Femtocell Access Control in the TDMA/OFDMA Uplink

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Abstract—This paper investigates open vs. closed access in the uplink of femtocell networks, for the particular case of orthogonal multiple access protocols like TDMA or OFDMA. Open access reduces near-far interference and provides an inexpensive way to expand the capacity of the operator’s network. Additionally, with a cap on the amount of resources allocated to the cellular users, open access ensures the femtocell owner still monopolizes a large portion of the femtocell’s capacity and backhaul. Seemingly open access is the appropriate approach for both two parties. We show mathematically and through simulations that the reality is more complicated and depends heavily on cellular user density: for example, closed access is typically preferable at high user densities in orthogonal multiple access. The results of this paper suggest that OFDMA or TDMA femtocells should adapt their access mechanism to the average cellular user density, which changes slowly.

I. INTRODUCTION

Femtocell access points (FAPs) are short-range low-power and extremely low-cost home base stations with third party backhaul (e.g. DSL or cable modem). They are usually deployed and controlled by end-users who desire better indoor signal transmission and reception [1]. FAPs are attractive not only as home base stations, but as an essential component of 3G and 4G networks: they extend high quality coverage inside peoples’ houses and would appear to improve the network capacity in a much cheaper way than normal base stations.

Despite FAPs promise, many concerns still remain, especially cross-tier interference [2], [3]. Macrocell users nearby, but not allowed access to the FAPs (i.e. in femtocell closed access), can cause significant macro-to-femto interference in the uplink and likewise suffer strong femtocell interference in the downlink. These affects are akin to the well known near-far problem, and endanger the successful co-existence of two-tier networks. Thus femtocell open access is proposed and under intense investigation [4], [5]. Seemingly open access is the preferred approach: by allowing nearby cellular users to use the femtocell, it eliminates strong interference experienced by the femtocell and expands the operator’s network capacities as well. However, the precise tradeoffs of open vs. closed access have not been carefully scrutinized, and in particular there is a shortage of analytical models and results on the topic.

A. Related Work

Initial simulation-centric studies on femtocell access control are accomplished by the 3GPP RAN 4 group [6]–[8], which all show that with adaptive open access, the interference in two-tier networks is mitigated and the deployment of co-channel femtocells becomes feasible. However, open access can possibly starve the subscribers since it allows nearby visitors to share the limited femtocell resources, especially the backhaul capacity. Thus it is necessary to incorporate femtocell resource allocation into closed and open access.

The increased handover frequency in open access is a possible challenge to its implementation. A technique combining intracell handovers with power control was proposed in [5], and a hybrid access model – open access with a cap on the amount of resources allocated to the cellular users – was simulated in [4]. Both of these approaches substantially reduce the number of handovers in open access while mitigating the cross-tier interference. In this paper, we simply call the hybrid access model open access, since our open access approach has an upper limit of K users.

B. Contributions

This paper evaluates the relative merits of open and closed access in the uplink of femtocell-overlaid cellular networks, from the viewpoint of both the femtocell owner (measured by owner’s achieved rate) and the network operator (cellular users’ sum throughput). We derive the cumulative distribution function (CDF) for uplink cross-tier interference in orthogonal multiple access schemes (TDMA or OFDMA). The capacity tradeoffs for both the femtocell owner and cellular users are then presented. In TDMA or OFDMA, the preferences of the femtocell owner and the network operator highly depends on cellular user density. Thus, for 4G networks (LTE & WiMAX) that use OFDMA, femtocell access control should be a function of the cellular user density. Note that the authors also have studied open vs. closed access for non-orthogonal multiple access (i.e. CDMA), which results in the more simple conclusion that open access is always preferred [9].

II. SYSTEM MODEL

We consider a macrocell area of radius R, inside which N cellular users, denoted as $U_1, U_2, \ldots, U_N$, are uniformly and independently located. Since the home user is transmitting and receiving inside the house, we could assume it is located at a deterministic position, with a distance of $d$ from the FAP. Femtocell-to-femtocell interference has been neglected.
for reasons of analytical tractability. However, because uplink femtocell transmissions typically originate and terminate indoors and are of low power, their contribution to the overall interference is expected to be negligible compared to the more numerous and high power outdoor (macro-cellular) users.

A. Channel Model and Interference

Considering the path loss effects, our channel model is given by ($\alpha$ is larger than $\beta$ to incorporates wall penetration)

\[
H(|x|) = \begin{cases} 
|x|^{-\beta} & \text{indoor transmission} \\
|x|^{-\alpha} & \text{outdoor & cross-wall transmission}
\end{cases}
\]

(1)

Here, $|x|$ is the distance from the transmitter to the respective base station. In the channel model, we ignore short-term fading effects, since it does not have a large effect in a wideband system with sufficient diversity, e.g. multi-antenna diversity or distributed subcarrier allocation (OFDMA).

Definition 1: For $U_j \in \{U_0, \ldots, U_N\}$, its interference factor $I_j$ is defined as $h_j|g_j|$, where $h_j$ and $g_j$ are its channels to the FAP and macrocell BS respectively. $\{I_1, I_2, \ldots, I_N\}$ are i.i.d. random variables, and we define their ordered statistics as

\[
I_{(1)} = \min(I_1, \ldots, I_N), \ldots, I_{(N)} = \max(I_1, \ldots, I_N)
\]

Cellular users are reordered as $\{U_{(1)}, \ldots, U_{(N)}\}$, correspondingly.

Assumption 1: We assume $I_0 \geq I_{(N)}$ holds, because the home user is closer to the FAP, and the indoor channel has a smaller path loss exponent.

Assumption 2: We assume that there is no coordination between the FAP and the macrocell BS, nor between different FAPs, in terms of power control or resource scheduling.

Denote $P_c$ and $P_f$ as the received power at the macrocell BS and the FAP respectively. User $U_j$ in the macrocell (femtocell) will cause interference of $P_cI_j$ ($P_f/I_j$) to the FAP (the macrocell BS).

The following lemma is stated without proof (see [10] for the proof).

Lemma 1: The cumulative distribution function for the interference factor of each cellular user is

\[
F_I(i) = \begin{cases} 
\left(\frac{r}{R}\right)^2 & 0 \leq i < \left(\frac{R}{R-D}\right)^\alpha \\
\left(\frac{R-R-D}{R-D}\right)^\alpha & i = 1 \\
\left(\frac{R-D}{R-D}\right)^\alpha & 1 < i \leq \left(\frac{R}{R-D}\right)^\alpha \\
1 - \frac{L(i)}{\pi} & \frac{1}{1-\frac{2}{\sqrt{\alpha}}} \leq \frac{2}{\sqrt{\alpha}} \leq \pi, \\
1 - \left(\frac{1}{r/R}\right)^2 & \frac{1}{1-\frac{2}{\sqrt{\alpha}}} < i
\end{cases}
\]

(2)

where $r, L(i), \varphi$ are given by:

\[
r = \frac{1}{1-\frac{2}{\sqrt{\alpha}}} D \quad \varphi = \arccos \left(\frac{D}{2R}\right),
\]

\[
L(i) = \frac{\pi}{\pi R^2} \left(\theta - 0.5 \sin 20\right)r^2 + (\phi - 0.5 \sin 2\phi)R^2,
\]

where $x_e = \frac{D^2}{2(1-2\alpha)}$, $\theta = \arccos \left(\frac{r^2 + x_e^2 - R^2}{2r x_e}\right)$ and $\phi = \arccos \left(\frac{x_e^2 + R^2 - r^2}{2x_e R}\right)$.

B. Handover Metric in Open Access

Suppose the maximum number of additional cellular users that the FAP can serve is $K$. A typical handover metric in open access is given in below.

Assumption 3: When cellular users cause outage to the home user, the FAP picks the most noisy interferer from the macrocell to serve. This hand over procedure continues as long as the home user still experiences outage and the number of handed over cellular users does not exceed $K$.

This assumption meets the interests of both the femtocell owner and the network operator by maximally reducing the interference. Additionally, it reduces the overheads signaling as stated in [4].

C. Resource Allocation and Uplink Rate

Macrocell BS. We assume the macrocell BS fairly allocates the time resources among its users, since they have the same rate requirement (stated below) and are i.i.d. located.

Femtocell BS. Since the FAP needs to differentiate the subscriber from the visitors, it is not necessarily to fairly allocate its resources, i.e. the time resources and backhaul capacity (the FAP backhaul capacity, denoted as $C_b$, is often modest and thus possibly constrains its users’ rates). In general as the FAP serves additional $L$ ($0 \leq L \leq K$) cellular users, the home user (each of the $L$ cellular users) is allocated with a portion $\lambda_L$ ($\mu_L$) of the time resources and backhaul capacity.

Assumption 4: Each macrocell user has a rate requirement $C$, and each femtocell user has a rate requirement of $\min(C, \lambda_L C_b)$, where $C_b$ is its allocated backhaul capacity. The user achieves its required rate when the received SIR at or above its corresponding SIR target. Otherwise it is in outage and the rate is zero.

Definition 2: The event $A_L = \{L \in \{0, 1, 2, \ldots, K\}\}$ is defined as the FAP provides service to $L$ additional cellular users. In event $A_L$, denote the SIR targets of the home user, handed over cellular users and the remaining macrocell users as $\Gamma_{f,L}, \Gamma_{h,L}$ and $\Gamma_{c,L}$ respectively. Their success probabilities are denoted as $p_{f,L}, p_{h,L}$ and $p_{c,L}$ accordingly.

In this paper, we evaluate open vs. closed access from the viewpoint of the femtocell owner – the home user’s ergodic rate $C_0 = \sum_{L=0}^{K} \min(C, \lambda_L C_b) p_{f,L}$, and the viewpoint of the network operator – cellular users’ sum throughput $C_{sum} = \sum_{L=0}^{K} [(N - L) C_{p,c,L} + L \min(C, \mu_L C_b) p_{h,L}]$.

III. CAPACITY CONTOURS

In this section, we analyze a TDMA scenario, which can also be viewed as OFDMA on a per subband basis, since each subband is orthogonal and allocated in a TDMA fashion.

Theorem 1: The capacity contours in closed access (i.e. the FAP does not serve cellular users) are given by

\[
C_0 = \min(C, \lambda_0 C_b) F_I \left(\frac{P_f}{P_f + \Gamma_{f,0}}\right)
\]

\[
C_{sum} = NCP \left(\frac{P_c}{P_f + \Gamma_{f,0}} \geq \Gamma_{c,0}\right)
\]

(3)
Proof: Each cellular user causes interference to the home user during its time slot, namely $1/N$, the ergodic rate of the home user is

$$C_0 = \min(C, \lambda_0 C_b) \sum_{j=1}^{N} \frac{1}{N} \mathbb{P} \left( \frac{P_f}{P_c I_{f,j}} \geq \Gamma_{f,0} \right)$$

$$= \min(C, \lambda_0 C_b) F_I \left( \frac{P_f}{P_c I_{f,0}} \right) \tag{4}$$

On the other hand, each cellular user experiences an interference of $P_f/I_0$ from the home user. Their sum rate is

$$C_{sum} = NC p_{c,0} = NCP \left( \frac{P_c}{P_f / I_0} \geq \Gamma_{c,0} \right) \tag{5}$$

Remark 1: In TDMA the interference is time shared, so the average outage probability of the home user and consequently its ergodic rate are not scaled by $N$ – the goal of closed access.

The SIR target of macrocell users (in both open and closed access) is an increasing function of their density. As more macrocell users added, each of them has a smaller time fraction and must increase its SIR target to achieve a given rate requirement.

In closed access, since the received SIR of a cellular user in the macrocell is a constant value of $P_f/I_0$, there is a cutoff user loading $N_c^*$, such that: 1) when $N < N_c^*$, each cellular user’s SIR target constraint is satisfied and $C_{sum} = NC$; 2) when $N \geq N_c^*$, each cellular user’s SIR target is infeasible and $C_{sum} = 0$. The value of $N_c^*$ is governed by the inequality $\frac{P_f}{P_f / I_0} \leq \Gamma$ (SIR target), which for example in a Gaussian channel is $N_c^* = \left\lceil \frac{1}{\log_2(1 + \frac{P_f}{P_f / I_0})} \right\rceil$.

Remark 2: The network operator prefers closed access when $N < N_c^*$, while embracing open access when $N \geq N_c^*$.

From Assumption 3, FAP will pick the strongest interferers sequentially in open access, so

$$\{A_L\} = \begin{cases} \{ \sum_{j=0}^{L-1} P_f / I_{f,(N-j)}, \sum_{j=0}^{L-1} P_f / I_{f,(N-L)} \geq \Gamma_{f,L} \} & L < K \\ \{ \sum_{j=0}^{K-1} P_f / I_{f,(N-j)}, \sum_{j=0}^{K-1} P_f / I_{f,(N-L)} \geq \Gamma_{f,j} \} & L = K \end{cases} \tag{6}$$

Lemma 2: The success probabilities for the home user, the supported cellular users at the femtocell and the remaining macrocell users are given by

$$p_{f,L} = \begin{cases} \mathbb{P}(A_L) & L < K \\ \frac{1}{N-K} \sum_{j=1}^{N-K} \mathbb{P} \left( \frac{P_f}{P_c I_{f,(j)}} \geq \Gamma_{f,K}, A_K \right) & L = K \end{cases} \tag{7}$$

$$p_{h,L} = \frac{1}{N-L} \sum_{j=1}^{N-L} \mathbb{P} \left( \frac{P_f}{P_c I_{f,(j)}} \geq \Gamma_{h,L}, A_L \right) \tag{8}$$

$$p_{c,L} = \lambda_L \mathbb{P} \left( \frac{P_c}{P_f / I_0} \geq \Gamma_{c,L} \right) \mathbb{P}(A_L) + \mu_L \sum_{j=N-L+1}^{N} \mathbb{P} \left( \frac{P_c}{P_f / I_{f,(j)}} \geq \Gamma_{c,L}, A_L \right) \tag{9}$$

Proof: Denote $S_f$ as the event that the home user succeeds in its communication process. When $L < K$, we have

$$p_{f,L} = \mathbb{P}(S_f, A_L) = \mathbb{P}(S_f | A_L) P(A_L) = \mathbb{P}(A_L) \tag{10}$$

When the FAP serves only $L (L < K)$ cellular users, the home user experiences no outage. Therefore $\mathbb{P}(S_f | A_L) = 1$ and the last equality holds. When $L = K$, the remaining $N - K$ macrocell users are equally likely interfering the home user with probability $\frac{1}{N-K}$. The probability of success of the home user $p_{f,K}$ is thus given by

$$p_{f,K} = \sum_{j=1}^{N-K} \frac{1}{N-K} \mathbb{P} \left( \frac{P_f}{P_c I_{f,(j)}} \geq \Gamma_{f,K}, A_K \right) \tag{11}$$

Similar arguments hold for $p_{h,L}$ and $p_{c,L}$.

In open access, due to the random femto-to-macro interference, the cellular users’ sum throughput is strictly less than $NC$, the sum throughput in closed access when $N < N_c^*$. The femto-to-macro interference is $P_f/I_0$ in closed access, which in open access after handover will increase to (due to Assumption 1) $P_f/I_0$ in the time slot of $U_{(i)}$, the cellular user served by the FAP. The increased femto-to-macro interference indeed bottlenecks the performance of open access by reducing sum throughput in low user density.

Note that the FAP predictably allocates a large portion of OTA resources to the home user. Additionally, the cellular users served by the femtocell must be very close to the FAP according to the handover criteria, which greatly reduces the randomness of their location. For these two reasons, the femto-to-macro interference is also nearly deterministic in open access. Therefore a cutoff user loading $N_0^*$ occurs in open access, upper and lower bounds of which are given in below and later in Remark 4 respectively.

Remark 3: When the amount of cellular users in the macrocell is $N_c^*$, the femto-to-macro interference in closed access causes their sum throughput to be zero, which should also be true in open access due to the increased femto-to-macro interference. Considering the at most $K$ cellular users served by the FAP, an upper bounder of the cutoff value $N_0^*$ should be given by $N_0^* \leq N_c^* + K$.

Theorem 2: When the FAP is set to serve at most one cellular user, namely $K = 1$, the closed-form capacity contours in open access are given by

$$C_0 = \min(C, \lambda_1 C_b) F_I N \left( \frac{P_f}{P_f / I_{f,0}} \right) + \min(C, \lambda_1 C_b) p_{f,1}$$

$$C_{sum} = NCP \left( \frac{P_c}{P_f / I_0} \geq \Gamma_{c,0} \right) F_I \left( \frac{P_f}{P_f / I_{f,0}} \right) + C(N-1)p_{c,1} + \min(C, \mu_1 C_b) p_{h,1} \tag{12}$$

where the success probabilities are

$$p_{f,1} = \frac{N}{N-1} F_I \left( \frac{P_f}{P_c I_{f,1}} \right) \left( 1 - F_I^{N-1} \left( \frac{P_f}{P_c I_{f,0}} \right) \right)$$

$$p_{h,1} = \frac{N}{N-1} F_I \left( \frac{P_f}{P_c I_{h,1}} \right) \left( 1 - F_I^{N-1} \left( \frac{P_f}{P_c I_{f,0}} \right) \right)$$

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help network operator significantly because the macrocell BS is still overloaded even after the femtocell serves $K$ cellular users for it. Thus concerns on privacy and security favor closed access in high user density.

**Femtocell Resource Allocation.** There exists a minimum value of $\lambda^*$ in femtocell resource allocation to ensure the home user benefits from open access. It is shown in Fig. 3 that $\lambda^*$ is an increasing function of cellular user density $N$. Therefore the FAP resource allocation in TDMA or OFDMA should be adaptive to $N$.

**Discussion on Shadowing.** We investigate the impact of large-scale random channel effects in this subsection by incorporating lognormal shadowing with standard deviation $\sigma_s = 10$ dB in the channel model. Consider the home user’s *ergodic rate* in TDMA for instance. By comparing Fig. 1 (path loss only) and Fig. 4 (path loss and shadowing), it is seen that shadowing lowers the rates by approximately 5% ~ 10%, but the main trends of the curves are preserved. Further simulation results (not included due to space limitations) confirm that our main conclusions are unchanged in view of shadowing.

**Conclusion for TDMA/OFDMA Access.** In orthogonal multiple access, the choices of the two parties are highly dependent on the cellular user density, with both preferring open access in medium density, closed access in high density, and they are in disagreement at low density. Therefore, our results suggest that when deploying OFDMA, femtocell access control should be adaptive based on the estimated cellular user density. Note that this conclusion is probably contingent on the assumption of no coordination between the femtocells and the macrocell BS. Future work should (and surely will) consider inter-BS coordination both among and across the two tiers, which is surely helpful and our conjecture will be especially important to open access in high user densities.

**APPENDIX**

**A. Proof of Theorem 2**

The key in the proof is the calculation of $p_{f,1}$, $p_{h,1}$ and $p_{c,1}$. Applying (7), we have

$$p_{f,1} = \frac{1}{N-1} \sum_{j=1}^{N-1} P\left( I(j) \leq \frac{P_f}{P_c \Gamma_{f,1}}, I(N) > \frac{P_f}{P_c \Gamma_{f,0}} \right)$$  

(13)

Use the bins and balls technique, we denote $P(I \leq \frac{P_f}{P_c \Gamma_{f,j}})$ as $p$, and $P(I \leq \frac{P_f}{P_c \Gamma_{f,0}})$ as $q$ to solve

$$N-1 \sum_{j=1}^{N-1} P(I(j) \leq \frac{P_f}{P_c \Gamma_{f,1}}, I(N) > \frac{P_f}{P_c \Gamma_{f,0}})$$

$$= \sum_{j=1}^{N-1} \sum_{k=j}^{N-1} \left\{ \binom{N}{k} p^k (1-p)^{N-k} \right\}$$

$$= Np(1-q)^{N-1}$$  

(14)

The expression of $p_{f,1}$ follows by substituting back for $p$ and $q$. Similar technique applies to $p_{h,1}$ and $p_{c,1}$. 

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REFERENCES


TABLE I
NOTATIONS AND PARAMETERS

<table>
<thead>
<tr>
<th>Description</th>
<th>Sim. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>macrocell radius $R$</td>
<td>300 meters</td>
</tr>
<tr>
<td>Reference distances $D$, $d$</td>
<td>200,5 meters</td>
</tr>
<tr>
<td>Path loss exponents $\alpha$, $\beta$</td>
<td>4, 2</td>
</tr>
<tr>
<td>Received power $P_f$, $P_c$</td>
<td>1, 1</td>
</tr>
<tr>
<td>Spreading factor $G$ (for CDMA)</td>
<td>64</td>
</tr>
<tr>
<td>User rate requirement $C$</td>
<td>0.5 bps/Hz</td>
</tr>
<tr>
<td>Femtocell backhaul capacity $c_b$</td>
<td>2 bps/Hz</td>
</tr>
<tr>
<td>FAP resources allocation $\lambda_L$, $\mu_L$</td>
<td>$1 - L/N$, $1/N$ for fair comparison</td>
</tr>
<tr>
<td>SIR target of the home user $\gamma_{f,L}$</td>
<td>$2^{\min(C/\lambda_L,c_b)} - 1$ (TDMA)</td>
</tr>
<tr>
<td>SIR target of handed over user $\gamma_{h,L}$</td>
<td>$2^{\min(C/\mu_L,c_b)} - 1$ (TDMA)</td>
</tr>
<tr>
<td>SIR target of macrocell user $\gamma_{c,L}$</td>
<td>$2^{(N-L)/N} - 1$ (TDMA)</td>
</tr>
</tbody>
</table>

Fig. 1. The home user’s ergodic rate versus cellular user density in TDMA. We have $\lambda_L = 1 - \frac{L}{N}$ and $\mu_L = \frac{1}{N}$ for fair comparison, $0 \leq L \leq K$.

Fig. 2. Cellular users’ sum throughput versus cellular user density in TDMA. We have $\lambda_L = 1 - \frac{L}{N}$ and $\mu_L = \frac{1}{N}$ for fair comparison, $0 \leq L \leq K$.

Fig. 3. The value of $\lambda^*$, i.e. the minimal proportion of femtocell resources required by the home user in TDMA open access, versus cellular user density.

Fig. 4. The home user’s ergodic rate in TDMA by incorporating the shadowing effect into the channel model. We assume a lognormal shadowing with standard deviation $\sigma_s = 10$ dB. For the purpose of comparison with Fig. 1 (which includes path loss attenuation only), we have $\lambda_L = 1 - \frac{L}{N}$ and $\mu_L = \frac{1}{N}$, $0 \leq L \leq K$. 

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