Primary User Capacity Maximization in Cooperative Detection Network using $m$ out of $N$ Fusion Rule

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Abstract—Cooperative detection is a well accepted notion for alleviating the “hidden terminal problem” in cognitive radio (CR) networks. Furthermore, a number of CRs cooperating with each other significantly boosts the reliability of identification of any spectral opportunity. We consider a situation in which the primary user (PU) of the spectrum tries to communicate at a maximum possible rate with the primary receiver which could be a base station (BS) in presence of an $N$ number of energy detector based CRs. Deploying the $m$ out of $N$ fusion rule at the BS, we formulate the problem of allocating the total PU power across its different transmission slots in a bid to maximize the PU-BS capacity such that the fused probability of detection is lower bounded by a specific level guaranteeing a prescribed reliability of detection. The problem is shown to possess convexity and is solved to allocate the available PU transmission power optimally. Analysis of $m$ out of $N$ fusion rule on the PU capacity and on the fused detection probability is presented. The effect of time-bandwidth product of the energy detector on the system metrics is also analyzed.

I. INTRODUCTION

Cognitive radio (CR) emerged as an aspiring technology to solve the problem of communication spectrum “exhaustion”, which is more of an underutilization rather than physical exhaustion. It is a device with a capability to identify opportunities to use radio spectrum exclusively licensed to the primary user (PU) and communicates over them ensuring that no harmful intervention to the PU transmission exists. A CR fulfills the task of spectrum sensing and can adjust its parameters according to the information obtained by sensing its radio environment [1]. A challenge for spectrum sensing is the “hidden terminal problem” in which the CRs are shadowed from PU, cannot detect its presence and start to transmit thus destructively interfering with PU transmission. This problem is alleviated by the idea of “cooperative spectrum sensing” [2] where numerous CRs in space aid a central network controller such as the base station (BS) to detect the presence or absence of PU. Efficient use of the total available power in a communication network is of utmost importance in the ever persistent energy crisis scenario thus hinting towards the need for “Green Communication” networks. We direct our study to address this issue of efficient power utilization in cooperative detection networks.

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Energy detector [3] is a low complexity detector requiring no information about the PU signal and is very suitable in situations that need to maintain PU signal security. An energy detector filters the received signal over a prescribed bandwidth, squares it, integrates it over a period of time and then compares the resulting statistic against a threshold to decide upon the presence or absence of PU. The time-bandwidth product of the energy detector [3] is an important factor affecting the CR’s detection performance. Cooperative detection involves a number of CRs aiming to detect the PU signal. Their individual detection performance metrics need to be fused together to obtain the overall performance metrics of cooperative detection thus improving the reliability of detection. A likelihood ratio test (LRT) based fusion is considered to be the optimal one [4]. Due to the complexity of the LRT based fusion, several suboptimal fusion schemes are considered for practical implementation. One of such rules is the $m$ out of $N$ fusion rule [5] which indicates the presence of PU only if at least $m$ out of the total $N$ CRs detect that the PU is present. Most common suboptimal fusion schemes are the OR and AND based fusion which correspond to $m = 1$ and $m = N$ respectively [4]. Another variation is the MAJORITY rule which essentially is $m = \frac{N}{2}$ for even $N$ and $m = \frac{N+1}{2}$ for odd $N$. Our focus is on the generic $m$ out of $N$ based fusion.

A previous work in [9] presented the concept of PU aided cooperative detection with an OFDM based CR where the issue was maximization of cooperative detection probability under a PU capacity constraint. However, the work was valid only for OR based fusion and for unity value of time-bandwidth product. In contrast to this work, we propose a notion that the foremost task of a CR network is to let the PU communicate at its maximum possible rate under the condition that the PU aided cooperative detection is reliable enough. In other words, as long as a desired amount of the cooperative probability of correct detection aided by PU is maintained, the PU can enjoy its connection to, say, BS at the maximum possible rate. We focus on this issue of maximizing the PU capacity (communication rate) to the BS under a constraint on the cooperative detection probability being greater than a pre-specified value. The constraint is also on the total PU transmission power which needs to be optimally allocated across its transmission slots. In addition, we use a more generic suboptimal scheme, the $m$ out of $N$ fusion scheme and then formulate a convex optimization problem. We solve it to study...
the effect of the fusion rule on the optimal PU capacity and on the corresponding fused detection probability. Furthermore, the effect of different values of the time-bandwidth product of the energy detector on the optimal PU capacity and on the corresponding fused detection probability is also analyzed.

The setting of our system is briefed in section II. The \( m \) out of \( N \) fusion scheme is discussed in section III. The problem of PU capacity maximization is addressed in section IV. Pertinent numerical analysis is presented in section V. We make the concluding remarks in section VI.

II. SYSTEM SETTING

Consider a PU communicating with a BS and at the same time being detected by an \( N \) number of CRs which are independently distributed in space as depicted in Fig. 1. The PU transmits for a total of \( N \) time slots with a power \( p_k \) in the \( k^{th} \) slot which is received only by the BS and by the \( k^{th} \) CR, \( \forall k \in \{1, ..., N\} \). We assume this to be possible due to existence of some coordination mechanism among the CRs (that could be facilitated by the BS) thus letting the PU transmission in a particular slot to be received only by a particular CR apart from the BS. The total available PU transmission power for \( N \) slots is \( P_{\text{tot}} \). The PU-BS channel for each transmission slot and each PU-CR channel is modeled as AWGN corrupted. We assume that each CR is connected to the BS by some sort of feedback existing between them and hence the BS is aware of any \( k^{th} \) CR’s probability of detection \( P_{d_k} \), and its probability of false alarm \( P_{f_k} \) which are equal to,

\[
P_{d_k} = Q_u\left(\frac{2p_k \Gamma(u)}{n_k^{cr} \sqrt{\lambda}}\right) \tag{1}
\]

\[
P_{f_k} = \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \tag{2}
\]

with \( \lambda \) being the energy detector threshold, \( n_k^{cr} \) is AWGN variance on the \( k^{th} \) PU-CR channel and \( Q_u(\cdot, \cdot) \) is generalized Marcum-Q function of order \( u, u \) being the time-bandwidth product [6].

The PU transmission in \( k^{th} \) slot is also received at the BS \( \forall k \in \{1, ..., N\} \). Then, the capacity of PU to BS can be expressed as,

\[
C = \sum_{k=1}^{N} \log_2\left(1 + \frac{p_k}{n_k^{bs}}\right) \tag{3}
\]

where we have assumed the bandwidth to be 1 and \( n_k^{bs} \) is the variance of AWGN which corrupts the transmission of PU in \( k^{th} \) slot.

III. FUSION USING \( m \) OUT OF \( N \) RULE

The received power at a CR would not be equal to that at any other CR in general. So, the individual detection probabilities at the CRs are not necessarily equal. We assume that the BS is responsible in fusing the detection probabilities and false alarm probabilities of \( N \) CRs using the \( m \) out of \( N \) fusion rule. The overall fused detection probability, \( P_d \) in this case will be equal to the complement of cumulative distribution function (CDF) of the Poisson-Binomial distribution which can be written as [7],

\[
P_d = 1 - \frac{m}{N+1} - \sum_{n=1}^{N} \left[1 - e^{-\pi n/N(N+1)}\right] \prod_{k=1}^{N} \left[1 - P_{d_k} (1 - e^{\pi n/N(N+1)})\right]. \tag{4}
\]

As the individual \( P_{f_k} \) in (2) does not depend upon the power levels \( p_k \), the false alarm probability will be same for all CRs. Then, the overall fused false alarm probability, \( P_f \) would be equal to [5],

\[
P_f = \sum_{l=m}^{N} \frac{N!}{(N-l)!!} \left\{ P_{f_k} (1 - P_{f_k})^{N-l} \right\}^l \tag{5}
\]

IV. PRIMARY USER CAPACITY MAXIMIZATION

Our main purpose is to maximize the PU capacity to the BS with a restriction on the fused probability of detection to be above a specified threshold, say, \( P_{d,thr} \). Additionally, the sum of all \( p_k \) would never exceed \( P_{\text{tot}} \). Thus, the problem is to,

\[
\text{Maximize : } \quad C \quad \text{subject to : } \quad P_{d} \geq P_{d,thr} \quad \text{and} \quad \sum_{k=1}^{N} p_k \leq P_{\text{tot}}. \tag{6}
\]

The solution is to find optimal PU transmission power levels. Here, we note that the objective function in (6) is convex (Appendix A). The feasible set for the power constraints is convex (Appendix B) and that for the overall detection

Fig. 1: Cooperative detection of a primary user (PU) communicating with the base station (BS).
probability constraint is also convex (Appendix C). Hence, (6) is a convex optimization problem.

The Lagrangian for this problem can be written as,

\[ \Lambda(p, \mu) = - \sum_{k=1}^{N} \log_2 \left( 1 + \frac{p_k}{n_k} \right) + \mu_0 \left[ \sum_{k=1}^{N} p_k - P_{tot} \right] 
- \sum_{k=1}^{N} \mu_k p_k + \mu_{N+1} \left[ \log(P_{d,thr}) - \log(P_d) \right]. \]  

(7)

Applying KKT conditions for Lagrangian optimality [8], we may write,

\[ \mu_0^* = \frac{1}{p^*_k + n_k} + \mu_k^* + \mu_{N+1}^* \frac{\partial \log(P_d)}{\partial p_k} \bigg|_{p_k=p^*_k}. \]  

(8)

From KKT conditions for Lagrange multipliers, \( \mu_k^* \geq 0 \), \( \mu_{N+1}^* \geq 0 \) and \( \frac{\partial \log(P_d)}{\partial p_k} > 0 \) (Appendix D) such that \( \mu_0^* > 0 \). From complementary slackness,

\[ \mu_0^* \cdot \left[ \sum_{k=1}^{N} p_k^* - P_{tot} \right] = 0. \]  

(9)

Since, \( \mu_0^* > 0 \), \( \sum_{k=1}^{N} p_k^* = P_{tot} \) which means that all the available PU transmission power is allocated.

V. NUMERICAL ANALYSIS

The task is to obtain optimal power levels across different transmission slots of the PU. Then, we study the effect of fusion rule \( m \) and time-bandwidth product on the optimal PU capacity and on the fused detection probability resulting from the optimal power allocation. We characterize the cooperative detection performance in terms of receiver operating characteristic (ROC) curves whereas PU capacity is plotted against the detector threshold \( \lambda \).

We take \( N=4 \) cooperating CRs and hence the same number of PU transmission slots. We set \( P_{tot} = 20 \) and the lower threshold on fused detection probability \( P_{d,thr} \) is fixed to 80% of the maximum possible detection probability that is obtained by maximizing \( P_d \) subject to power constraints without any inclusion of PU capacity. We set the AWGN variance levels as \( n_{bs}^{nu} = n_{cr}^{nu} = [1 2 2 1] \) on the PU-BS and PU-CR channels respectively. The problem in (6) is then solved numerically using MATLAB.

The variation of the optimal PU capacity against the CR detector threshold is depicted in Fig. 2 and the ROC for the optimal power allocation is plotted in Fig. 3. We note from Fig. 2 that the capacity for \( m = \{1, 2\} \) decreases with increasing threshold whereas for \( m = \{3, 4\} \) remains unchanged with threshold. In Fig. 3, the detection probability observed at a specific false alarm probability is higher for \( m = \{1, 2\} \) compared to \( m = \{3, 4\} \). This can be explained by the fact that the gain in cooperative detection probability is obtained when PU sacrifices some of its capacity to help improve the cooperative detection probability [9]. The gain in detection probability for \( m = \{1, 2\} \) compared to \( m = \{3, 4\} \) in our case comes from the reduction in PU capacity for \( m = \{1, 2\} \) compared to \( m = \{3, 4\} \). For \( m = \{3, 4\} \), as the detection probability is lower compared to that for \( m = \{1, 2\} \), the PU is not bothered to improve it and communicates to BS selfishly at the maximum possible rate. We also notice from Fig. 3 that the detection probability is best for \( m = 2 \) out of 4 rule which is the MAJORITY rule followed by \( m = 1 \) out of 4 which is the OR rule. The worst detection performance is for \( m = 4 \) out of 4 which is the AND rule. So, if the need is to maximize the PU communication rate to the BS, any of \( m = \{3, 4\} \) would help. If the best detection performance is needed, MAJORITY would be the choice whereas if there is a need to balance the PU capacity-fused detection probability tradeoff, OR based fusion seems to fulfill the requirement to
In Fig. 4, the maximized PU capacity against the detector threshold is plotted across different values of the time-bandwidth product $u$ of the energy detector for the MAJORITY fusion rule. Clear improvement in PU capacity at higher values of $u$ is depicted at a fixed threshold of the detector. For explaining this, we need to take into account the fact that the fused detection probability degrades with increasing values of $u$ as deduced from the ROC plots in Fig. 5. The fused detection performance is best for $u = 1$ which is obtained at the expense of PU capacity which is lowest at $u = 1$. As $u$ is increased, fused detection probability observed at a specific fused false alarm probability decreases and this decrease appears as an increase in the PU capacity.

VI. CONCLUSION

An $N$ number of energy detector based cognitive radios (CRs) cooperatively detecting the presence or absence of a primary user (PU) communicating with the base station (BS) was considered. The aim was to optimally allocate power across different PU transmission slots in order to maximize the PU capacity to the base station (BS) and at the same time guarantee a reliable detection of PU signal by maintaining the fused probability of detection to be at least or above a prespecified value. Assuming a suboptimal $m$ out of $N$ fusion rule deployed at the BS, we analyzed the effect of the fusion rule, $m$ on the optimal PU capacity and on the detection performance corresponding to the optimal power allocations. Our analysis suggested that the MAJORITY rule is the best in terms of detection performance but had the lowest PU capacity. This tradeoff between PU capacity and cooperative detection probability was found to be balanced somewhat by the OR fusion rule. We also illustrated the effect of the time-bandwidth product ($u$) parameter of the energy detector on the PU-BS capacity and on the fused detection probability. The detection performance was upper bounded by $u=1$ case which correspondingly had the minimum PU capacity. Again, PU capacity-cooperative detection probability tradeoff existed across variation of $u$ and capacity was found to increase with increasing $u$ but at the cost of decreased reliability of cooperative detection.

REFERENCES

The complementary cumulative distribution function \[ \mathcal{F}(x) = 1 - \mathcal{F}(x) \] is log-concave. Hence, \( \mathcal{F}(x) \) must be positive definite. By the argument that \( \frac{\partial \mathcal{F}_d}{\partial p_k} > 0 \) (Appendix E), we conclude that \( \frac{\partial \log(P_d)}{\partial p_k} > 0 \).

**E. Proof of Positive Definiteness of \( \frac{\partial P_{d,k}}{\partial p_k} \)**

Rewriting the probability of detection for the \( k \)-th CR in (1) using the alternative expression for Marcum-Q function in (4.37) of [13],

\[
P_{d,k} = e^{-\frac{r_k}{n_k^2} + \frac{1}{2}} \sum_{\nu=-u+1}^{\infty} \left( \frac{2p_k}{\lambda n_k^2} \right)^\nu I_\nu \left( \frac{2p_k \lambda}{n_k^2} \right) \tag{12}
\]

where \( I_\nu(z) \) is the modified Bessel function of first kind. Differentiating (12) partially w.r.t. \( p_k \) using the product rule and then applying \( \frac{\partial (\nu I_\nu(z))}{\partial z} = z^{-\nu} I_{\nu-1}(z) \) from (9.6.28) of [14], we get,

\[
\frac{\partial P_{d,k}}{\partial p_k} = -\frac{1}{n_k^2} e^{-\frac{r_k}{n_k^2} + \frac{1}{2}} \sum_{\nu=-u+1}^{\infty} \left( \frac{2p_k}{\lambda n_k^2} \right)^\nu I_\nu \left( \frac{2p_k \lambda}{n_k^2} \right) + \frac{1}{n_k^2} e^{-\frac{r_k}{n_k^2} + \frac{1}{2}} \sum_{t=-u}^{\infty} \left( \frac{2p_k}{\lambda n_k^2} \right)^t I_t \left( \frac{2p_k \lambda}{n_k^2} \right) = I_{-u} \left( \frac{2p_k \lambda}{n_k^2} \right) \tag{13}
\]

For \( p_k > 0 \), (13) implies that \( \frac{\partial P_{d,k}}{\partial p_k} > 0 \). For \( p_k = 0 \), we take the limit on both sides of (13) and then replace \( I_{-u}(z) \) with its infinite series representation from (8.445) on p. 919 of [15] as \( I_{-u}(z) = \sum_{p=0}^{\infty} \frac{1}{p!(u+p+1)} \left( \frac{z}{2} \right)^{u+p+1} \) (also using the fact \( I_{-u}(z) = I_u(z) \) for \( u \neq 0 \)), (13) can consequently be expressed as,

\[
\lim_{p_k \to 0} \frac{\partial P_{d,k}}{\partial p_k} = \lim_{p_k \to 0} e^{-\lambda/2} \left( \frac{\lambda}{2} \right) u \sum_{p=0}^{\infty} \frac{1}{p!(u+p+1)} \left( \frac{\lambda p_k}{2n_k^2} \right)^{2p} > 0 \tag{14}
\]

where \( \Gamma(u+1) \) was replaced by \( u! \) for \( u \) being an integer. Hence, from (13) and (14) we conclude that \( \frac{\partial P_{d,k}}{\partial p_k} > 0 \).