven separation principle argument. In [11], a passivity approach was utilized to develop an asymptotically stabilizing, proportional-“dirty” derivative setpoint controller, which eliminated velocity measurements via the filtering of the unit quaternion. The passivity-based, velocity-free setpoint controller of [11] was later applied to the three-parameter kinematic representation problem in [18].

In this paper, we provide a solution to the adaptive attitude tracking control problem without angular velocity measurements using the Modified Rodrigues parameter kinematic representation. In particular, we exploit the control designs proposed in [3, 6] for robot manipulators to formulate an adaptive output feedback control law that compensates for unknown spacecraft inertia. The output feedback nature of the controller stems from the use of a velocity estimator, with a high-pass filter form that is motivated by the Lyapunov-type analysis, which generates a velocity-related signal from attitude measurements. The overall structure of the control law is formed by a desired compensation adaptation law-type [13] term and a proportional-“dirty” derivative-type feedback term. The proposed controller is shown to guarantee the asymptotic convergence of the attitude and angular velocity tracking errors.

## 2 Rigid Spacecraft Model

The mathematical model of a rigid spacecraft is given by the following equations [7, 8]

\[
\begin{align*}
\dot{\omega} &= -\omega \times J \omega + \tau = -S(\omega)J \omega + \tau \\
\dot{q} &= T(q)\omega
\end{align*}
\]  

(1)

where (1) represents the rigid body dynamics and (2) is the kinematic equation. In (1), $\omega(t) \in \mathbb{R}^3$ is the angular velocity of the spacecraft in a body-fixed frame, $J \in \mathbb{R}^{3 \times 3}$ is the spacecraft’s constant inertia matrix, $\tau(t) \in \mathbb{R}^3$ is the control torque input, and $S \in \mathbb{R}^{3 \times 3}$ is the skew-symmetric matrix representing the cross product operation

\[
S(a) \triangleq [a \times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad \forall a \in \mathbb{R}^3.
\]  

(3)
In (2), \( q(t) \in \mathbb{R}^3 \) represents the Modified Rodrigues parameters \([16]\) describing the spacecraft attitude with respect to an inertial frame, given by

\[
q(t) = \hat{\kappa}(t) \tan \left( \frac{\phi(t)}{4} \right), \quad \phi \in [0, 2\pi) \text{ rad}
\]  

(4)

where \( \hat{\kappa}(t) \in SO(2) \) \([7]\) and \( \phi(t) \in \mathbb{R} \) are the Euler eigenaxis and eigenangle, respectively.\(^1\) The Jacobian matrix \( T \in \mathbb{R}^{3 \times 3} \) for the Modified Rodrigues parameters is given by \([16]\)

\[
T(q) = \frac{1}{2} \left[ \left( \frac{1 - q^T q}{2} \right) I_{3 \times 3} + S(q) + qq^T \right].
\]  

(5)

To facilitate the subsequent control formulation, we combine (1) and (2) to form the following second-order, nonlinear dynamic equation \([17]\)

\[
J^*(q) \ddot{q} + C^*(q, \dot{q}) \dot{q} = P^T(q) \tau
\]  

(6)

where the matrices \( P, J^*, C^* \in \mathbb{R}^{3 \times 3} \) are defined as

\[
P(q) \triangleq T^{-1}(q), \quad J^*(q) \triangleq P^T J P, \quad C^*(q, \dot{q}) \triangleq -J^* \dot{P} - P^T S(JP \dot{q}) P.
\]  

(7)

Note from (3), (5), and (7) that \( J^* \) and \( C^* \) are bounded provided their arguments are bounded. The dynamic model of (6) shares similar properties with the rigid-link robot manipulator dynamics \([10]\). Specifically, the inertia matrix \( J^*(q) \) is positive-definite and symmetric, and can be bounded as follows

\[
J_1 ||x||^2 \leq x^T J^*(q) x \leq J_2(||q||)||x||^2, \quad \forall x \in \mathbb{R}^3
\]  

(8)

where \( J_1 \) is a positive constant, and \( J_2 (||q||) \) is a positive nondecreasing function. Also, the matrices \( J^* \) and \( C^* \) satisfy the following skew-symmetric relationship (see \([17]\) for proof)

\[
x^T \left( \frac{1}{2} J^* - C^* \right) x = 0, \quad \forall x \in \mathbb{R}^3.
\]  

(9)

Finally, the left-hand side of (6) is linear in the parameters in the sense that

\[
J^*(x) \dot{x} + C^*(x, \dot{x}) \dot{x} = W(x, \dot{x}, \ddot{x}) \theta, \quad \forall x \in \mathbb{R}^3
\]  

(10)

where \( W \in \mathbb{R}^{3 \times 6} \) is the regression matrix, and \( \theta \in \mathbb{R}^6 \) is the constant parameter vector given by

\[
\theta \triangleq [J_{11} \quad J_{22} \quad J_{33} \quad J_{12} \quad J_{13} \quad J_{23}]^T
\]  

(11)

with \( J_{ij} \) being the \( ij \)-th component of the spacecraft inertia matrix \( J \) defined in (1). The construction of \( W \) in (10) is facilitated if one notices that \([1]\)

\[
J \nu = \Lambda(x) \theta, \quad \forall x = [x_1, x_2, x_3]^T \in \mathbb{R}^3
\]  

(12)

where \( \Lambda \in \mathbb{R}^{3 \times 6} \) is defined as

\[
\Lambda(x) \triangleq \begin{bmatrix}
x_1 & 0 & 0 & x_2 & x_3 & 0 \\
0 & x_2 & 0 & x_1 & 0 & x_3 \\
0 & 0 & x_3 & 0 & x_1 & x_2
\end{bmatrix}.
\]  

(13)

\(^1\)Note that the modified Rodrigues parameters allows for eigenaxis rotations up to 360°, i.e., a kinematic singularity will only occur when \( \phi = 2\pi \) rad.
3 Problem Statement

The attitude control problem in this paper is to design the control input $\tau$ such that $\lim_{t \to \infty} q(t) = q_d(t)$, where $q_d(t) \in \mathbb{R}^3$ denotes a desired attitude trajectory. To facilitate the control development, we make the following technical assumptions regarding the desired trajectory: i) $q_d(t)$ and its first three time derivatives are bounded, and ii) $q_d(t)$ is constructed to avoid the kinematic singularity associated with the Modified Rodrigues parameters.

The above control objective is to be met under the constraints of i) no direct velocity feedback (i.e., $\dot{q}$ or $\omega$ are not measured) and ii) no knowledge of the spacecraft inertia parameters (i.e., $\theta$ defined in (11) is unknown). The control objective will be quantified by the attitude tracking error $e(t) \in \mathbb{R}^3$, defined as

$$e(t) \triangleq q_d(t) - q(t). \quad (14)$$

Since $\theta$ is unknown, the subsequent controller will contain an adaptation law that generates a dynamic parameter estimate $\hat{\theta}(t) \in \mathbb{R}^6$. The mismatch between the actual and estimated parameters is defined as

$$\tilde{\theta}(t) \triangleq \theta - \hat{\theta}(t) \quad (15)$$

where $\tilde{\theta}(t) \in \mathbb{R}^6$ denotes the parameter estimation error. Furthermore, if $\omega_d(t) \in \mathbb{R}^3$ is the desired angular velocity of the spacecraft in the body-fixed frame, defined from (2) as

$$\omega_d \triangleq T^{-1}(q_d)\dot{q}_d, \quad (16)$$

then an angular velocity tracking error in the body-fixed frame, $\tilde{\omega}(t) \in \mathbb{R}^3$, can be defined as

$$\tilde{\omega}(t) \triangleq \omega_d(t) - \omega(t). \quad (17)$$

4 Adaptive Control Design

4.1 Velocity Filter Formulation

Since the velocity signal $\dot{q}$ is not measured, a filter will be introduced to generate a velocity tracking error-related signal from attitude measurements (i.e., the filter attempts to capture the behavior of $\dot{e}$). The filter is given by the following dynamic relationship [3]

$$e_f = -ke + p, \quad \dot{p} = -(k + 1)p + (k^2 + 1)e, \quad p(0) = ke(0) \quad (18, 19)$$

where $e_f(t) \in \mathbb{R}^3$ is the filter output, $p(t) \in \mathbb{R}^3$ is an auxiliary filter variable, and $k$ is a positive scalar gain defined as

$$k = 1 + k_s \quad (20)$$

with $k_s$ being an additional positive scalar gain to be specified later.

To provide some insight on the design of the above filter, while also motivating the subsequent controller design, we now derive the dynamics of the filter output $e_f$. To do this, we first differentiate (18) with respect to time and then use (19) to substitute for $\dot{p}$ to produce

$$\dot{e}_f = -k\dot{e} - (k + 1)p + (k^2 + 1)e. \quad (21)$$
After solving for $p$ from (18) and substituting the resulting expression into (21), we obtain the dynamics for $e_f$ as follows

$$\dot{e}_f = -e_f - k\eta + e$$  \hspace{1cm} (22)$$

where $\eta(t) \in \mathbb{R}^3$ is an auxiliary error variable defined as

$$\eta = \dot{e} + e_f + e.$$  \hspace{1cm} (23)$$

The signal $\eta$ will be the basis for the formulation of the controller. In addition, note that (23) can be rearranged as

$$\dot{e} = -e + \eta - e_f$$  \hspace{1cm} (24)$$

to give the dynamics for $e$.

### 4.2 Torque Input Formulation

We begin the control formulation by developing the open-loop dynamics for $\eta$. We take the time derivative of (23) and pre-multiply the resulting expression by $J^*$ to yield

$$J^*\dot{\eta} = J^*\ddot{q}_d + C^* (\dot{q}_d - \dot{e}) - k_s J^* \eta - 2J^* e_f - P^T \tau$$  \hspace{1cm} (25)$$

where (6), (20), (22), and (24) have been used, and $\dot{q}$ has been replaced by $\dot{q}_d - \dot{e}$ in accordance with the time derivative of (14). Based on (10), we now define a desired linear parametrization as follows

$$W_d(t) \theta \triangleq J^* (q_d) \ddot{q}_d + C^* (q_d, \dot{q}_d) \dot{q}_d.$$  \hspace{1cm} (26)$$

Adding and subtracting (26) to the right-hand side of (25) and substituting (24) for $\dot{e}$, we can rewrite the open-loop dynamics of $\eta$ as

$$J^*\dot{\eta} = -C^* \eta - k_s J^* \eta + W_d \theta + \chi - P^T \tau$$  \hspace{1cm} (27)$$

where the auxiliary variable $\chi \in \mathbb{R}^3$ is defined as

$$\chi(q, \dot{q}, e_f, t) \triangleq J^* (q) \ddot{q}_d + C^* (q, \dot{q}) \dot{q}_d - J^* (q_d) \ddot{q}_d - C^* (q_d, \dot{q}_d) \dot{q}_d + C^* (q, \dot{q}) (e + e_f) - 2J^* (q) e_f.$$  \hspace{1cm} (28)$$

Based on the structure of (27) and the subsequent stability analysis, we propose the following adaptive controller for the torque input

$$\tau = T^T (W_d \dot{\theta} - k e_f + e)$$  \hspace{1cm} (29)$$

where $k$ was defined in (20), and the dynamic parameter estimate $\dot{\theta}(t)$ is updated by the following adaptation rule

$$\dot{\theta}(t) = \Gamma \int_0^t W_d^T (\sigma) (e(\sigma) + e_f(\sigma)) d\sigma + \Gamma W_d^T e - \Gamma \int_0^t \dot{W}_d^T (\sigma) e(\sigma) d\sigma$$  \hspace{1cm} (30)$$

with $\Gamma \in \mathbb{R}^{6 \times 6}$ being a diagonal, positive-definite, adaptation gain matrix. After substituting (29) into (27), we obtain the following closed-loop dynamic equation for $\eta$

$$J^*\dot{\eta} = -C^* \eta - k_s J^* \eta + W_d \ddot{\theta} + k e_f - e + \chi$$  \hspace{1cm} (31)$$

where (15) has been utilized.
Remark 1: Note that an upper bound can be placed on the variable $\chi$ of (28) as follows (see [6] for a similar proof)
\[
\|\chi\| \leq \rho(\zeta_{dp}, \zeta_{dv}, \zeta_{da}, \|y\|) \|y\| \tag{32}
\]
where $\zeta_{dp}, \zeta_{dv}, \zeta_{da}$ are positive constants defined as
\[
\|q_d(t)\| \leq \zeta_{dp}, \quad \|\dot{q}_d(t)\| \leq \zeta_{dv}, \quad \|\ddot{q}_d(t)\| \leq \zeta_{da}, \tag{33}
\]
$\rho(\cdot)$ is some positive nondecreasing function, and $y \in \mathbb{R}^9$ is defined as
\[
y \triangleq \begin{bmatrix} e^T & e_f^T & \eta^T \end{bmatrix}^T. \tag{34}
\]

5 Stability Analysis

The adaptive control law described by (18), (19), (29), and (30) ensures the asymptotic stability of the position and velocity tracking errors as delineated by the following theorem.

Theorem 1: Let the gain $k_s$ of (20) be given by
\[
k_s = \frac{1}{J_1} (1 + k_n) \tag{35}
\]
where $J_1$ was defined in (8) and $k_n$ is an additional gain. If $k_n$ is selected sufficiently large such that
\[
k_n \geq \frac{1}{4} \rho^2(\zeta_{dp}, \zeta_{dv}, \zeta_{da}, \sqrt{\frac{\lambda_2(q(0))}{\lambda_1}} \|z(0)\|), \tag{36}
\]
then
\[
\lim_{t \to \infty} e(t), \dot{e}(t) = 0 \quad \text{and} \quad \lim_{t \to \infty} \tilde{\omega}(t) = 0 \tag{37}
\]
where $\rho(\cdot)$ and $\tilde{\omega}$ were defined in (32) and (17), respectively, $z \in \mathbb{R}^{15}$ is defined as
\[
z \triangleq \begin{bmatrix} e^T & e_f^T & \eta^T & \tilde{\theta}^T \end{bmatrix}^T, \tag{38}
\]
and
\[
\lambda_1 \triangleq \frac{1}{2} \min \left\{ 1, j_1, \lambda_{\min}(\Gamma^{-1}) \right\} > 0, \quad \lambda_2(q) \triangleq \frac{1}{2} \max \left\{ 1, j_2(q), \lambda_{\max}(\Gamma^{-1}) \right\} > 0 \tag{39}
\]
with $j_2(q)$ being defined in (8). Note that $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively.

Proof. We define the non-negative function
\[
V(e, e_f, \eta, \tilde{\theta}) \triangleq \frac{1}{2} e^T e + \frac{1}{2} e_f^T e_f + \frac{1}{2} \eta^T J^* \eta + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{40}
\]
which satisfies the following bounds
\[
\lambda_1 \|y\|^2 \leq \lambda_1 \|z\|^2 \leq V \leq \lambda_2(q) \|z\|^2 \tag{41}
\]
where $y$, $z$, $\lambda_1$, and $\lambda_2$ were defined in (34), (38), and (39). Differentiating (40) along (22), (24), and (31), and noting from the definitions of (15) and (23) that $\dot{\tilde{\theta}} = -\Gamma W_d^T \eta$, we get
\[
\dot{V} = -e^T e - e_f^T e_f + \eta^T (-k_s J^* \eta + \chi) \tag{42}
\]
where (9) has been used. Utilizing (8), \( \dot{V} \) can be upper bounded by
\[
\dot{V} \leq -\|e\|^2 - \|e_f\|^2 - k_{s,1} \|\eta\|^2 + \|\eta\| \|\chi\|
\] (43)
which, after the use of (32) and (35), becomes
\[
\dot{V} \leq -\|e\|^2 - \|e_f\|^2 - \|\eta\|^2 + \left[\rho(\|y\|) \|y\| \|\eta\| - k_n \|\eta\|^2\right].
\] (44)
Applying the nonlinear damping argument [5] to the bracketed term of (44) produces
\[
\dot{V} \leq -\|y\|^2 + \rho^2(\|y\|) \|y\|^2
\] (45)
where (34) has been used. Due to the nondecreasing nature of \( \rho(\cdot) \), (45) and (41) can be used to further upper bound \( \dot{V} \) as
\[
\dot{V} \leq -\beta \|y\|^2,
\] (46)
where \( \beta \) is some positive constant; hence,
\[
V(e(t), e_f(t), \eta(t), \hat{\theta}(t)) = V(z(t)) \leq V(z(0)), \quad \forall t \geq 0
\] (48)
where (38) has been used. From (48), a sufficient condition for (47) is given by
\[
\dot{V} \leq -\beta \|y\|^2 \quad \text{for} \quad k_n \geq \frac{1}{4} \rho^2\left(\sqrt{\frac{\lambda_1}{\lambda}}\right); \quad (49)
\] hence, by using (41), we can obtain a sufficient condition for (49) as follows
\[
\dot{V} \leq -\beta \|y\|^2 \quad \text{for} \quad k_n \geq \frac{1}{4} \rho^2\left(\sqrt{\frac{\lambda_2(q(0))}{\lambda_1}} \|z(0)\|\right). \quad (50)
\]
From (41) and (50), it is clear that \( y(t) \in \mathcal{L}_\infty \). Standard signal chasing arguments can be used to show that all other signals remain bounded. This information can then be utilized to illustrate that \( \dot{y}(t) \in \mathcal{L}_\infty \). From (50), it is easy to show that \( y(t) \in \mathcal{L}_2 \); hence, we can now use Barbalat’s Lemma [5, 17] to conclude that
\[
\lim_{t \to \infty} y(t) = 0 \quad \text{for} \quad k_n \geq \frac{1}{4} \rho^2\left(\sqrt{\frac{\lambda_2(q(0))}{\lambda_1}} \|z(0)\|\right) \quad (51)
\] which implies from (34) and (24) that
\[
\lim_{t \to 0} e(t), \dot{e}(t) = 0 \quad \text{for} \quad k_n \geq \frac{1}{4} \rho^2\left(\sqrt{\frac{\lambda_2(q(0))}{\lambda_1}} \|z(0)\|\right). \quad (52)
\]
To prove the second result of (37), notice from (2), (14), and (16) that the velocity tracking \( \dot{e} \) can be written as
\[
\dot{e} = \ddot{q}_d - \dot{q} = T(q_d)\omega_d - T(q)\omega.
\] (53)
\footnote{To reduce the notational complexity in the following derivations, the dependency of \( \rho(\cdot) \) on the constants \( \zeta_{dp}, \zeta_{dv}, \zeta_{da} \) will be omitted from the function’s argument.}
\footnote{Note that \( T(q_d) \) is bounded since \( q_d \) is constructed to avoid the kinematic singularity.
After adding and subtracting the term $T(q_d)\omega$ to the right-hand side of (53), we get

$$\dot{\omega} = T^{-1}(q_d)\dot{e} - T^{-1}(q_d)(T(q_d) - T(q))\omega$$

(54)

where (17) has been used. From (52), we know that

$$\lim_{t \to \infty} q(t) = q_d(t) \implies \lim_{t \to \infty} [T(q_d(t)) - T(q(t))] = 0;$$

(55)

hence, the right-hand side of (54) goes to zero asymptotically, which indicates that

$$\lim_{t \to \infty} \tilde{\omega}(t) = 0 \quad \text{for} \quad k_n \geq \frac{1}{4} \rho^2(\sqrt{\frac{\lambda_2(q(0))}{\lambda_1}}|\|z(0)|||). \quad \Box$$

(56)

6 Simulation Results

The adaptive control law given by (18), (19), (29), and (30) was simulated for the rigid spacecraft dynamics of (1) and (2) with the following inertia matrix [1]

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix} \text{kg} \cdot \text{m}^2,$$

(57)

and the initial conditions set to $q(0) = 0$ rad and $\omega(0) = 0$ rad/sec. The desired attitude trajectory, $q_d(t) \equiv \hat{\kappa}_d(t) \tan \left(\frac{\phi_d(t)}{4}\right)$ with $\hat{\kappa}_d(t) \in SO(2)$ and $\phi_d(t) \in \mathbb{R}$, was selected as

$$\hat{\kappa}_d(t) = \frac{1}{2} \begin{bmatrix} \cos (0.2t) & \sin (0.2t) & \sqrt{3} \end{bmatrix}^T, \quad \phi_d(t) = \pi \text{ rad.}$$

(58)

The control and adaptation gains in the following simulations were selected by trial-and-error until a good tracking performance was obtained. This procedure resulted in the following values for the gains: $k = 50$ and $\Gamma = \text{diag\{5, 5, 5, 10, 10\}} \times 10^4$. The parameter estimates were initialized to zero (i.e., $\hat{\theta}(0) = 0$). Figures 1 and 2 show the attitude tracking error $e(t)$ and angular velocity tracking error $\tilde{\omega}(t)$, respectively. The components of the parameter estimate vector $\hat{\theta}(t)$ are depicted in Figure 3, where the top plot contains the estimates of the diagonal terms of the inertia matrix (i.e., $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$) and the bottom plot contains the estimates of the off-diagonal terms (i.e., $\hat{\theta}_4, \hat{\theta}_5, \hat{\theta}_6$). It is interesting to note that, for the desired trajectory of (58), the off-diagonal estimates converged to their corresponding, actual parameter values shown in (57). The control torque input $\tau(t)$ is shown in Figure 4.

7 Conclusion

In this paper, we developed an adaptive tracking control strategy for rigid spacecraft. The controller required no angular velocity measurements nor knowledge of the spacecraft inertia matrix. A high-pass filter was used to generate a pseudo-velocity tracking error signal, while a gradient-type, adaptive update law accounted for the inertia uncertainty. The controller was shown to ensure asymptotic attitude and angular velocity tracking. Simulation results illustrated the controller performance.
References


Figure 1: Attitude tracking error $e(t)$
Figure 2: Angular velocity tracking error $\tilde{\omega}(t)$

Figure 3: Parameter estimates $\hat{\theta}(t)$
Figure 4: Control torque input $\tau(t)$