Revised Adomian decomposition method for solving systems of ordinary and fractional differential equations

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Abstract

A modification of the Adomian decomposition method applied to systems of linear/nonlinear ordinary and fractional differential equations, which yields a series solution with accelerated convergence, has been presented. Illustrative examples have been given.

Keywords: System of ordinary/fractional differential equations; Adomian polynomials; Revised Adomian decomposition; Caputo fractional derivative

1. Introduction

Numerous problems in Physics, Chemistry, Biology and Engineering science are modeled mathematically by systems of ordinary and fractional differential equations, e.g. series circuits, mechanical systems with several springs attached in series lead to a system of differential equations. On the other hand motion of an elastic column fixed at one end and loaded at the other, can be formulated in terms of a system of fractional differential equations [4]. Since most realistic differential equations do not have exact analytic solutions approximation and numerical techniques, therefore, are used extensively. Recently introduced Adomian Decomposition Method (ADM) [2] has been used for solving a wide range of problems. This new iterative method has proven rather successful in dealing with both linear as well as nonlinear problems, as it yields analytical solutions and offers certain advantages over standard numerical methods. It is free from rounding off errors since it does not involve discretization, and is computationally inexpensive. Biazar et al. [6] have applied this method to a system of ordinary differential equations. Daftardar-Gejji and Jafari [7,8] have explored this method to obtain solutions of a system of linear and nonlinear fractional differential equations. Further in [9] they have suggested a modification (termed as “revised ADM”) of this method and have applied revised ADM for solving a system of nonlinear algebraic equations. In the present paper we use the revised Adomian decomposition method to obtain solutions of systems of linear/nonlinear ordinary and fractional differential equations.

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We demonstrate that the series solution thus obtained converges faster relative to the series obtained by standard ADM. Several illustrative examples have been presented.

The present paper has been organized as follows. In Section 2, we give basic definitions and preliminaries. Section 3 deals with the analysis of ADM applied to a system of ordinary differential equations. In Sections 4 and 5, we introduce revised ADM for systems of ordinary and fractional differential equations, respectively. Section 6 compares the revised ADM and standard ADM with illustrative examples. This is followed by the conclusions in Section 7.

2. Definitions and preliminaries

Definition 2.1. A real function \( f(x) \), \( x > 0 \) is said to be in the space \( C_x, \alpha \in \Re \) if there exists a real number \( p \) (\( > \alpha \)), such that \( f(x) = x^p f_1(x) \) where \( f_1(x) \in C[0, \infty) \). Clearly \( C_x \subset C_\beta \) if \( \beta \leq \alpha \).

Definition 2.2. A function \( f(x) \), \( x > 0 \) is said to be in the space \( C^m_x, m \in N \cup \{0\} \), if \( f^{(m)}(x) \in C_x \).

Definition 2.3. The (left sided) Riemann–Liouville fractional integral of order \( \mu \geq 0 \) [9–12] of a function \( f \in C_x, \alpha \geq -1 \) is defined as

\[
I^\mu f(x) = \frac{1}{\Gamma(\mu)} \int_0^x f(t) \frac{dt}{(x-t)^{1-\mu}}, \quad \mu > 0, \quad x > 0, \tag{1}
\]

\[
I^0 f(x) = f(x). \tag{2}
\]

Definition 2.4. The (left sided) Caputo fractional derivative of \( f \in C^m_{-1}, m \in N \) is defined as [9,10]

\[
D^\mu f(x) = \begin{cases} 
[I^{m-\mu}f^{(m)}(x)] & m-1 < \mu < m, \\
\frac{d^\mu}{dx^\mu}f(t) & \mu = m.
\end{cases} \tag{3}
\]

Note that [9,11]

\[
I^\mu I^\nu f = I^\mu+\nu f, \quad \mu, \nu \geq 0, \quad f \in C_x, \alpha \geq -1, \tag{4}
\]

3. System of ordinary differential equations and Adomian decomposition

Consider the following system of ordinary differential equations:

\[
y'_i(x) = \sum_{j=1}^{n} b_{ij}(x)y_j + N_i(x,y_1,y_2,\ldots,y_n) + g_i(x), \quad y_i(0) = c_i, \quad i = 1,2,\ldots,n, \tag{5}
\]

where \( b_{ij}(x), g_i(x) \in C[0, T] \) and \( N_i \)'s are nonlinear continuous functions of its argument. Integrating both side of Eq. (5) from 0 to \( x \) and the using initial conditions, we get

\[
y_i(x) = c_i + \int_0^x g_i(x)dx + \int_0^x \sum_{j=1}^{n} b_{ij}(x)y_j dx + \int_0^x N_i(x,y_1,y_2,\ldots,y_n)dx, \quad \text{for } i = 1,2,\ldots,n. \tag{6}
\]

The standard ADM [2] yields the solution \( y_i(x) \) by the series

\[
y_i(x) = \sum_{m=0}^{\infty} y_{im}(x) \tag{7}
\]

and the nonlinear terms by an infinite series of Adomian polynomials.
defines the components $y_{im}$

We approximate the solution $y(x)$ by the truncated series

$$f_k(x) = \sum_{m=0}^{k-1} y_{im}(x) \quad \text{and} \quad \lim_{k \to \infty} f_k(x) = y(x), \quad i = 1, 2, \ldots, n.$$  

4. Revised ADM for a system of ordinary differential equations

In this section we propose a modification of the Adomian decomposition. We set

$$y_{10}(x) = c_i + \int_0^x g_i(x)dx,$$

$$y_{1,m+1}(x) = \int_0^x \sum_{j=1}^n b_{ij}(x)y_{jm} dx + \int_0^x A_{im} dx,$$

$$y_{10}(x) = c_l + \int_0^x g_l(x)dx + \int_0^x \sum_{j=1}^{l-1} b_{lj}(x)y_{j0} dx, \quad l = 2, \ldots, n,$$

$$y_{l,m+1}(x) = \int_0^x \sum_{j=1}^{l-1} b_{lj}(x)y_{jm+1} dx + \int_0^x \sum_{j=l}^{n} b_{lj}(x)y_{jm} dx + \int_0^x A_{im} dx,$$

where $A_{im}^*$ is defined as

$$A_{im}^* = \begin{cases} A_{lm+1}, & \text{if } N_l \text{ are independent of } y_1, y_{i+1}, \ldots, y_n, \\ 1A_{lm+1} + 2A_{lm}, & \text{if } N_l(y_1, \ldots, y_n) = 1N_l(y_1, \ldots, y_{l-1}) + 2N_l(y_1, \ldots, y_n), \quad l = 2, 3, \ldots, \end{cases}$$

Here $1A_{lm+1}, 2A_{lm}$ are Adomian polynomials corresponding to $1N_l$ and $2N_2$ as defined in Eq. (9).

5. Revised ADM for a system of fractional differential equations

We consider the following system of fractional differential equations:

$$D^\alpha y_i(x) = \sum_{j=1}^n (\phi_{ij}(x) + \gamma_{ij}D^\alpha u_j)y_j + N_i(x, y_1, \ldots, y_n) + g_i(x),$$

$$y_i^{(k)}(0) = c_i^k, \quad 0 \leq k \leq m_i, \quad \alpha_j \leq \alpha_i, \quad m_i < \alpha_i \leq m_i + 1, \quad 1 \leq i \leq n.$$
where \( N_i \)'s are nonlinear functions of \( x, y_1, \ldots, y_n \) and \( x_i, x_{ij} \in \mathbb{R}^+ \). Applying \( I^n \) to both the sides of Eq. (14), we get
\[
y_i = \sum_{k=0}^{m_i} c_k \frac{x^k}{k!} + I^n \sum_{j=1}^{n} \left( \phi_{ij}(x) + \gamma_{ij} D^{x_j} \right) y_j + I^n N_i(x, y_1, \ldots, y_n), \quad 1 \leq i \leq n,
\]
where \( m_i < x_i \leq m_i + 1 \) and \( m_{ij} < x_{ij} \leq m_{ij} + 1 \). Here \( 0 \leq x_{ij} < x_i \), for \( 1 \leq i, j \leq n \), \( \gamma_{ij} \)'s are constants and \( \phi_{ij}(x) \), \( g_i(x) \in C[0, T] \).

Using standard ADM we get
\[
\sum_{m=0}^{\infty} y_{im} = \sum_{k=0}^{m_i} c_k \frac{x^k}{k!} + I^n g_i(x) - \sum_{j=1}^{n} \sum_{k=0}^{m_{ij}} c_k \frac{x^{x_{ij}+k}}{k!} T(x_i - x_{ij} + k + 1) + \sum_{m=0}^{\infty} \sum_{j=1}^{n} \left( I^n \phi_{ij}(x) + \gamma_{ij} I^n D^{x_j} y_{jm}(x) \right) + I^n A_{im}, \quad 1 \leq i \leq n, m = 0, 1, \ldots
\]

and the recurrence relations are
\[
y_{i0}(x) = \sum_{k=0}^{m_i} c_k \frac{x^k}{k!} + I^n g_i(x) - \sum_{j=1}^{n} \sum_{k=0}^{m_{ij}} c_k \frac{x^{x_{ij}+k}}{k!} T(x_i - x_{ij} + k + 1),
\]
\[
y_{i,m+1}(x) = \sum_{j=1}^{n} \left( I^n \phi_{ij}(x) + \gamma_{ij} I^n D^{x_j} y_{jm}(x) \right) + I^n A_{im}, \quad 1 \leq i \leq n, \quad m = 0, 1, \ldots
\]

In view of the revised Adomian decomposition, we set
\[
y_{10}(x) = \sum_{k=0}^{m_1} c_k \frac{x^k}{k!} + I^n g_1(x) - \sum_{j=1}^{n} \sum_{k=0}^{m_{1j}} c_k \frac{x^{x_{1j}+k}}{k!} T(x_1 - x_{1j} + k + 1),
\]
\[
y_{1,m+1}(x) = \sum_{j=1}^{n} \left( I^n \phi_{1j}(x) + \gamma_{1j} I^n D^{x_j} y_{1jm}(x) \right) + I^n A_{1m},
\]
\[
y_{10}(x) = \sum_{k=0}^{m_1} c_k \frac{x^k}{k!} + I^n g_1(x) - \sum_{j=1}^{n} \sum_{k=0}^{m_{1j}} c_k \frac{x^{x_{1j}+k}}{k!} T(x_1 - x_{1j} + k + 1) + \sum_{l=1}^{n} \left( I^n \phi_{1l}(x) + \gamma_{1l} I^n D^{x_l} y_{1m}(x) \right),
\]
\[
y_{1,m+1}(x) = \sum_{j=1}^{n} \left( I^n \phi_{1j}(x) + \gamma_{1j} I^n D^{x_j} y_{1m+1}(x) \right) + \sum_{l=1}^{n} \left( I^n \phi_{1l}(x) + \gamma_{1l} I^n D^{x_l} y_{1m}(x) \right) + I^n A_{1m}, \quad l = 2, 3, \ldots, m = 0, 1, 2, \ldots,
\]

where \( A_{im} \) are defined in Eq. (13). For the convergence of the above method we refer the reader to [1, 5, 7].

6. Illustrative examples

To give a clear overview of the revised method, we present the following examples. We apply the revised ADM and compare the results with the standard ADM.

(i) Consider the following system of linear equations:
\[
\begin{align*}
y_1' &= y_2 - \cos x, \quad y_1(0) = 1, \\
y_2' &= y_3 - e^x, \quad y_2(0) = 0, \\
y_3' &= y_1 - y_2, \quad y_3(0) = 2.
\end{align*}
\]
This system is equivalent to the following system of integral equations:

\[ y_1 = y_1(0) - \int_0^x \cos x \, dx + \int_0^x y_3, \]
\[ y_2 = y_2(0) - \int_0^x e^x \, dx + \int_0^x y_3, \]
\[ y_3 = y_3(0) + \int_0^x (y_1 - y_2) \, dx. \]

The revised Adomian procedure would lead to:

\[ y_{10} = 1 - \sin x, \quad y_{1m+1} = \int_0^x y_{3m} \, dx, \]
\[ y_{20} = 1 - e^x, \quad y_{2m+1} = \int_0^x y_{3m} \, dx, \]
\[ y_{30} = 2 + \int_0^x (y_{10} - y_{20}) \, dx, \quad y_{3m+1} = \int_0^x (y_{1m+1} - y_{2m+1}) \, dx = e^x + \cos x. \]

It should be noted that after one iteration only, we get the exact solution \( y_1 = e^x, \ y_2 = \sin x \) and \( y_3 = e^x + \cos x \). This example has been solved by Biazar et al. [6] using the standard ADM, by which they obtain the exact solution after two iterations.

(ii) Consider the following system of nonlinear differential equations:

\[ y_1' = 2y_2^2, \quad y_1(0) = 1, \]
\[ y_2' = e^{-x}y_1, \quad y_2(0) = 1, \]
\[ y_3' = y_2 + y_3, \quad y_3(0) = 0. \]

Integrating this we get

\[ y_1 = 1 + 2 \int_0^x y_2^2 \, dx, \]
\[ y_2 = 1 - \int_0^x e^{-x}y_1 \, dx, \]
\[ y_3 = \int_0^x (y_2 + y_3) \, dx. \]

The revised Adomian procedure leads to

\[ y_{10} = 1, \quad y_{1m+1} = 2 \int_0^x A_{1m} \, dx, \]
\[ y_{20} = 1 + \int_0^x e^{-x}y_{10} \, dx = 2 - e^{-x}, \quad y_{2m+1} = \int_0^x e^{-x}y_{1m+1} \, dx, \]
\[ y_{30} = \int_0^x y_{20} \, dx = e^{-x} + 2x - 1, \quad y_{3m+1} = \int_0^x (y_{2m+1} + y_{3m}) \, dx. \]

The subsequent terms are

\[ y_{11} = 2 \int_0^x A_{10} \, dx = 2 \int_0^x y_{20}^2 \, dx = -7 - e^{-2x} + \frac{8}{e^x} + 8x, \]
\[ y_{21} = \int_0^x e^{-x}y_{11} \, dx = \frac{14}{3} + \frac{1}{e^{2x}} - \frac{4}{e^x} + \frac{8x}{e^x}, \]
\[ y_{31} = \int_0^x (y_{21} - y_{30}) \, dx = -\frac{89}{9} - \frac{1}{9e^{3x}} + \frac{2}{e^{2x}} + \frac{8}{e^x} + \frac{11x}{3} + \frac{8x}{e^x} + x^2. \]
Biazar et al. [6] have solved this example using the standard ADM. We draw below graphs of $y_1(x)$, $y_2(x)$ and $y_3(x)$ and compare the exact solution, solution given by the standard Adomian method and solution given by the revised Adomian method. In Figs. 1–3, we plot $e^{2x}$, $e^x$, $xe^x$, which are the exact solutions, $y_1$, $y_2$, $y_3$ denote solutions obtained by revised ADM and $y_1^r$, $y_2^r$, $y_3^r$ denote solutions obtained by the standard ADM.

Fig. 1. (Example (ii)).

Fig. 2. (Example (ii)).

Fig. 3. (Example (ii)).
(iii) Consider the following system of linear differential equations:

\[ y_1' = y_1 + y_2, \quad y_1(0) = 0, \]
\[ y_2' = -y_1 + y_2, \quad y_2(0) = 1. \]

This system is equivalent to

\[ y_1 = \int_0^x (y_1 + y_2) \, dx, \]
\[ y_2 = 1 + \int_0^x (-y_1 + y_2) \, dx. \]

In view of the revised Adomian scheme,

\[ y_{10} = 0, \quad y_{1,m+1} = \int_0^x (y_{1m} + y_{2m}) \, dx, \]
\[ y_{20} = 1 - \int_0^x y_{10} \, dx, \quad y_{2,m+1} = \int_0^x (-y_{1m+1} + y_{2m}) \, dx, \quad m = 0, 1, \ldots \]

In the first iteration we have

\[ y_{11} = \int_0^x (y_{10} + y_{20}) \, dx = x \quad \text{and} \quad y_{21} = \int_0^x (-y_{11} + y_{20}) \, dx = x - \frac{x^2}{2}. \]

The subsequent terms are

\[ y_{12} = x^2 - \frac{x^3}{3!}, \quad y_{22} = \frac{x^2}{2!} - \frac{x^3}{2!} + \frac{x^4}{4!}, \]
\[ y_{13} = \frac{x^3}{3!} - \frac{x^4}{4!} + \frac{x^5}{5!}, \quad y_{23} = \frac{x^3}{3!} - \frac{x^4}{4!} + \frac{x^5}{5!} - \frac{x^6}{6!}, \]
\[ y_{14} = \frac{x^4}{4!} - \frac{x^5}{12} + \frac{x^6}{5!} - \frac{x^7}{7!}, \quad y_{24} = \frac{x^4}{4!} - \frac{x^5}{12} + \frac{x^6}{48} - \frac{x^7}{6!} + \frac{x^8}{8!}, \]
\[ \vdots \]

After five iterations, we get

\[ y_1 \approx x + x^2 + \frac{2x^3}{3!} - \frac{4x^5}{5!} - \frac{4x^6}{6!} + \frac{20x^7}{7!} - \frac{x^8}{9!} + \frac{x^9}{9!}, \]
\[ y_2 \approx 1 + x - \frac{2x^3}{3!} - \frac{4x^4}{4!} - \frac{4x^5}{5!} - \frac{x^6}{6!} + \frac{28x^7}{7!} - \frac{27x^8}{8!} - \frac{9x^9}{9!} + \frac{x^{10}}{10!}. \]
This example has been solved by Shawagfeh and Kaya [13] using the standard ADM. In Figs. 4 and 5, we plot $e^x$ and $e^x \cos x$ which are the exact solutions, $y_i$, $i = 1, 2$, denote solutions obtained by revised ADM and $y_i^*$, $i = 1, 2$ denotes solutions obtained by the standard ADM.

(iv) Consider the following system of linear fractional differential equations:

\begin{align*}
D^{1.5}y_1 &= y_2, \quad y_1(0) = y'_1(0) = 1, \\
D^{0.5}y_2 &= -y_2 - y_1 + 1 + t, \quad y_2(0) = 0.
\end{align*}

The standard ADM leads to the following scheme:

\begin{align*}
y_{10} &= 1 + t, \quad y_{1,m+1} = \Gamma^{1.5}y_{2m}, \\
y_{20} &= \frac{\Gamma^{0.5}}{\Gamma(1.5)} - \frac{\Gamma^{1.5}}{\Gamma(2)} y_{2m+1} = \Gamma^{0.5}(-y_{1m} - y_{2m}), \quad m = 0, 1, \ldots.
\end{align*}

In the first iteration we get

\begin{align*}
y_{11} &= \Gamma^{1.5}y_{20} = \frac{\Gamma^2}{\Gamma(3)} + \frac{\Gamma^3}{\Gamma(4)}, \\
y_{21} &= \Gamma^{0.5}(-y_{10} - y_{20}) = -\frac{\Gamma^{0.5}}{\Gamma(1.5)} + \frac{\Gamma^{1.5}}{\Gamma(2)} - \frac{\Gamma^1}{\Gamma(2)} - \frac{\Gamma^2}{\Gamma(3)}.
\end{align*}

The subsequent terms are

\begin{align*}
y_{12} &= \Gamma^{1.5}y_{21} = -\frac{\Gamma^2}{\Gamma(3)} - \frac{\Gamma^3}{\Gamma(4)} - \frac{\Gamma^{2.5}}{\Gamma(3.5)} - \frac{\Gamma^{3.5}}{\Gamma(4.5)}, \\
y_{22} &= \Gamma^{0.5}(-y_{11} - y_{21}) = -\frac{\Gamma^{3.5}}{\Gamma(4.5)} + \frac{\Gamma^1}{\Gamma(2)} + \frac{\Gamma^2}{\Gamma(3)} + \frac{\Gamma^{1.5}}{\Gamma(2.5)}, \\
y_{13} &= \Gamma^{1.5}y_{22} = -\frac{\Gamma^6}{\Gamma(6)} + \frac{\Gamma^{2.5}}{\Gamma(3.5)} + \frac{\Gamma^3}{\Gamma(4)} + \frac{\Gamma^{3.5}}{\Gamma(4.5)}, \\
y_{23} &= \Gamma^{0.5}(-y_{12} - y_{22}) = \frac{\Gamma^{3.5}}{\Gamma(4.5)} + \frac{\Gamma^3}{\Gamma(4)} + 2\frac{\Gamma^4}{\Gamma(5)} - \frac{\Gamma^{1.5}}{\Gamma(2.5)} - \frac{\Gamma^2}{\Gamma(3)}.
\end{align*}

Using the above terms,

\begin{align*}
y_1 &= 1 + t + \frac{\Gamma^2}{\Gamma(4)} - \frac{\Gamma^5}{\Gamma(6)} + \cdots, \\
y_2 &= -\frac{\Gamma^2}{\Gamma(3)} + \frac{\Gamma^3}{\Gamma(4)} + 2\frac{\Gamma^4}{\Gamma(5)} + \cdots.
\end{align*}
The revised ADM leads to the following scheme:

\[ y_{10} = 1 + t, \quad y_{20} = \frac{t^{0.5}}{\Gamma(1.5)} + \frac{t^{1.5}}{\Gamma(2)} - t^{0.5}y_{10} = 0, \]
\[ y_{1,m+1} = I^{1.5}y_{2m}, \quad y_{2,m+1} = \frac{t^{0.5}}{\Gamma(2)} - t^{0.5}(y_{1m+1} - y_{2m}), \quad m = 0, 1, \ldots. \]

In the first iteration we have

\[ y_{11} = I^{1.5}y_{20} = 0, \]
\[ y_{21} = t^{0.5}(-y_{11} - y_{20}) = 0. \]

Using the above terms, we find \( y_{1,m+1} = y_{2,m+1} = 0, m \geq 0 \). Hence we \( y_{1}(t) = 1 + t \) and \( y_{2}(t) = 0 \).

**Remark.** In this example, we get exact solution in only one iteration of the revised ADM, whereas even after several iterations, standard ADM does not yield exact solution.

(v) Consider the following system of nonlinear fractional differential equations:

\[ D^{2.5}y_{1} = y_{2}, \quad y_{1}(0) = y_{1}'(0) = 0, \quad y_{1}''(0) = 2, \]
\[ D^{0.5}y_{2} = -y_{2} - y_{1}' + t^{4}, \quad y_{2}(0) = 0. \]

The standard ADM leads to the following recurrence relations:

\[ y_{10} = t^{2}, \quad y_{1,m+1} = I^{2.5}y_{2m}, \]
\[ y_{20} = \frac{t^{4.5}}{\Gamma(5.5)}, \quad y_{2,m+1} = -t^{0.5}y_{2m} - t^{0.5}A_{2m}, \quad m = 0, 1, \ldots, \]

where \( A_{2m} \) are Adomian polynomials. In this example we have \( N_{2}(t,y_{1},y_{2}) = y_{1}^{2} \) and using Eq. (9), we get

\[ A_{20} = y_{10}^{2}, \]
\[ A_{21} = 2y_{10}y_{11}, \]
\[ A_{22} = y_{11}^{2} + 2y_{10}y_{12}, \]
\[ A_{23} = 2y_{10}y_{13} + 2y_{11}y_{12}, \]
\[ \vdots \]

Thus the standard ADM yields

\[ y_{11} = t^{2}y_{20} = \frac{t^{7}}{\Gamma(8)}, \]
\[ y_{21} = -t^{0.5}y_{20} - t^{0.5}A_{20} = -\frac{t^{5}}{\Gamma(6)} - \frac{t^{4.5}}{\Gamma(5.5)}, \]
\[ y_{12} = t^{2.5}y_{21} = -\frac{t^{7.5}}{\Gamma(8)} - \frac{t^{7}}{\Gamma(8)}, \]
\[ y_{22} = -t^{0.5}y_{21} - t^{0.5}A_{21} = \frac{t^{5.5}}{\Gamma(6.5)} + \frac{t^{5}}{\Gamma(6)} - 144 \frac{t^{0.5}}{\Gamma(10.5)}, \]
\[ y_{13} = t^{2.5}y_{22} = \frac{t^{8}}{\Gamma(9)} + \frac{t^{7.5}}{\Gamma(8.5)} - 144 \frac{t^{12}}{\Gamma(13)}, \]
\[ \vdots \]

In view of the revised ADM we get

\[ y_{10} = t^{2}, \quad y_{20} = t^{4.5} - t^{0.5}A_{20} = 0, \]
\[ y_{1,m+1} = t^{2.5}y_{2m}, \quad y_{2,m+1} = -t^{0.5}y_{2m+1} - t^{0.5}A_{2m+1}, \quad m = 0, 1, \ldots. \]
In the first iteration we have
\begin{align*}
y_{11} &= I^{2.5}y_{10} = 0, \\
y_{21} &= -I^{0.5}y_{20} - I^{0.5}A_{21} = 0.
\end{align*}
Using the above terms, we find \( y_{1,m+1} = y_{2,m+1} = 0, \ m \geq 1 \). Hence \( y_1(t) = t^2 \) and \( y_2(t) = 0 \). Thus in one iteration only we get exact solution using revised ADM. In Fig. 6, \( y_1 \) denotes the solution obtained by revised ADM (which is equal to the exact solution) and \( y'_1 \) denotes the solution obtained by standard ADM (obtained using five iterations).

7. Conclusions

Adomian decomposition is a powerful method which yields a convergent series solution for linear/nonlinear problems. This method is better than numerical methods, as it is free from rounding off errors, and does not require large computer power. We have suggested a modification of this method, termed as “revised ADM”. In the present paper we employ the revised ADM for solving a system of ordinary/fractional differential equations. The revised method yields a series solution which converges faster than the series obtained by standard ADM. The illustrative examples clearly demonstrate this. Mathematica has been used for computation of Adomian polynomials and graphs presented in this paper.

Acknowledgements

Hossein Jafari thanks University Grants Commission, New Delhi, India for the award of Junior Research Fellowship and acknowledges Y. Talebi, University of Mazandaran, Babolsar, Iran for encouragement.

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