Vertically Polarized Dipoles and Monopoles, Directivity, Effective Height and Antenna Factor

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Abstract—The effective length ($L_{e}$) of linear antennas like dipoles and the effective height ($H_{e}$) of monopoles shorter than a wavelength are determined using a transmitting Hertz Dipole as a reference. Effective Receiving Area ($A_{eR}$) has been calculated in the receiving mode and hence the Antenna Receiving Directivity ($D_{R}$) or Gain ($G_{R}$) can be determined. From these calculations, it can be seen that the Received Power ($W_{R}$) in a link between two dipole antennas in free space or between two monopole antennas over a perfect ground is of the same value in the far field region. As a result, the directivity or gain of the ideal dipole antenna in free space has the same value in transmission and reception; but, a different value for monopole and dipole antennas installed over a perfect ground plane. This result is fulfilled in the ideal as well as the real case where losses and mismatchings are involved.

Index Terms—Antenna factor, antenna resonant height, antenna resonant length, dipole antenna, effective antenna height, effective antenna length, folded monopole antenna, ground plane, ground plane equivalent loss resistance, ground plane losses, LF MF AM broadcast antennas, LF MF AM transmitting antennas, monopole antenna, natural ground plane, quarter wave monopole antenna, short dipole antenna, short monopole antenna.

I. INTRODUCTION

OME problems arise when linear antennas like monopoles and dipoles are installed over a perfect or imperfect ground plane. They were pointed out by the late Kenneth Norton [1], [2] but it seems little importance was given by the antenna and propagation community. Mr. Ermi Roos was consulting the author over the problem and a study was performed in order to get a clearer view. Other papers on the subject were published by J. W. Ames and W. A. Edson [3], but it has controversial results not easily to be believed and with no theoretical evidences.

The effective length of a dipole antenna can be obtained from the current distribution along the antenna structure in both the transmitting and receiving modes. In both cases the effective length yields the same result since the current distribution depends only on the physical antenna length and is independent of the feed power.

To obtain the effective height of monopole antennas the same concept that has been used from early days in wireless communication for the lowest end of the radio spectrum is applied.

Several A&P and EMC papers [6], [8]–[29], [31]–[33], [35]–[51] and [53] were browsed in order to know if this problem was a concern to the scientific community; but, no evidence was found.

II. EFFECTIVE LENGTH

Effective length is a very old term applied to antennas and this parameter allows to know the antenna radiation efficiency in the transmitting mode or as an electromagnetic energy collector in the receiving mode.

For any linear antenna in the $z$ or vertical axis oriented, according to Fig. 1, the vector potential ($A$) has only one component in the surrounding space ($A_z$), to be:

$$A_z = \frac{\mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} I(z) \left[ e^{-i\frac{2\pi}{\lambda}z} - e^{-i\frac{2\pi}{\lambda}L} \right] dz \quad (1)$$

- $I(z)$ is the current value in each antenna point with a time variation $e^{j\omega t}$.
- $L$ is the antenna physical length whose geometrical center is located at the origin of the coordinate system.
• R is the distance from any current element \( I(z')dz' \) on the antenna structure to a point in space where the vector potential is evaluated.
• \( a \) is the antenna wire radius intended to be smaller than the antenna length \( (L) \) and the wavelength \( \lambda \) (\( a \ll L, a \ll \lambda \)). The distance \( R \) according to the Pythagorean Theorem is:

\[
R = \left[ (z - z')^2 + r^2 \right]^{1/2}
\]

\( r \) is the distance from the coordinate origin to a point in the horizontal plane \( x, y \) as can be seen in Fig. 1.

Taking into account that \( z = r \cos \theta \) and \( \rho = r \sin \theta \), the distance \( R \) is found to be:

\[
R = \sqrt{\left( z - z' \right)^2 + \rho^2}
\]

(3)

The effective length of a linear antenna \( L < \lambda \) with a current distribution \( I(z) \) is found to be:

\[
R \approx r - z' \cos(\theta)
\]

(4)

And the vector potential results:

\[
A_z = \frac{\mu_0}{4\pi} e^{-j\frac{R}{\rho}} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} I(z') e^{jz' \cos(\theta)} dz'
\]

(5)

Using the Euler identity:

\[
e^{jz' \cos(\theta)} = \cos(jz' \cos(\theta)) + jsin(jz' \cos(\theta))
\]

(6)

If \((\beta L = \beta L_2 < 1) \) or \((L_2 / \lambda < 1/\pi) \) it can be seen that:

\[
e^{-j\beta' z' \cos(\theta)} \approx 1 + j0
\]

(7)

In these conditions the vector potential for \( r > L/2 \) and \( L/2 < \lambda < 1/\pi \), is found to be:

\[
A_z = \frac{\mu_0}{4\pi} e^{-j\frac{R}{\rho}} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} I(z)dz.
\]

(8)

This expression is of practical value with a small error if \( r > \lambda \) and \( L < \lambda \) that can perfectly be fulfilled.

In the Hertz Dipole case and for \( z = 0 \) the antenna current is \( I_0 = I(z) = I(0) \), for instance:

\[
A_z = \frac{\mu_0 I_0}{4\pi} \left( \frac{1}{I_0} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} I(z)dz \right) e^{-j\frac{R}{\rho}}
\]

(9)

For a Hertz Dipole \( I(z) = I_0 = \text{constant} \) and the \( z \) component of the vector potential results:

\[
A_z(\text{Hertz}) = \frac{\mu_0 I_0 dL e^{-j\frac{R}{\rho}}}{4\pi}
\]

(10)

To get any linear antenna (\( L < \lambda \)) producing the same vector potential as a Hertz Dipole in any space point or:

\[
A_z = A_z(\text{Hertz})
\]

(11)

Results:

\[
\frac{\mu_0 I_0}{4\pi} \left( \frac{1}{I_0} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} I(z)dz \right) e^{-j\frac{R}{\rho}} = \frac{\mu_0 I_0 dL e^{-j\frac{R}{\rho}}}{4\pi}
\]

(12)

In this last expression the Hertz Dipole length must be

\[
dL = \frac{1}{I_0} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} I(z)dz
\]

(13)

Both the linear antenna and the Hertz Dipole produce the same vector potential and, of course, the same magnetic and electric field intensity, as well as the same power density in the same space point if these conditions are fulfilled.

By definition, the effective length \( L_e \) of any linear antenna \( L < \lambda \) is the length with a constant current distribution, like in the Hertz Dipole, producing the same electromagnetic field in the surrounding space if \( r > \lambda \).

The effective length of a linear antenna \( L < \lambda \) with a current distribution \( I(z) \) is found to be:

\[
I_e = \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} I(z)dz \quad (m)
\]

(14)

\( I_0 \) is the feeding point current and if \( L < \lambda \), \( I_0 \) will be the maximum current value for a center fed dipole antenna.

With this effective length definition for a Hertz dipole antenna results:

\[
I_e = \frac{1}{I_0} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} I_0 dz \quad (m)
\]

(15)

or:

\[
I_e = dL \quad (m)
\]

(16)

A Hertz dipole antenna has an effective length \( I_e \) exactly of the same value as its physical length \( dL \) because of the constant current distribution along its physical structure. The antenna effective length has been obtained in the transmitting condition.

In the receiving case, the incoming wave has a Poynting vector or power density \( P_1 \). This Poynting vector \( P_1 \) produces an incoming power flux through the surface \( \Sigma \) around the receiving antenna conductor as is shown in Fig. 2.

Power density \( P_1 \) induces a circulating current \( I(z) \) on the receiving dipole antenna, scattering or reradiating part of the received power as a scattered power density \( P_s \) in the surrounding space.

The electrical and magnetic fields in the surrounding space must be:

\[
E = E_t + E_s \quad (17)
\]

\[
H = H_t + H_s \quad (18)
\]

- \( E \) is the total electric field around the receiving antenna.
- \( E_t \) is the incoming wave electric field.
- \( E_s \) is the scattered wave electric field.
or:

\[ V_L \Gamma^w = \frac{1}{2} \oint (E \times H) \cdot d\sigma + \frac{1}{2} \oint (E_s \times H_s^* ) \cdot d\sigma \]

\[ + \oint (E \times H) \cdot d\sigma + \oint (E_s \times H_s^* ) \cdot d\sigma \quad (22) \]

- \( \oint (E_0 \times H_s^* ) \cdot d\sigma \) is the incoming power on the surface \( \Sigma \) and it is always radial to this surface.
- \( \oint (E_s \times H_s^* ) \cdot d\sigma \) is the outgoing or reradiated complex power onto the surrounding space.

Additionally the expression

\[ \oint (E \times H) \cdot d\sigma + \oint (E_s \times H_s^* ) \cdot d\sigma = 0 \quad (23) \]

Because the incoming power is equal to the outgoing power in the receiving system as this signal just passes through and continues on beyond the surface.

For these reasons the power balance will be:

\[ V_L \Gamma^w = \oint (E \times H^* ) \cdot d\sigma + \oint (E_s \times H_s^* ) \cdot d\sigma \quad (24) \]

On the surface \( \Sigma \) the electric and magnetic fields are:

\[ E_i = 1_z E_i \quad (25) \]

representing the incident electric field of the incoming wave.

\[ E_s = -1_z E_s(z) \quad (26) \]

representing the scattered electric field by the receiving dipole antenna.

\[ H_s = 1_\rho H_s(z) \quad (27) \]

representing the scattered magnetic field by the receiving dipole antenna.

The incident electric field \( E_i \) is independent of the coordinate \( z \), because it depends on the incoming or plane wave, far away from its origin. This wave is travelling perpendicularly to the \( z \) axis as can be seen in Fig. 2.

The electric and magnetic scattered fields \( [E_s(z)] \) and \( [H_s(z)] \) are functions of the coordinate \( z \) because they depend on the current distribution impressed on the receiving antenna by the incoming wave. These scattered fields can be calculated from the \( z \) component of the vector potential as previously shown.

Performing the vectorial products yields the following result:

\[ E_i \times H_s^* = -1_\rho E_i H_s^*(z) \quad (28) \]

\[ E_s \times H_s^* = 1_\rho E_s(z) H_s^*(z) \quad (29) \]

Taking into account that the surface differential vector \( (d\sigma) \) is pointing inward, or

\[ d\sigma = -1_\rho d\sigma \quad (30) \]
The power balance expression is found to be

\[
V_L I_n^a = \oint E_3 \Phi_n^a(z) d\sigma - \oint E_s(z) \Phi_n^a(z) d\sigma \tag{31}
\]

where:

\[
\oint E_3 \Phi_n^a(z) d\sigma \tag{32}
\]

Is the incoming power.

\[
\oint E_s(z) \Phi_n^a(z) d\sigma \tag{33}
\]

Is the scattered or reradiated power.

The incoming power must be the sum of the load impedance power and the scattered power as given by the expression:

\[
\oint E_3 \Phi_n^a(z) d\sigma = V_L I_n^a + \oint E_s(z) \Phi_n^a(z) d\sigma \tag{34}
\]

This occurs in any point \( z \) along the antenna conducting wire. The Ampere Law must be fulfilled giving:

\[
H_s(z) = \frac{I(z)}{2\pi a} \tag{35}
\]

\( a \) is the antenna conducting wire radius.

Introducing this value in the power balance expression and taking into account the incoming electric field independence from the \( z \) coordinate, the antenna length being shorter than the wavelength, the current phase on the different antenna points is negligible.

\[
I^*(z) = I(z)
\]

When the phase reference is taken at a point when \( z = 0 \), the power balance becomes:

\[
E_3 I_0 \int_{-\frac{a}{2}}^{\frac{a}{2}} dz = V_L I_0 + \frac{I_0^2}{2} Z_a \tag{36}
\]

\[
\frac{I_0^2}{2} Z_a = \int_{-\frac{\Phi}{2}}^{\frac{\Phi}{2}} E_s(z) I_0 dz \tag{37}
\]

The above expressions is the complex power scattered when the receiving antenna impedance is \( Z_a = R_a + jX_a \) where in the ideal case losses are zero.

Dividing this expression by \( I_0 \) and performing the indicated integration, the power balance is found to be:

\[
E_3 dL = V_L + I_0 Z_a \tag{38}
\]

The first term is the voltage \( V_i \) induced by the incoming wave on the Hertz Dipole antenna, thus

\[
V_i = V_L + I_0 Z_a \tag{39}
\]

This is the voltage Kirchhoff Law fulfilled by the Hertz Dipole antenna equivalent circuit when it is used as a receiving antenna.

Fig. 3 shows this circuit.

For any other linear dipole antenna (\( L < \lambda \)) a similar equivalent circuit to the Hertz dipole antenna will be obtained when this antenna is used in the receiving mode. In this specific case the equivalent circuit is obtained applying the Thevenin Theorem as can be shown by Fig. 3.

Considering the antenna input terminals, the Thevenin Theorem establishes the necessity to know the voltage \( V_i \) with open terminals and the current \( I_{sc} \) with short circuited terminals.

Equivalent Thevenin circuit will have a generator of voltage \( V_i \) with an internal impedance \( Z_{ta} = V_i / I_{sc} \) as shown in Fig. 3. This circuit has a load impedance \( Z_L \) where part of the incoming wave power will be dissipated.

In order to represent the incoming wave, a far away Hertz dipole radiator can be imagined as having a network of four terminals as shown in Fig. 3.

Their equations are

\[
V_1 = Z_{11} I_1 + Z_{12} I_2 \tag{40}
\]

\[
V_2 = Z_{21} I_1 + Z_{22} I_2 \tag{41}
\]

Having short circuit current \( I_{sc} = -I_2 \) on the antenna 2 and making \( V_2 = 0 \), yields

\[
I_2 = -\frac{Z_{21}}{Z_{22}} I_1 \tag{42}
\]

or

\[
I_{sc} = \frac{Z_{21}}{Z_{22}} I_1 \tag{43}
\]
Having open circuit voltage $V_1 = V_2$ on the antenna 2 and making $I_2 = 0$ produces:

$$V_1 = Z_{21}I_1 \quad (44)$$

The impedance $Z_{\alpha}$ of the Thevenin equivalent generator is given by the expression

$$Z_{\alpha} = \frac{V_1}{I_{\text{oc}}} = Z_{21}I_1 \frac{Z_{22}}{Z_{21}I_1} = Z_{22} \quad (45)$$

And

$$Z_{\alpha} = \frac{V_2}{I_2} \bigg|_{I_1=0} \quad (46)$$

$Z_{\alpha}$ is the input antenna 2 impedance when it is in the transmitting mode and the antenna 1 current (Hertz dipole) is zero. In this case the Hertz dipole is not radiating. To obtain the open circuit induced voltage on the antenna 2 the reciprocity theorem is applied

$$Z_{21} = Z_{12} \quad (47)$$

In this case this is found to be

$$\frac{V_1}{I_2} \bigg|_{I_1=0} = \frac{V_2}{I_1} \bigg|_{I_2=0} \quad (48)$$

The open circuit voltage $V_1$ on the antenna 1 (Hertz dipole) is

$$V_1 = E_2L_1 \quad (49)$$

$E_2$ is the incoming electric field on the Hertz dipole due to the transmitting antenna 2 and $L_1$ is the effective length of the Hertz dipole, $L_1 = dL$, as was seen previously.

At the same time the open circuit voltage $V_2$ on the antenna 2 due to the transmitting antenna 1 is:

$$V_2 = E_1L_2 \quad (50)$$

$E_1$ is the incoming electric field on the antenna 2 due to the transmitting antenna 1 and $L_2$ is the unknown effective length of the antenna 2.

Having expression (48) the following relationship arises:

$$\frac{E_2L_2}{I_2} = \frac{E_1L_2}{I_1} \quad (51)$$

or

$$L_2 = \frac{I_1E_2}{I_2E_1}L_1 \quad (52)$$

This expression allows calculating the effective length of antenna 2 necessary in solving for the value of the open circuit voltage on the receiving antenna.

The Hertz dipole antenna’s far field for maximum radiation ($\theta = \pi/2$ rad) and vertical polarization is

$$E_1 = j\frac{\beta}{4\pi}I_1dL e^{-j\beta r} \quad (53)$$

where $r$ is the distance between antennas 1 and 2 according to Fig. 3.

If antenna 2 physical length is shorter than the wavelength $\lambda$ the Hertz far field expression can be used, replacing the differential length $dL$ by the antenna 2 effective length $L_e$, calculated by:

$$L_e = \frac{1}{\beta} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} I(z)dz \quad (54)$$

$I(z)$ is the antenna 2 current distribution in the transmitting mode.

Antenna 2 far field in the transmitting mode for maximum radiation ($\theta = \pi/2$ rad) and in vertical polarization is

$$E_2 = j\frac{\beta}{4\pi}I_2L_e e^{-j\beta r} \quad (55)$$

Replacing the fields $E_1$ and $E_2$ in expression (52) is found to be:

$$L_2 = \frac{I_1I_2L_e}{I_2I_1dL}L_1 \quad (56)$$

But, because the Hertz dipole antenna effective length is $L_1 = dL$, the expression (56) produces

$$L_2 = L_e \quad (57)$$

This is the antenna 2 effective length in the receiving mode. Substituting the effective length in expression (50) and taking into account that the $E_2$ field is the incoming field $E_3$ and the open voltage $V_2$ is the induced voltage $V_1$ now becomes

$$V_1 = E_3L_e \quad (58)$$

This expression is another definition for an antenna effective length in the receiving mode.

$$I_e = \frac{V_1}{E_3} \quad (59)$$

The effective length $L_e$ is the relationship between the induced open circuit voltage $V_1$ and the incoming electric field $E_3$, where a perfect polarization match is intended.

The Thevenin equivalent circuits of Fig. 3 fulfills the Voltage Kirchhoff Law:

$$V_1 = I_aZ_{\alpha} + I_3Z_L \quad (60)$$

That when multiplied by $I_a$ produces the received power $W_R$ on the load impedance $Z_L$:

$$W_R = |I_a|^2Z_L \quad (61)$$

$I_a$ is the effective current value flowing into the load impedance $Z_L$. Also, the received power is represented by the expression

$$W_R = V_1I_a* - I_a^2Z_{\alpha} \quad (62)$$
The first term is the power delivered by the equivalent Thevenin generator; the second is the power delivered on the receiving antenna impedance \( Z_a \).

The received power \( W_R \) according to the power balance equation is:

\[
W_R = V_i |I_a|^2
\]

\[
= E_i \int_{-\frac{L}{2}}^{\frac{L}{2}} \Gamma(z) dz - \int_{-\frac{L}{2}}^{\frac{L}{2}} E_s(z) \Gamma^*(z) dz \quad (63)
\]

Utilizing equations (62) and (63) it follows:

\[
E_i \int_{-\frac{L}{2}}^{\frac{L}{2}} \Gamma(z) dz - \int_{-\frac{L}{2}}^{\frac{L}{2}} E_s(z) \Gamma(z) dz = V_i |I_a|^2 - |I_a|^2 Z_a \quad (64)
\]

The first integral is the true power delivered by the incoming wave to the receiving antenna. The second integral is the receiving antenna scattered power into the surrounding space. This is the effect of the current distribution \( I(z) \) on the antenna conductor because it is the true current induced by the incoming wave and according to the load impedance \( Z_L \) connected on the antenna terminals.

In equation (64) it is not possible to affirm that the power delivered by the incoming wave is equal to the power delivered by the Thevenin generator to the equivalent circuit. At the same time it is not possible to ascertain that the scattered power is equal to the delivered power by the Thevenin generator to the antenna impedance \( Z_a \).

This means that the delivered power to the antenna impedance \( Z_a \) is not the same as the power scattered into the surrounding space.

This is only valid when the impedance matching is fulfilled, or, \( Z_L = Z_a \).

The lossless system case under matched conditions are given by the expressions:

\[
R_L = R_a = R_{\text{rad}} \quad (65)
\]

And

\[
X_L = -X_a \quad (66)
\]

The dissipated power across both impedances (\( Z_a \) and \( Z_a \)) are of the same value. The active received power \( W_R \) is of the same value as the scattered power \( W_s \) or “dissipated” by the receiving antenna radiation resistance \( R_{\text{rad}} \).

This is valid only for a dipole antenna in free space and whose length \( L \) is shorter than the wavelength. In any other case, the active scattered power is not necessarily dissipated on the radiation resistance of the equivalent Thevenin circuit and a more careful analysis must be made up especially to the incoming wave power on the receiving antenna system.

It was shown previously in the Hertz dipole antenna case the effective length is equal to the physical length (\( L_e = dL \)) and the induced open circuit voltage \( V_i \) by the incoming wave effective electric field \( E_i \) is found to be:

\[
V_i = E_i dL \quad (V) \quad (67)
\]

The power available by the receiving antenna \( W_{ra} \) due to the incoming wave is:

\[
W_{ra} = \frac{E_i^2 L^2}{4R_{\text{rad}}} \quad (68)
\]

And the power delivered to the load impedance is given by:

\[
W_R = \frac{V_i^2}{R_L} \quad (69)
\]

In a perfectly matched system and with no losses, the received power \( W_R \) on the load impedance \( Z_L \) is of the same value of the power available \( W_{ra} \) from the receiving antenna because \( V_L = V_i/2 \) to give:

\[
W_R = W_{ra} = \frac{V_i^2}{4R_{\text{rad}}} = \frac{V_i^2}{4R_L} = \frac{V_i^2}{R_L} \quad (W) \quad (70)
\]

or

\[
W_R = W_{ra} = \frac{E_i^2 L^2}{4R_{\text{rad}}} \quad (W) \quad (71)
\]

### III. Receiving Antenna Effective Area

The power available from the receiving antenna can be expressed according to the maximum effective receiving area \( A_{eRM} \) and the incoming wave power density \( P_i \) as:

\[
W_{ra} = A_{eRM} P_i \quad (W) \quad (72)
\]

The incoming wave power density \( P_i \) is related to the effective electric field strength \( E_i \) through the intrinsic space impedance \( Z_{00} \) to give:

\[
W_{ra} = \frac{E_i^2}{Z_{00}} A_{eRM} \quad (73)
\]

Equations (71) and (73) are the relationship between the maximum effective receiving area and the receiving antenna effective length, given by:

\[
A_{eRM} = \frac{Z_{00} E_i^2}{4R_{\text{rad}}} \quad (m^2) \quad (74)
\]

and

\[
I_e = 2 \sqrt{\frac{R_L A_{eRM}}{Z_{00}}} \quad (m) \quad (75)
\]

These relationships are obtained through the receiving antenna radiation resistance \( R_{\text{rad}} \) and the free space intrinsic impedance \( Z_{00} \), where the antenna is resonant.

In the case of a resonant antenna but not perfectly matched for \( R_L \gg R_{\text{rad}} \) the equations are:

\[
A_{eRM} = \frac{Z_{00} E_i^2}{R_L} \quad (m^2) \quad (76)
\]

and

\[
I_e = \sqrt{\frac{R_L A_{eRM}}{Z_{00}}} \quad (m) \quad (77)
\]
In case of monopole antennas the effective length $L_e$ must be replaced by the effective height $H_e$ in all the previous equations.

In order to calculate the receiving antenna directivity or gain the logical way is determining the effective antenna length $L_e$, obtained from the receiving antenna current distribution $I(z)$.

Maximum receiving antenna directivity when no losses are involved is found to be:

$$G_{RM} = D_{RM} = \frac{4\pi}{\lambda} A_{eRM}$$  \hspace{1cm} (78)

Also

$$G_{RM} = D_{RM} = \frac{\pi Z_0}{R_{load}} \left( \frac{L_e}{\lambda} \right)^2$$  \hspace{1cm} (79)

or:

$$G_{RM} = D_{RM} = \left( \frac{2 L_e}{\lambda} \right)^2 \left( \frac{120}{R_{load}} \right)$$  \hspace{1cm} (80)

It can be seen clearly obtaining the receiving antenna directivity is different from the transmitting case.

Transmitting antenna directivity $D_T$ or gain $G_T$ is obtained by the power density radiation function $F(\theta, \phi)$ integration in the surrounding space. For a dipole antenna oriented in the $z$ axis is given by:

$$D_T = \frac{U_{\text{max}}}{U_0}$$  \hspace{1cm} (81)

$$D_T = \frac{4\pi U_{\text{max}}}{\int_0^{2\pi} d\phi \int_0^\pi U_{\text{max}} F(\theta) \sin(\theta) d\theta}$$  \hspace{1cm} (82)

- $F(\phi)$ is an angle $\phi$ function and constant in this case.
- $F(\theta)$ is an angle $\theta$ function.
- $U_{\text{max}}$ is the maximum radiation intensity (W/sq rad).
- $U_0$ is the average radiation intensity (W/sq rad).

IV. Transmitting Antenna Effective Area

In the transmitting case the maximum equivalent effective transmitting area for zero losses is found to be:

$$A_{eTM} = \frac{\lambda^2}{4\pi} G_{TM} = \frac{\lambda^2}{4\pi} D_{TM} \quad (\text{m}^2)$$  \hspace{1cm} (83)

$G_{TM} = D_{TM}$ is the maximum transmitting gain or maximum directivity occurring when angle $\theta$ is 90 degrees or $\pi/2$ radians (efficiency $\eta = 1$).

In the transmitting case the transmitting antenna area is determined knowing the directivity or gain. On the contrary, in the receiving antenna case the directivity or gain is determined knowing the receiving antenna effective length or the receiving antenna area.

This statement is very important:

“Only in free space the antenna directivity or gain is the same in the transmitting and receiving case if both dipole antennas are physically identical, if not, this cannot be fulfilled. This is also valid for the transmitting and receiving antenna areas”.

V. Antenna Factor

Receiving antenna factor $A_f$ is a similar concept as the effective length or height except the incoming wave received voltage is evaluated on the load impedance instead at the receiving antenna open terminals. In this case, this concept can include the receiving antenna and matching losses.

If no losses are involved and having a perfect matched conditions the load impedance voltage $V_L$ is found to be:

$$V_L = \frac{V_i}{2}$$  \hspace{1cm} (84)

In these conditions the minimum receiving antenna factor $A_{fRm}$ is obtained and it is given by:

$$A_{fRm} = \frac{V_i}{V_L} = \frac{2 F_4}{V_i} = \frac{2}{I_e} \left( \frac{1}{\text{in}} \right)$$  \hspace{1cm} (85)

In free space the effective half wave theoretical dipole antenna length is given by:

$$L_e = \frac{1}{I_0} \int_0^H I(z) dz = \frac{1}{I_0} \int_0^H I_M \sin(\beta(H - z)) dz$$  \hspace{1cm} (86)

For $z = 0$, $I(z) = I(0) = I_0 = I_M \sin(\beta H)$:

$$L_e = \frac{2}{\sin(\beta H)} \int_0^H \sin(\beta(H - z)) dz$$  \hspace{1cm} (87)

$$L_e = \frac{\lambda}{\pi \sin(\beta H)} (1 - \cos(\beta H))$$  \hspace{1cm} (88)

For $L = \lambda/2$, $H = \lambda/4$, $\beta H = \pi/2$ (rad) or $\beta H = 90^\circ$:

$$L_e = \frac{\lambda}{\pi}$$  \hspace{1cm} (89)

or

$$L_e = 0.3183 \lambda$$  \hspace{1cm} (90)

where $H = L/2$.

Its minimum antenna factor is found to be:

$$A_{fRm} = \frac{2}{I_e} \frac{6.2832}{\lambda} \left( \frac{1}{\text{m}} \right)$$  \hspace{1cm} (91)

This receiving antenna factor ($A_{fRm}$) is useful for field strength measurements because by taking into account the load impedance voltage ($V_L$), the losses included in the balanced system, in the circuit resistances and in the matching factors can be included on it. This, of course, will increase the minimum antenna factor value.

It will be shown, that the transmitting or receiving antennas effective length ($L_e$) and consequently the minimum antenna factor ($A_{fRm}$) are of the same value in free space, or, over a perfect ground plane. The relevant parameters depend only on the antenna current distribution $I(z)$ as it is independent of the input power generated by a generator or by an incoming wave.
The received to transmitted power relationship utilizes the maximum transmitting and receiving area as applied by the Friis Equation:

\[
\frac{W_R}{W_T} = A_{eRM}A_{eTM} \left(\frac{1}{\lambda}\right)^2 \tag{92}
\]

- \(W_R\) is the maximum received power (W).
- \(W_T\) is the transmitted power (W).
- \(A_{eRM}\) is the maximum effective receiving area (m²).
- \(A_{eTM}\) is the maximum effective transmitting area (m²).
- \(r\) is the distance between both antennas (m).
- \(\lambda\) is the operating wavelength (m).

Also

\[
A_{eTM}A_{eRM} = \frac{W_R}{W_T}(r\lambda)^2 \tag{93}
\]

and

\[
G_{TM}G_{RM} = \frac{W_R}{W_T} \left(\frac{4\pi\lambda}{\lambda}\right)^2 \tag{94}
\]

With no losses (\(\eta = 1\)), \(G_{TM} = D_{TM}\), \(G_{RM} = D_{RM}\), and:

\[
D_{TM}D_{RM} = \frac{W_R}{W_T} \left(\frac{4\pi\lambda}{\lambda}\right)^2 \tag{95}
\]

or

\[
D_{TM}(d\Delta i) + D_{RM}(d\Delta i) = A_{e}(d\Delta) - A_{w}(d\Delta) \tag{97}
\]

where:

\[
A_{w} = \frac{W_T}{W_R}
\]

Is the transmitted-received power relationship.

\[
A_{e} = \left(\frac{4\pi\lambda}{\lambda}\right)^2
\]

Is the free space loss. Receiving antenna efficiency \(\eta = G_{RM}/D_{RM}\) is the relationship between the delivered power \(W_R\) at the input receiver impedance and the maximum power \(W_{R_{\text{max}}}\) delivered in the perfectly matched theoretical condition.

### VI. MONOPOLE

A theoretical monopole is installed on a perfect ground plane of infinite surface and conductivity. This is shown on Fig. 4. Its physical height is \(H\) and half of the dipole length \((H = L/2)\).

Monopole antennas have been in operation since early wireless telegraphic communications and its effective height was defined in the transmitting case as:

\[
H_e = \frac{1}{I_0} \int_{0}^{H} I(\xi) d\xi \quad (\text{m}) \tag{98}
\]

For a Hertz monopole as seen in Fig. 4 whose height is \(dH\) and with a constant current distribution \(I(\xi) = I_0\) the effective height is given by:

\[
H_e = dH \quad (\text{m}) \tag{99}
\]

Its radiation resistance is:

\[
R_{\text{rad}} = 160\pi^2 \left(\frac{H}{\lambda}\right)^2 \tag{100}
\]

For a non top loaded short monopole \((H < \lambda/8)\) the effective height \(H_e\) is found to be:

\[
H_e = \frac{1}{I_0} \int_{0}^{H} I(\xi) d\xi = \frac{\lambda}{2\pi\sin(\lambda/4)} \left(1 - \cos(\lambda H)\right) \tag{101}
\]

\[
H_e = \frac{H}{2} \tag{102}
\]

Its radiation resistance is:

\[
R_{\text{rad}} = 40\pi^2 \left(\frac{H}{\lambda}\right)^2 = 160\pi^2 \left(\frac{H_e}{\lambda}\right)^2 \tag{103}
\]
The monopole antenna effective height is obtained to be:

\[
H_e = \frac{1}{I_0} \int_0^H I_M \sin(\beta(H-z)) \, dz
\]

\[
= \frac{\lambda}{2\pi \sin(\beta H)} \left( 1 - \cos(\beta H) \right) \quad (104)
\]

Equation (104) describes the effective height of a quarter-wave monopole, when \( H = \lambda/4 \)

\[
\beta H = 90^\circ \quad \text{or} \quad \beta H = \frac{\pi}{2} \text{ rad}
\]

\[
H_e = \frac{A}{2\pi} \quad (105)
\]

or

\[
H_e = 0.15915\lambda \quad (106)
\]

Its radiation resistance is \( R_{\text{rad}} \approx 36.5(\Omega) \) and depends slightly on the \( H/\lambda \) relationship as pointed out by Trainotti [52], diminishing its value.

Monopole transmitting directivity is:

\[
D_T = \frac{4\pi U_{\text{max}}}{\int_0^{\pi/2} U_{\text{max}} F(\theta) \sin(\theta) \, d\theta} \quad (107)
\]

The maximum transmitting area is:

\[
A_{\text{TM}} = \frac{\lambda^2}{4\pi} D_{\text{TM}} \quad (108)
\]

The maximum receiving area \( A_{\text{RM}} \), knowing the effective height \( H_e \), can thus be calculated with:

\[
A_{\text{RM}} = \frac{Z_0 I_R^2}{4R_{\text{rad}}} \quad (109)
\]

The maximum receiving directivity is given by:

\[
D_{\text{RM}} = \frac{4\pi A_{\text{RM}}}{\lambda^2} \quad (110)
\]

also

\[
D_{\text{RM}} = \frac{\pi Z_0}{R_{\text{rad}}} \left( \frac{H_e}{\lambda} \right)^2 \quad (111)
\]

or using the Friis equation the directivity can be calculated with

\[
D_{\text{RM}} = \frac{W_T}{W_{\text{TM}}} \left( \frac{4\pi I}{\lambda} \right)^2 \quad (112)
\]

In the monopole antenna case always the transmitting and receiving directivities are of different value. Also of different value are the effective area for transmitting and receiving antennas.

However, the effective height and antenna factors are always the same in the transmitting and receiving cases because they are functions of current distribution and are not dependent on the transmitting and receiving power. Of course, this is valid for physically identical antennas, in both the transmitting and receiving cases perfectly matched.
The current relationship is given by the relationship between the current \( I_T \) flowing in the antenna circuits of the transmitter and the current \( I_L \) on the receiver load impedance:

\[
A_c = \frac{I_T}{I_L}
\]  

(117)

and:

\[
A = 20 \log \left( \frac{I_T}{I_L} \right)
\]  

(118)

Of course, the three definitions must yield practically the same value. If not the problem must be carefully analyzed.

The site attenuation \( A \) is the result of the space attenuation \( A_s \) and the antenna directivities or gains, obtaining:

\[
A = \frac{A_s}{D_{TM}D_{RM}}
\]  

(119)

The space attenuation is given by:

\[
A_s = \left( \frac{4\pi r}{\lambda} \right)^2
\]  

(120)

or:

\[
A_s = 20 \log \left( \frac{4\pi r}{\lambda} \right)
\]  

(121)

Resulting in:

\[
A(dB) = A_s(dB) - [G_{TM}(dB) + G_{RM}(dB)]
\]  

(122)

For no losses \( (\eta = 1) \):

\[
A(dB) = A_s(dB) - [D_{TM}(dB) + D_{RM}(dB)]
\]  

(123)

In actual practice, it is very easy to measure the transmitting antenna input voltage \( V_T \) or the output generator voltage and the voltage \( V_L \) measured on the receiving antenna load impedance, especially using an accurate field strength meter or spectrum analyzer, obtaining this way the site attenuation \( A_c \) very precisely if both antennas are physically identical. Alternatively the input power \( P_T \) of a transmitting antenna can easily be measured and the received power \( P_R \) can precisely be measured by means of a good spectrum analyzer.

Free space attenuation \( A_s \) is calculated knowing the distance \( r \) and the operating frequency or wavelength.

The result obtained is the transmitting and receiving antenna directivity product. If both antennas, dipoles or monopoles are physically identical, does not represent that they are electromagnetically identical, as generally they can get different directivity or gain in the transmitting and receiving modes.

It is important not to assign the same directivity value to both of them. Nevertheless, the effective length or height is the same in the transmitting and receiving case if the antennas are physically identical.

The transmitting antenna directivity can be calculated accurately when half wave dipoles or quarter wave monopoles are used. Once, the received voltage \( V_L \) or received power \( P_R \) is known, this directivity can be determined either using the Friis Equation or else by utilizing the effective length or height of the receiving antenna.

Following a similar analysis, such as Smith [14], and considering the antennas in the receiving and transmitting modes as shown in Fig. 5, the power density in space for a radio link in free space, or over a perfect ground plane, is given by:

\[
P_i = \frac{W_T D_{TM}}{4\pi r^2} e^{-j2\pi r}
\]  

(124)

For the effective electric field strength:

\[
E_i = \sqrt{\frac{P_i}{Z_{00}}} \left( \frac{e^{-j2\pi r}}{r} \right)
\]  

(125)

\( Z_{00} \) is the free space intrinsic impedance.

The transmitted power radiated into space by the transmitting antenna, in ideal and matched conditions, is obtained as:

\[
W_T = \frac{I_T^2 R_{rad}}{2} = \frac{P_i}{2} R_g
\]  

(126)

\[
R_{rad} = R_0 T = R_g
\]

Replacing the transmitted power in the effective electric field expression, it follows that:

\[
E_i = I_T \sqrt{\frac{R_{rad} D_{TM} Z_{00}}{4\pi}} \left( \frac{e^{-j2\pi r}}{r} \right)
\]  

(127)

The receiving antenna factor \( A_f R \) was defined previously as:

\[
A_f = \frac{E_i}{V_L} = \frac{V_i}{I_c V_L}
\]  

(128)

The load impedance current \( I_L \) is:

\[
I_L = \frac{V_L}{R_L} = \frac{V_i}{R_L I_c A_f R}
\]  

(129)

The receiving available power \( W_{ra} \) is:

\[
W_{ra} = \frac{E_i^2 A_f^2}{4\pi^2 r^2 D_{RM}}
\]  

(130)

Now, under matching conditions \( R_{rad} = R_L (R_{rad} = R_{atT}) \) and \( W_{ra} = W_R \), thus, the load voltage \( V_L \) is:

\[
V_L = \frac{E_i A_f}{2\pi} \sqrt{\frac{R_{rad} D_{RM}}{120}} = \frac{V_i}{2}
\]  

(131)

The induced voltage \( V_i \) of the incoming wave with power density \( P_i \) in the receiving antenna Thevenin circuit is obtained from the following equation:

\[
V_i = \frac{E_i A_f}{\pi} \sqrt{\frac{R_{rad} D_{RM}}{120}} = E_i I_c
\]  

(133)
Where the effective length is:

\[ I_e = \frac{\lambda}{\pi} \sqrt{\frac{R_{rad}D_{RM}}{120}} \]  
(134)

or

\[ D_{RM} = \left( \frac{\pi I_e}{\lambda} \right)^2 \left( \frac{120}{R_{rad}} \right) \]  
(135)

For a theoretical half wavelength dipole, the maximum directivity in the receiving mode is obtained from

\[ D_{RM}(L = \lambda/2) = \frac{120}{R_{rad}} \]  
(136)

Because \( I_e = \lambda/\pi \).

The current flowing through the input receiver load \( R_L \) is shown with the following:

\[ I_L = \frac{V_i}{R_L A_f R} = \frac{V_i \pi}{R_L A_f R} \sqrt{\frac{120}{R_{rad}D_{RM}}} \]  
(137)

or

\[ I_L = \frac{V_i}{R_L A_f R} \sqrt{\frac{Z_{00}}{4R_{rad}A_{eRM}}} \]  
(138)

Invoking the reciprocity theorem, in the transmitting mode (shown in Fig. 5), the transmitting antenna current \( I_T \) flowing through the transmitting antenna radiation resistance, with a matched condition, is given as follows:

\[ I_T = \frac{V_g \pi}{R_g A_f T} \sqrt{\frac{120}{R_{rad}D_{RM}}} \]  
(139)

also,

\[ I_T = \frac{V_g \pi}{R_g A_f T} \sqrt{\frac{Z_{00}}{4R_{rad}A_{eRM}}} \]  
(140)

With this current flowing into the transmitting antenna the incoming wave effective electric field \( E_i \) in space, becomes:

\[ E_i = \frac{60\pi V_g}{R_g A_f T} \sqrt{\frac{D_{TM}}{D_{RM}}} \left( \frac{e^{-j\varphi}}{r} \right) \]  
(141)

Taking into account that \( V_g = 2V_T \), the field strength \( E_i \) is:

\[ E_i = \frac{120\pi V_T}{R_g A_f T} \sqrt{\frac{D_{TM}}{D_{RM}}} \left( \frac{e^{-j\varphi}}{r} \right) \]  
(142)

Or, in terms of the effective areas:

\[ E_i = \frac{120\pi V_T}{R_g A_f T} \sqrt{\frac{A_{eTM}}{A_{eRM}}} \left( \frac{e^{-j\varphi}}{r} \right) \]  
(143)

From a previous definition, the voltage site attenuation \( A_v \) is:

\[ A_v = \frac{V_T}{V_L} \]  
(144)

And the voltage \( V_L \) as measured with the field strength meter is:

\[ V_L = \frac{E_i}{A_f R} \]  
(145)

The answer for the voltage site attenuation is:

\[ A_v = \frac{V_T A_f R}{E_i} \]  
(146)

Now, putting the electric field strength parameter in the voltage site attenuation equation, results in:

\[ |A_v| = \frac{A_f R A_f T R_g A_f T}{120\pi} \sqrt{\frac{D_{RM}}{D_{TM}}} \]  
(147)

also

\[ |A_v| = \frac{A_f R A_f T R_g A_f T}{120\pi} \sqrt{\frac{A_{eRM}}{A_{eTM}}} \]  
(148)

As a function of frequency the voltage site attenuation results:

\[ |A_v| = \frac{7.06 \times 10^6 A_f R A_f T R_g A_f T}{120\pi} \sqrt{\frac{D_{RM}}{D_{TM}}} \]  
(149)

or

\[ |A_v| = \frac{7.06 \times 10^6 A_f R A_f T R_g A_f T}{120\pi} \sqrt{\frac{A_{eRM}}{A_{eTM}}} \]  
(150)

For physically identical antennas \( A_f R = A_f T \) resulting:

\[ |A_v| = \frac{7.06 \times 10^6 A_f R A_f T R_g A_f T}{120\pi} \sqrt{\frac{A_{eRM}}{A_{eTM}}} \]  
(151)

The site attenuation in dB is obtained as:

\[ A = 20 \log |A_v| \]  
(152)

VIII. PRACTICAL EXAMPLES

For practical examples the following equations will be applied for vertically polarized dipoles or monopoles.

**Dipole transmitting antenna directivity**

\[ D_T = \frac{U_{max}}{U_0} = \frac{4\pi}{\int_0^{2\pi} d\phi \int_0^{\pi/2} F(\theta) \sin(\theta) d\theta} \]  
(153)

**Monopole transmitting antenna directivity**

\[ D_T = \frac{U_{max}}{U_0} = \frac{4\pi}{\int_0^{2\pi} d\phi \int_0^{\pi/2} F(\theta) \sin(\theta) d\theta} \]  
(154)

**Transmitting antenna effective area**

\[ A_{eTM} = \frac{\lambda^2}{4\pi} D_{TM} \ (m^2) \]  
(155)

**Power density in space**

\[ P_i = \frac{W_T D_{TM}}{4\pi r^2} \ (\frac{W}{m^2}) \]  
(156)

**Effective electric field intensity**

\[ E_i = \sqrt{P_i Z_{00}} \ (\frac{V}{m}) \]  
(157)
Effective length

\[ I_e = \frac{1}{L_0} \int_{-H}^{H} 1(Z) \, dz \quad (\text{m}) \quad (158) \]

Effective height

\[ H_e = \frac{1}{L_0} \int_{0}^{H} 1(Z) \, dz \quad (\text{m}) \quad (159) \]

Dipole receiving antenna induced voltage in open circuit

\[ V_i = E_d I_e \quad (160) \]

Monopole receiving antenna induced voltage in open circuit

\[ V_i = E_d H_e \quad (161) \]

Receiving antenna equivalent Thevenin circuit current under matching conditions

\[ I_a = \frac{V_i}{2R_{rad}} = \frac{V_i}{2R_L} \quad (162) \]

Received power

\[ W_R = I_a^2 R_L = I_e^2 R_L = \frac{V_i^2}{R_L} \quad (163) \]

Receiving antenna effective area

\[ A_{eRM} = \frac{W_R}{P_i} \quad (164) \]

For a dipole antenna

\[ A_{eRM} = \frac{Z_{00} L_e^2}{4R_{rad}} \quad (165) \]

For a monopole antenna

\[ A_{eRM} = \frac{Z_{00} H_e^2}{4R_{rad}} \quad (166) \]

Receiving antenna directivity or gain \((\eta = 1)\)

\[ D_{RM} = G_{RM} = \frac{W_R}{W_T D_{TM}} \left( \frac{4\pi \lambda}{\lambda} \right)^2 \quad (167) \]

Transmitting and receiving antenna factor (minimum value) for physically identical dipole antennas perfectly matched.

\[ A_f = A_f R = A_f = \frac{2}{I_e} \left( \frac{1}{\text{m}} \right) \quad (168) \]

Transmitting and receiving antenna factor (minimum value) for physically identical monopole antennas perfectly matched.

\[ A_f = A_f R = A_f = \frac{2}{H_e} \left( \frac{1}{\text{m}} \right) \quad (169) \]

Voltage site attenuation

\[ A_v = \frac{V_T}{V_L} \quad (171) \]

Current site attenuation

\[ A_c = \frac{I_T}{I_a} = \frac{I_T}{I_L} \quad (172) \]

Power site attenuation

\[ A_w = \frac{W_T}{W_R} \quad (173) \]

Site attenuation

\[ A = 10 \log \left( \frac{W_T}{W_R} \right) \]

\[ = 20 \log \left( \frac{V_T}{V_L} \right) \]

\[ = 20 \log \left( \frac{I_T}{I_a} \right) = 20 \log \left( \frac{I_T}{I_L} \right) \quad (174) \]

Important: These three \(A\) values must yield the same result.

From field strength measurements it can be determined:

Field strength

\[ E_i = V_L A_f R \quad (\text{V/m}) \quad (175) \]

Power density

\[ P_i = \frac{E_i^2}{Z_{00}} \left( \frac{W}{\text{m}^2} \right) \quad (176) \]

Maximum receiving antenna effective area

\[ A_{eRM} = \frac{W_R}{P_i} \quad (\text{m}^2) \quad (177) \]

Maximum receiving antenna directivity or gain \((\eta = 1)\)

\[ D_{RM} = G_{RM} = \frac{4\pi}{\lambda^2} A_{eRM} \quad (178) \]

IX. Example 1

Radio link between two very short dipole antennas as shown in Fig. 6(a) in free space \((L = 0.1\lambda)\).

Data:

- \(f = 100\) (MHz)
- \(\lambda = 3\) (m)
- \(L = 0.1\lambda = 0.3\) (m)
- \(W_T = 1\) (W)
- \(r = 30\) (m)

Results:

- \(D_{TM} = 1.50 (1.76 \text{ dBi})\)
- \(R_{rad} = R_{rad T} = 1.97\) (\(\Omega\))
- \(A_{eTM} = 1.07\) (m²)
- \(V_T = 1.40\) (V)
X. Example 2

Radio link between two short monopole antennas as shown in Fig. 6(b) over perfect ground plane (H = 0.05 \lambda).

**Data:**
- f = 100 (MHz)
- \lambda = 3 (m)
- H = 0.05 \lambda = 0.15 (m)
- W_T = 1 (W)
- r = 30 (m)

**Result:**
- D_{TM} = 3 (4.77 dBi)
- R_{rad} = R_{uT} = 0.99 (\Omega)
- A_{eTM} = 2.15 (m²)
- V_T = 0.99 (V)
- I_T = 1.01 (A)
- P_i = 2.65 \cdot 10^{-1} (W/m²)
- E_i = 3.16 \cdot 10^{-1} (V/m)
- H_e = 0.075 (m)
- V_i = 2.37 \cdot 10^{-2} (V)
- R_{rad} = R_{uR} = 0.99 (\Omega)
- I_L = 1.20 \cdot 10^{-2} (A)
- W_R = 1.43 \cdot 10^{-1} (W)
- A_{eRM} = 0.537 (m²)
- [\begin{array}{c}
  D_{RM} = 0.75 (-1.25 dBi) \\
  V_L = 1.18 \cdot 10^{-2} (V) \\
  A_f = 26.67 (AF = 28.52 dB/m) \\
  A_v = 84.15 (38.50 dB) \\
  A_c = 83.92 (38.48 dB) \\
  A_w = 6993 \\
  A = 38.45 (dB) \\
  A_n = 1.58 \cdot 10^4 (41.98 dB)
\end{array}]
- [\begin{array}{c}
  A_{e} - A \equiv 41.98 - 38.45 = 3.53 (dB) \\
  D_{TM} + D_{RM} = 4.77 - 1.25 = 3.53 (dB)
\end{array}]
- D_{TM} \cdot D_{RM} = 2.25
- D_{TM}/D_{RM} = A_{eTM}/A_{eRM} = 4 (6 dB)

**Note:** Short monopoles have different transmitting and receiving directivities or D_{TM} = 4.77 dBi and D_{RM} = -1.25 dBi. Received power W_R is of the same value as for two short dipoles in free space. Directivity product is of the same value as for two short dipoles (H = L/2). Transmitting and receiving directivity relationship corresponds to D_{TM} - D_{RM} = 4.77 - (-1.25) = 6.02 dB or the transmitting antenna directivity is 6.02 dB higher than the receiving antenna directivity, but the power balance is exactly the same as the free space case, where the directivities are of the same value.

XI. Example 3

Radio link between two theoretical half wave dipole antennas in free space (H/\lambda \approx \infty) (similar to Fig. 6(a)).

**Data:**
- f = 100 (MHz)
- \lambda = 3 (m)
- L = 0.5 \lambda = 1.5 (m)
- W_T = 1 (W)
- r = 30 (m)

**Results:**
- D_{TM} = 1.64 (2.15 dBi)
Radio link between two quarter wave monopole antennas over perfect ground plane (similar to Fig. 6(b)).

Data:
- \( f = 100 \) (MHz)
- \( \lambda = 3 \) (m)
- \( H = 0.25 \lambda = 0.75 \) (m)
- \( W_T = 1 \) (W)
- \( r = 30 \) (m)

Results:
- \( D_{TM} = 3.273 \) (5.15 dB)
- \( R_{rad} = R_{AT} = 30.5 \) (\( \Omega \))
- \( A_{ETM} = 23420 \) (m\(^2\))
- \( V_T = 191 \) (V)
- \( I_T = 1.26 \cdot 10^{-1} \) (A)
- \( P_T = 2.90 \cdot 10^{-4} \) (W/m\(^2\))
- \( E_3 = 3.31 \cdot 10^{-1} \) (V/m)
- \( H_R = 4.77 \cdot 10^{-1} \) (m)
- \( V_L = 1.58 \cdot 10^{-1} \) (V)
- \( R_{rad} = R_{AR} = 30.5 \) (\( \Omega \))
- \( I_L = 2.16 \cdot 10^{-3} \) (A)
- \( W_R = 1.71 \cdot 10^{-4} \) (W)
- \( A_{ARM} = 0.59 \) (m\(^2\))
- \( D_{RM} = 0.826 \) (-0.83 dB)
- \( V_L = 7.88 \cdot 10^{-3} \) (V)
- \( A_f = 4.19 \) (AF = 12.44 dB/m)
- \( A_v = 76.65 \) (37.69 dB)
- \( A_c = 76.85 \) (37.71 dB)
- \( A_w = 5848 \)
- \( A = 37.67 \) (dB)

- \( A_o = 1.58 \cdot 10^4 \) (41.98 dB)
- \( \frac{A_o - A}{A} = 41.98 - 37.67 = 4.31 \) (dB)
- \( D_{TM} + D_{RM} = 5.15 + 0.83 = 6.32 \) (dB)
- \( D_{TM} \cdot D_{RM} = 2.08 \)
- \( D_{TM}/D_{RM} = A_{ETM}/A_{ARM} = 4 \) (6 dB)

Note: Two quarter wave monopole antennas have different transmitting and receiving antenna directivities. Received power is of the same value as for two half wave dipole antennas in free space. Directivity product has the same value as for two half wave dipole antennas in free space. Transmitting and receiving directivity relationship corresponds to \( D_{TM} - D_{RM} = 5.15 - (-0.83) = 5.97 \approx 6 \) dB or the transmitting antenna directivity is 6 dB higher than the receiving antenna directivity, but the power balance is exactly the same as the free space case, where the directivities are of the same value.
free space. Transmitting and receiving directivity relationship corresponds to $D_{TM} - D_{RM} = 5.15 - (-0.89) = 6.04 \approx 6 \text{ dB}$ or the transmitting antenna directivity is 6 dB higher than the receiving antenna directivity.

XIV. EXAMPLE 6

Radio link between two vertically polarized resonant dipole antennas over perfect ground plane according to Fig. 7.

Center dipole antenna height $h_m$ is varied from $0.2\lambda_{07}$ to $2.5\lambda$. Transmitting and receiving antenna effective length were obtained by current integration. WIPL-D software [35] was used to obtain the dipole antenna resonance and its input impedance as a function of the antenna height over ground.

Data:
- $f = 100$ (MHz)
- $H/\lambda = 100$
- $\lambda = 3$ (m)
- $L = 0.4667 \lambda$
- $W_T = 1$ (W)
- $r = 30$ (m)

Results are represented in Fig. 8(a)–(e).

Transmitting and receiving vertical dipole antenna resonance average value for $H/\lambda = 100$ was found to be:

$$L = 0.4667 \lambda$$

Average values of transmitting and receiving antenna effective length $L_e$ and antenna factors $AF$ (minimum value) have been determined to be practically equal varying the center dipole antenna height over perfect ground, to be:

$$L_e = 0.31 \lambda$$

$$AF_T = AF_R = AF = 6.65 \text{ dB}$$

Maximum transmitting and receiving directivity have been found of different value but its first maximum value is found at a half wavelength center antenna height over ground:

$$D_{TM} = 8.39 dBi$$

$$D_{RM} = 2.36 dBi$$

$$D_{TM} - D_{RM} \approx 6 \text{ dB}$$
These same results can be obtained at any other frequency for the same H/a relationship.

XV. EXAMPLE 7

Radio link between two vertical resonant dipole antennas over a perfect ground plane varying the operation frequency in the VHF spectrum according to Fig. 7.

Data:
- H/a = 100
- L = 0.4667 λ
- Iw = 0.31 λ
- hTm = 2 m
- hRm = 0.5 λ
- W = 1 (W)
- r = 30 (m)

Results can be seen Fig. 9 and Table I.

Site attenuation results are practically the same as obtained by NBS (FitzGerrell [5]) up to 300 MHz. Here the receiving antenna center height was adopted for the maximum directivity or hRm = 0.5 λ. The transmitting and receiving antennas over ground are really an array of two elements considering the antenna images and for this reason maximum directivity is found to be at 90 degrees with the z axis or along the ground plane for any antenna heights. At the same time increasing the transmitting antenna height it doesn’t reach the free space directivity value but always a value practically four times its free space value (DiTM ≈ 8 dBm). At the same time, NBS measurements for frequencies higher than 200 MHz are not made in the minimum receiving antenna height (hRm = 0.5 λ), but in the second or third value in order to obtain a minimum site attenuation. The transmitting antenna height is maintained as constant of 2 meters in height by NBS. No H/a dipole antenna relationship value is mentioned in the NBS report.

XVI. EXAMPLE 8

Radio link between a quarter wave monopole antenna and a vertical resonant dipole antenna over perfect ground as shown in Fig. 10.

Data:
- f = 100 (MHz)
TABLE I
EXAMPLE 7 RESULTS

<table>
<thead>
<tr>
<th>Freq. [MHz]</th>
<th>44</th>
<th>65</th>
<th>100</th>
<th>143</th>
<th>210</th>
<th>311</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_{TM} [dBi]</td>
<td>7.15</td>
<td>8.35</td>
<td>8.07</td>
<td>8.12</td>
<td>8.11</td>
<td>8.11</td>
</tr>
<tr>
<td>L_e [m]</td>
<td>2.11</td>
<td>1.43</td>
<td>0.93</td>
<td>0.65</td>
<td>0.44</td>
<td>0.30</td>
</tr>
<tr>
<td>D_{RM} [dBi]</td>
<td>2.23</td>
<td>2.28</td>
<td>2.29</td>
<td>2.30</td>
<td>2.31</td>
<td>2.34</td>
</tr>
<tr>
<td>A_{FM} [dB/m]</td>
<td>-0.47</td>
<td>2.92</td>
<td>6.65</td>
<td>9.76</td>
<td>13.10</td>
<td>16.51</td>
</tr>
<tr>
<td>A [dB]</td>
<td>25.48</td>
<td>27.62</td>
<td>31.62</td>
<td>34.66</td>
<td>38.01</td>
<td>41.39</td>
</tr>
<tr>
<td>A(NBS) [dB]</td>
<td>25.20</td>
<td>28.00</td>
<td>31.60</td>
<td>36.70</td>
<td>38.50</td>
<td>46.1</td>
</tr>
<tr>
<td>D_{TM} + D_{RM} [dB]</td>
<td>9.39</td>
<td>10.63</td>
<td>10.36</td>
<td>10.42</td>
<td>10.42</td>
<td>10.45</td>
</tr>
</tbody>
</table>

Note: In this case the site attenuation can’t be calculated by the current or voltage relationship but only by the power relationship.

Voltage site attenuation calculated by (149) it doesn’t give good result either. It seems this equation is valid only between two physically identical antennas. Nevertheless, \((20 \log(39.37)+20 \log(72.49))/2 = 34.55 \text{ dB}\) or the average between current and voltage site attenuation has the same value as the site attenuation obtained as power relationship.

Transmitting antenna factor is found to be:

\[
H_{eT} = \frac{\lambda}{2\pi} = 0.159\lambda = 0.4775\text{ (m)}
\]

\[
A_{fT} = \frac{2}{H_{eT}}
\]

\[
A_{fT} = 4.19 \frac{1}{\lambda} \left( AF = 12.44 \text{ dB/m} \right)
\]

The receiving antenna factor \(A_{fR}\) obtained is 6.65 dB/m like in the Example 6, using a transmitting monopole instead of a physically identical antenna, like a resonant half wave dipole antenna.

XVII. EXAMPLE 9

Radio link between a transmitting monopole antenna and a receiving loop antenna in medium frequency band as can be shown in Fig. 11.

In this example the receiving antenna loop has a diameter of 0.25 m and three shielded turns.

The receiving antenna effective height, antenna factor and radiation resistance can be seen in Table II and Fig. 17, according to the physical antenna size [7].

Loop effective height can be calculated by:

\[
H_e = \frac{2\pi}{\lambda} n S = \frac{n\pi^2D^2}{2\lambda}
\]
Fig. 11. Radio link between a transmitting monopole antenna and a receiving loop antenna.

**A. Example 9.a**

The antenna radiation resistance of the theoretical loop is extremely small as can be seen in Table II (see Fig. 12). If the antenna is operating in open circuit the output voltage $V_L$ is equal to the induced voltage $V_i$ and the antenna factor will be:

$$A_{FR} = \frac{1}{H_e}$$

In this case the input impedance of the field strength meter must be infinite and the receiving power $W_R$ is $0$ so the directivity or gain is zero ($-\infty$ dB).

**B. Example 9.b**

In order to obtain the antenna matching the input impedance of the field strength meter must be equal to the receiving antenna radiation resistance (see Fig. 13). Of course, this is a pure theoretical case because the antenna losses are supposed to be zero, and the power delivered on $R_L$ is the maximum value $W_{R_{\text{max}}}$. In this case the antenna factor will be:

$$A_{FR} = \frac{2}{H_e}$$

In order to calculate the receiving antenna directivity a quarter wave monopole antenna is used with a transmitted power of $W_T = 1$ kW at a distance of 10 km. The obtained receiving antenna directivity is $D_{\text{RM}} = 1.5 (1.76$ dB) and independent of the operating frequency as shown in Fig. 16.

**C. Example 9.c**

The loop wire resistance can be calculated as a function of frequency to be (see Fig. 14):

$$R_p = \frac{1}{\sqrt{2\pi \left( \frac{\omega \mu_0}{2\sigma} \right)}}$$

<table>
<thead>
<tr>
<th>$f$ (MHz)</th>
<th>$R_e$ (m)</th>
<th>$AF_{R_{\text{oc}}}$</th>
<th>$AF_{R_{\text{m}}}$</th>
<th>$R_{\text{rad}}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$1.5 \cdot 10^{-3}$</td>
<td>56.24</td>
<td>62.24</td>
<td>$5.2 \cdot 10^{-9}$</td>
</tr>
<tr>
<td>0.6</td>
<td>$1.8 \cdot 10^{-3}$</td>
<td>54.65</td>
<td>60.65</td>
<td>$1.0 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>0.7</td>
<td>$2.1 \cdot 10^{-3}$</td>
<td>53.32</td>
<td>59.32</td>
<td>$2.0 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>0.8</td>
<td>$2.4 \cdot 10^{-3}$</td>
<td>52.16</td>
<td>58.16</td>
<td>$3.4 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>0.9</td>
<td>$2.7 \cdot 10^{-3}$</td>
<td>51.12</td>
<td>57.12</td>
<td>$5.4 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>1.0</td>
<td>$3.0 \cdot 10^{-3}$</td>
<td>50.22</td>
<td>56.24</td>
<td>$8.3 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>1.1</td>
<td>$3.3 \cdot 10^{-3}$</td>
<td>49.40</td>
<td>55.40</td>
<td>$1.2 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>1.2</td>
<td>$3.7 \cdot 10^{-3}$</td>
<td>48.63</td>
<td>54.63</td>
<td>$1.7 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>1.3</td>
<td>$4.0 \cdot 10^{-3}$</td>
<td>47.95</td>
<td>53.95</td>
<td>$2.3 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>1.4</td>
<td>$4.3 \cdot 10^{-3}$</td>
<td>47.28</td>
<td>53.30</td>
<td>$3.2 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>1.5</td>
<td>$4.6 \cdot 10^{-3}$</td>
<td>46.70</td>
<td>52.70</td>
<td>$4.2 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>1.6</td>
<td>$4.9 \cdot 10^{-3}$</td>
<td>46.16</td>
<td>52.16</td>
<td>$5.4 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>1.7</td>
<td>$5.2 \cdot 10^{-3}$</td>
<td>45.58</td>
<td>51.58</td>
<td>$6.9 \cdot 10^{-7}$</td>
</tr>
</tbody>
</table>

oc Receiving antenna in open circuit (9.a).

* Receiving antenna matched (9.b).

![Fig. 12. Equivalent circuit of a theoretical resonant loop antenna of example 9.a.](image)

where:
- $l$ is the loop wire length ($l = 2.356$ m).
- $a$ is the loop wire radius ($a = 2.5 \cdot 10^{-1}$ m).
- $\sigma$ is the loop wire conductivity.
- $\mu_0$ is the free space magnetic permeability.
- $\omega = 2\pi f$. 

Fig. 12. Equivalent circuit of a theoretical resonant loop antenna of example 9.a.
For copper wire:

$$R_p = 3.91 \cdot 10^{-4} \sqrt{f}$$

9.c.1: If the field strength meter has an infinite impedance the antenna directivity or gain is zero and the antenna factor is found to be:

$$A_{fr} = \frac{1}{R_e}$$

In order to obtain the antenna matching the input impedance of the field strength meter must be equal to the receiving antenna radiation resistance plus the loop wire resistance ($R_L = R_{rad} + R_p \approx R_p$). In this case the antenna factor will be the same value as in the previous case (9.b):

$$A_{fr} = \frac{2}{R_e}$$

9.c.2: In order to calculate the receiving antenna directivity a quarter wave monopole antenna is used as in the previous case. The obtained receiving antenna gain as a function of frequency is shown in Table III, where the theoretical directivity of 1.76 dBi has been decreased drastically to a very low gain as shown in Fig. 16.

**D. Example 9.d**

9.d.1: If the loop antenna is used with only the wire resistance and the field strength meter has an input resistance ($50 \Omega$) the equivalent circuit antenna current is extremely low and the operation is similar to the example 9a ($V_L \approx V_i$) and for this reason the obtained receiving antenna factor is (see Fig. 15):

$$A_{fr} = \frac{1}{R_e}$$

9.d.2: If the antenna loop is loaded with a 50 $\Omega$ resistor, in order to match the input field strength meter impedance, the receiving antenna factor is increased by 6 dB. In this case the obtained receiving antenna factor is:

$$A_{fr} = \frac{2}{R_e}$$

The 6 dB factor can be seen in the field strength meter putting the 50 $\Omega$ resistor in short circuit. In practice this value has been measured between 5 and 6 dB within the AM band. In both cases (9.d1) and (9.d2), the well known equation used by the FCC is fulfilled because the input resistance of the receiver is 50 $\Omega$ or:

$$A_{fr} = -29.78 + 20 \log (f_{\text{MHz}}) - 10 \log (G_{RM})$$
It is important to know that this equation is valid only when 50 Ω input impedance is used. The measured loaded loop antenna impedance with a Delta bridge is shown in Table IV where the resulting voltage standing wave ratio is practically lower than 2 within the AM band.

Fig. 16 is showing the resulting loop antenna directivity and gain according to the example 9a to 9d and the antenna factor within the AM band is shown in Fig. 17.

The site attenuation must be obtained by means of the power relationship as shown in the example 7 because the transmitting and receiving antenna are not physically identical. In this case at a frequency of 1 MHz with free space attenuation of 52.44 dB is found to be:

\[
\begin{align*}
A_V &= 3.96 \times 10^9 (131.95 \text{ dB}) \\
A_C &= 5.20 \times 10^9 (134.32 \text{ dB}) \\
A_w &= 2.15 \times 10^{13} \\
A &= 10 \log(A_w) = 133.32 \text{ dB} \\
A_s &= A - 52.44 - 133.32 = -80.88 \text{ dB} \\
D_{TM} + G_{RM} &= 5.15 + (-86.02) = -80.88 \text{ dB} \\
G_{RM} &= \eta D_{RM} \\
\eta &= G_{RM}/D_{RM} = 1.66 \times 10^{-9}
\end{align*}
\]

Free space attenuation and site attenuation permits to determine that the directivities or gains sum has the same value. Even in the physically identical antenna this permits to determine the individual directivity or gain of each antenna.

Measured small loop antenna gain is determined by means of field strength measurements of several transmitting stations within the MF AM band. These values can be seen in Fig. 18 where comparison with calculated values is shown.

### Table IV

<table>
<thead>
<tr>
<th>MHz</th>
<th>Z&lt;sub&gt;loop&lt;/sub&gt;</th>
<th>VSWR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>44 - j38</td>
<td>2.22</td>
</tr>
<tr>
<td>0.6</td>
<td>45 - j27</td>
<td>1.77</td>
</tr>
<tr>
<td>0.7</td>
<td>48 - j19</td>
<td>1.47</td>
</tr>
<tr>
<td>0.8</td>
<td>50 - j12</td>
<td>1.35</td>
</tr>
<tr>
<td>0.9</td>
<td>55 - j6</td>
<td>1.17</td>
</tr>
<tr>
<td>1.0</td>
<td>60 - j0.5</td>
<td>1.20</td>
</tr>
<tr>
<td>1.1</td>
<td>62 + j3.3</td>
<td>1.25</td>
</tr>
<tr>
<td>1.2</td>
<td>70 + j6</td>
<td>1.42</td>
</tr>
<tr>
<td>1.3</td>
<td>76 + j7.8</td>
<td>1.55</td>
</tr>
<tr>
<td>1.4</td>
<td>86 + j7.7</td>
<td>1.73</td>
</tr>
<tr>
<td>1.5</td>
<td>94 + j6</td>
<td>1.88</td>
</tr>
<tr>
<td>1.6</td>
<td>105 + j0.8</td>
<td>2.10</td>
</tr>
<tr>
<td>1.7</td>
<td>112 - j6.8</td>
<td>2.25</td>
</tr>
</tbody>
</table>
E. Example 9.e

Receiving loop antenna is used to determine the field strength produced by a medium frequency (MF) standard broadcast station. A practical example shown here corresponds to a 590 kHz AM station with a radiated power $W_T = 100$ KW and a transmitting half wave monopole antenna. The distance is 40 km. The calculated field strength over perfect ground is $E_i = 99.2 \text{ dB}$ V/m. The measured field strength with the receiving loop antenna is

$$E_t = V_L (\text{dB} V) + AF_R (\text{dB} V/m) = 32 + 60.8 = 92.8 \text{ dB} V/m$$

The difference between both values is due to the surface wave soil losses. These losses can be calculated according to the soil physical constants, i.e. conductivity and permittivity. For an average soil this attenuation factor (Sommerfeld-Norton-Trainotti) is found to be between 3.3 dB and 6.6 dB [30]. The resulting measurement is in good agreement with the theoretical predictions.

Another station of 990 kHz at a distance of 10 km was measured and a voltage of 48 dB $V$ with an antenna factor of 55.5 dB was obtained. The resulting field strength is $103.6 \text{ dB} V/m$ and compared to the theoretical value of $106.7 \text{ dB} V/m$ for a transmitted power of 40 kW and 0.42 $\lambda$ antenna height.

The corresponding average soil losses are 2.4 dB. It can be seen here also a good agreement with the theoretical predictions.

This 990 kHz example can be seen in Fig. 19.

XVIII. Example 10

A practical example of a folded monopole antenna 72 m in height is shown in the sketch of the Fig. 20.

The characteristics of a practical folded monopole antenna 72 m in height were obtained using the small loop antenna of the example 9. Measurements were performed at a distance of more than 100 km from a big city like Buenos Aires, Argentina, using the signals of several AM stations within the medium frequency band. Induced voltage obtained using this folded monopole was extremely high and for this reason the measurements with the small loop can’t be done close to this antenna because of the scattered field (Trainotti [54]).

The reference field strength was measured by means of three turns, 0.25 m in diameter small loop (3E25) at several points at convenient distances from the folded monopole antenna. Field strength $E_t$ can be found to be:

$$E_t (\text{dB} V/m) = V_L(\text{loop}) (\text{dB} V) + AF_R(\text{loop})(\text{dB} V/m)$$

The folded monopole antenna factor results:

$$AF_R(\text{mon}) (\text{dB} V/m) = E_t (\text{dB} V/m) - V_L(\text{mon})(\text{dB} V)$$

and

$$H_c(\text{mon})(\text{m}) = \frac{2}{AF_R(\text{mon})}$$
The characteristics of the folded monopole antenna can be seen in Tables V–VII and Fig. 21. Matched receiving mode monopole gain is obtained by knowing the monopole measured impedance.

Table VII values ** were calculated by means of the WIPL-D software [35] and the comparison with the measured values are supposing that the folded monopole antenna has exactly the calculated transmitting gain, very close to that in a practical case.

Current distribution was the same in the transmitted and received mode as shown by a folded monopole antenna model calculated by the WIPL-D software [35]. The antenna effective height is directly related to the current distribution and the measured values are very close to the calculated by the WIPL-D software [35].

It is important to note that the transmitted folded monopole antenna directivity or gain as shown by Table VII is practically 6 dB higher than the corresponding received gain as shown by the measured and calculated values. This value is increased as frequency is lower and higher than the frequencies where the folded monopole antenna impedance is close to
Fig. 22. Folded monopole antenna measured impedance (Jan. 16/2001). The antenna self-matched region can be appreciated.

Fig. 23. Radio link between a quarter wave transmitting monopole antenna and a very short receiving monopole antenna.

50 Ω as shown by Fig. 22, where the antenna is practically self-matched (Trainotti [54]).

It is important to consider that in the folded monopole antenna the current distribution is not zero at the top and this affect the antenna effective height and antenna factor.

XIX. EXAMPLE 11

Radio link between a quarter wave transmitting monopole antenna and a short monopole antenna in medium frequency band as can be shown in Fig. 23.

In this case the receiving monopole antenna height is \( H = 1 \text{ m} \).

TABLE VIII

<table>
<thead>
<tr>
<th>TABLE VIII</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>f (MHz)</th>
<th>( Z_a )</th>
<th>( R_s )</th>
<th>( R_{gp} )</th>
<th>( R_{XL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>( 1.2 \cdot 10^{-3} - j 33222 )</td>
<td>14.05</td>
<td>22.54</td>
<td>332.22</td>
</tr>
<tr>
<td>0.6</td>
<td>( 1.5 \cdot 10^{-3} - j 27700 )</td>
<td>15.39</td>
<td>24.27</td>
<td>277.22</td>
</tr>
<tr>
<td>0.7</td>
<td>( 2.0 \cdot 10^{-3} - j 23750 )</td>
<td>16.62</td>
<td>25.83</td>
<td>237.50</td>
</tr>
<tr>
<td>0.8</td>
<td>( 2.6 \cdot 10^{-3} - j 20790 )</td>
<td>17.77</td>
<td>27.25</td>
<td>207.90</td>
</tr>
<tr>
<td>0.9</td>
<td>( 3.3 \cdot 10^{-3} - j 18480 )</td>
<td>18.85</td>
<td>28.57</td>
<td>184.80</td>
</tr>
<tr>
<td>1.0</td>
<td>( 4.1 \cdot 10^{-3} - j 16640 )</td>
<td>19.87</td>
<td>29.80</td>
<td>166.40</td>
</tr>
<tr>
<td>1.1</td>
<td>( 5.0 \cdot 10^{-3} - j 15100 )</td>
<td>20.84</td>
<td>30.95</td>
<td>151.00</td>
</tr>
<tr>
<td>1.2</td>
<td>( 5.9 \cdot 10^{-3} - j 13850 )</td>
<td>21.77</td>
<td>32.04</td>
<td>138.50</td>
</tr>
<tr>
<td>1.3</td>
<td>( 6.9 \cdot 10^{-3} - j 12790 )</td>
<td>22.65</td>
<td>33.08</td>
<td>127.90</td>
</tr>
<tr>
<td>1.4</td>
<td>( 8.0 \cdot 10^{-3} - j 11880 )</td>
<td>23.50</td>
<td>34.06</td>
<td>118.80</td>
</tr>
<tr>
<td>1.5</td>
<td>( 9.2 \cdot 10^{-3} - j 11090 )</td>
<td>24.33</td>
<td>35.00</td>
<td>110.90</td>
</tr>
<tr>
<td>1.6</td>
<td>( 1.1 \cdot 10^{-2} - j 10380 )</td>
<td>25.13</td>
<td>35.91</td>
<td>103.80</td>
</tr>
<tr>
<td>1.7</td>
<td>( 1.2 \cdot 10^{-2} - j 9770 )</td>
<td>25.91</td>
<td>36.77</td>
<td>97.70</td>
</tr>
</tbody>
</table>

A. Example 11.a

The short monopole antenna radiation resistance is small and its impedance is capacitively very high as can be seen in Table VIII within the medium frequencies AM band, where the antenna losses are considered to be zero (see Fig. 24). If the antenna is operating in open circuit the output voltage \( V_L \) is practically equal to the induced voltage \( V_1 \) and the antenna factor will be:

\[
A_f R = \frac{E_4}{V_L} = \frac{1}{H_e}
\]

The antenna height \( H_e \) is found to be:

\[
H_e = \frac{\lambda}{2\pi \sin(\beta H)} \left(1 - \cos(\beta H)\right) \cong \frac{H}{2} = 0.5\text{m}
\]
The antenna factor is constant along the AM band or:

\[ \text{AF} = 2 \left( \frac{1}{\lambda} \right) \quad (\text{AF} = 6.02 \text{ dB/m}) \]

In this case the input impedance of the field strength meter must be infinite and the receiving power \( W_R = 0 \) so the directivity or gain is zero (\( -\infty \text{ dB} \)).

**B. Example 11.b**

11.b.1: In order to obtain the antenna matching the input impedance of the field strength meter must be equal to the receiving antenna radiation resistance as can be seen in Fig. 25. Of course, this is a pure theoretical case because the antenna losses are supposed to be zero, and the power delivered on \( R_L \) is the maximum value \( W_{R_{\text{max}}} \). In this case the antenna reactance is nullified by means of a proper inductive reactance \( X_{L5} \). The output voltage \( V_L \) is practically \( V_i/2 \) and the antenna factor will be:

\[ \text{AF}_R = \frac{2}{\text{He}} = 4 \left( \frac{1}{\lambda} \right) \quad (\text{AF} = 12 \text{ dB/m}) \]

This value is constant along the AM band.

In order to calculate the receiving antenna directivity a quarter wave monopole antenna

\( (D_{\text{TM}} = 3.27) \) is used with a transmitted power of \( W_T = 1 \text{ kW} \) at a distance of \( r = 10 \text{ km} \) over perfect ground. The obtained maximum receiving antenna directivity or gain using the Friis equation, is found to be:

\[ D_{\text{RM}} = \frac{W_R}{W_T D_{\text{TM}}} \left( \frac{4\pi r}{\lambda} \right)^2 \]

\( W_R \) is the received power on the load resistance \( R_L = R_{\text{rad}} \).

\( G_{\text{RM}} = D_{\text{RM}} = 0.75 (-12.7 \text{ dB}) \) and independent of the operating frequency as shown in Fig. 26. This value is 6 dB below the corresponding directivity or gain of a similar transmitting short monopole over perfect ground (3 or 4.77 dB).

The maximum antenna effective receiving area \( A_{\text{eRM}} \) is found to be:

\[ A_{\text{eRM}} = \frac{W_R}{P_i} = \frac{\lambda^2}{4\pi} D_{\text{RM}} \]

\( P_i \) is the wave power density produced by the transmitting antenna at a distance of 10 km. The maximum antenna effective receiving area \( A_{\text{eRM}} \) can be seen in Fig. 27. Free space attenuation \( (A_s) \) minus site attenuation \( (A) \) must be equal to the sum of the transmitting \( (D_{\text{TM}}) \) and receiving \( (D_{\text{RM}}) \) antenna directivity or gain. This can be seen as a constant along the AM band in Table IX. In this specific case the site attenuation must be calculated by the transmitting-receiving power relationship \( W_T/W_R \). A significant error can be achieved if voltage or current relationships are used and the statement is not fulfilled.

11.b.2: If the load resistance is \( R_L = 50 \Omega \) and \( X_{\text{in}} = X_{L5} \), the antenna equivalent Thevenin circuit current is practically
constant within the AM band as well as the delivered power to the load resistance because $R_L > R_{rad}$. In these conditions the obtained gain $G_{RM}$ by the Friis equation is shown in Fig. 26. The voltage on the load resistance is practically equal to the induced voltage $V_i$ and the antenna factor is found to be:

$$A_f = \frac{E_i}{V_L} \approx \frac{E_i}{V_i} = 2 \left( \frac{1}{\text{m}} \right) \quad (A_f = 6.02 \text{dB/m})$$

As a consequence the antenna effective receiving area is constant within the AM band as shown by Fig. 27 and equal to:

$$A_{eRM} = \frac{W_R}{\pi} = \frac{\lambda^2}{4\pi} G_{RM} = 1.88 \text{ m}^2$$

Here also it can be seen that:

$$A_e - A = D_{TM} + D_{RM} \quad (179)$$

### TABLE IX

<table>
<thead>
<tr>
<th>$f$ (MHz)</th>
<th>11.b1</th>
<th>11.b2</th>
<th>11.c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.90</td>
<td>3.90</td>
<td>-36.67</td>
</tr>
<tr>
<td>0.6</td>
<td>3.90</td>
<td>3.90</td>
<td>-35.09</td>
</tr>
<tr>
<td>0.7</td>
<td>3.90</td>
<td>3.90</td>
<td>-33.75</td>
</tr>
<tr>
<td>0.8</td>
<td>3.90</td>
<td>3.90</td>
<td>-32.59</td>
</tr>
<tr>
<td>0.9</td>
<td>3.90</td>
<td>3.90</td>
<td>-31.56</td>
</tr>
<tr>
<td>1.0</td>
<td>3.90</td>
<td>3.90</td>
<td>-30.65</td>
</tr>
<tr>
<td>1.1</td>
<td>3.90</td>
<td>3.90</td>
<td>-29.82</td>
</tr>
<tr>
<td>1.2</td>
<td>3.90</td>
<td>3.90</td>
<td>-29.07</td>
</tr>
<tr>
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<td>3.90</td>
<td>3.90</td>
<td>-27.73</td>
</tr>
<tr>
<td>1.6</td>
<td>3.90</td>
<td>3.90</td>
<td>-26.57</td>
</tr>
</tbody>
</table>

* $A_s - A$ (dB)
** $D_{TM} + D_{RM}$ (dB)

These results are shown in Table IX.

### C. Example 11.c

In the case of a real short monopole antenna it is important to take into account the equivalent resistance of the ground plane losses $R_{gp}$ as well as the losses in the inductor $X_{LS}$ ($R_{XL}$) in order to get resonance, as shown in Fig. 28. These ground plane losses are due to the circulating return current in the antenna circuit along the soil under the monopole antenna to a distance of half a wavelength in radius (Dorado-Trainotti [34]). The corresponding soil resistance $R_S$, ground plane equivalent resistance $R_{gp}$ for an average soil constants ($\sigma = 0.1 \text{ S/m} \epsilon_r = 10$) and the inductor resistance $R_{XL}$ are shown in Table VIII. Resistor $R_{XL}$ is supposed to be calculated from an inductor whose merit factor $Q$ is of 100 in value.

Maximum effective area and maximum directivity or gain values are shown in Figs. 26 and 27. It can be seen the losses effect due to the gain and effective area decrease compared to the theoretical case.

Here also the equation (179) is fulfilled as shown in Table IX (11.c).

### D. Example 11.d

**A) 11.d.1:** In practice it is very difficult to resonate the short monopole antenna due to a necessary continuous variable inductor. A practical way is using a non resonant short monopole, of course the circuit current will be lower due to the monopole high capacitive reactance. This effect decreases the antenna gain and antenna effective area because the power delivered on the terminal resistance will be decreased consequently. In this case, the losses are practically due to the ground plane effect and these losses are taken into account in the equivalent resistance $R_{gp}$ shown in the equivalent circuit of Fig. 29. Receiving antenna gain and effective area are shown in Figs. 26 and 27 and Table XI. The equation (179) is perfectly well fulfilled in the non resonant monopole antenna case as can be seen in Table X (11.d.1).

**B) 11.d.2:** In order to avoid standing waves on the transmission line connecting the field strength meter to the short monopole, a 50 $\Omega$ resistor $R_{LC}$ is installed in parallel with the monopole input terminals as shown in Fig. 30.
TABLE X
NON RESONANT SHORT MONOPOLE ANTENNA—H = 1 m

<table>
<thead>
<tr>
<th>f (MHz)</th>
<th>11.d1</th>
<th>11.d2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
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<td>0.6</td>
<td>−89.96</td>
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<td>−79.42</td>
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<td>1.2</td>
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<td>−77.91</td>
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<td>−76.53</td>
<td>−76.53</td>
</tr>
<tr>
<td>1.4</td>
<td>−75.24</td>
<td>−75.24</td>
</tr>
<tr>
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<td>−74.05</td>
<td>−74.05</td>
</tr>
<tr>
<td>1.6</td>
<td>−72.91</td>
<td>−72.91</td>
</tr>
<tr>
<td>1.7</td>
<td>−71.86</td>
<td>−71.86</td>
</tr>
</tbody>
</table>

* As − A (dB)
** D_{TM} + D_{RM} (dB)

Fig. 30. Non resonant short monopole antenna equivalent circuit with ground losses and an internal load resistance $R_C$.

Its effect is decreasing 6 dB the antenna gain as shown in Fig. 26 and Table XI. It can be seen in Table X that the equation (179) is also fulfilled (11.d2).

It is very important to install the short monopole over ground at a convenient distance from the measuring equipment and operator in order not to interfere the measuring field and avoiding significant errors. For this reason the 50 Ω transmission line is of paramount importance to connect it with the measuring equipment.

In both cases (11.d1) and (11.d2), the well known equation used by the FCC is fulfilled because the input resistance of the receiver is 50 Ω or:

$$AF_R = -29.78 + 20 \log(f MHz) - 10 \log(G_{RM})$$

It is important to know that this equation is valid only when 50 Ω input impedance is used.

E. Short Monopole Measurements

Short monopole ($H = 1 \text{ m}$) has been measured with and without an internal 50 Ω resistance ($R_C$) at several points in a wide open area at a distance of approximately 60 km from
downtown Buenos Aires City using a small loop (3E25), seen previously, as a reference antenna, within the MF AM band. The average obtained antenna factors can be seen in Table XI and Figs. 31 and 32 and compared with the calculated value as well as with $\lambda/\lambda_e$ and $2\lambda/\lambda_e$, according if this monopole is unloaded or loaded with $R_C = 50$ $\Omega$. As was pointed out previously, this monopole was installed over bare soil at a distance close to 10 m from the measuring field strength meter, using a RG58A/U coaxial transmission line.

Variations in the instrument indication were observed if a person was walking at a very short distance from the monopole feeding point and for this reason, no persons were near it during measurements.

This effect is practically non existent when the small shielded loop was employed.

$AF (\text{dB}) = 20 \log(\lambda/\lambda_e)$ is mention in NBS paper [4] and its calculated and measured values (55.58 and 55.5 dB) is very close to these results (around 1 dB) at a frequency of 1 MHz, the only value available in the MF AM band. In this case the antenna factors obtained are in dB/m and not in dB like those obtained by NBS. It is believed that these values are correct because they lead to the correct field strength unit ($dB\mu V/m$) when used to determine E field strength, measuring the input voltage ($dB\mu V$), in the corresponding instrument.

$$E(dB\mu V/m) = V_L(dB\mu V) + AF(dB/m)$$

The field strength meter (Singer NM-25 T) was previously calibrated using a synthesized generator and a spectrum analyzer and its indications were extremely accurate even though it was working more than 40 years in field strength tasks.

Its advantage is the carrier station value indication no matter of its modulation factor. This permits the field strength carrier measurement of any station within the MF AM band very precisely, practically with no variations in the needle of the measuring meter, if the carrier transmitted power is maintained constant, with or without modulation.

Measured short monopole gain and effective area for loaded and unloaded monopole antenna can be seen in Figs. 33 and 34. In this case the effective area is shown as:

$$A_{eRM}(dBsm) = 10 \log(A_{eRM})$$

**XX. CONCLUSIONS**

Several cases of receiving vertical polarized antennas directivity or gain has been analyzed. It can be pointed out that only in free space the transmitting and receiving antenna directivity or gain are of the same value if both antennas are physically identical.

Over perfect or natural ground plane, it was determined, that even for physically identical antennas their directivities or gains are of different values when the antennas are used as a transmitting or receiving ones.
This statement is valid for monopole as well as for dipole antennas over a ground plane.

The Friis Equation is fulfilled if the link is free of obstacle between the transmitting and receiving antenna in free space as well as in a link over perfect or natural ground plane. In the last case the soil losses must be taken into account.

The site attenuation can be calculated by the current, voltage or power relationship only if physically identical antennas are used, if not, the power relationship must be employed. It can be seen in the case of different physically antennas that the site attenuation must be determined by the power relationship and this value is the average of both the voltage and the current relationship.

Over a ground plane the supposing identical directivity or gain for physically identical antennas must be avoided because it leads to wrong results.

In the medium frequency band a small loop antenna is preferred over a short monopole antenna for performing field strength measurements. A small loop antenna is insensitive to persons and small objects in the vicinity of the antenna, whereas a short monopole antenna is sensitive to anything moving and cutting the near field or displacement current lines (see Table XII).

It can be seen in the Fig. 33 short monopole measured gain values are within 3 dB compared to the calculated ones while the small loop are closer to the predicted theoretical calculations as shown in Fig. 18.

Significant work must be done in order to obtain the characteristic of gain and antenna factor for low to very high frequencies or lower than 300 MHz, for vertically polarized antennas. However, very close between calculated and measured values are obtained. This permits to use calculated characteristics of measuring antennas with confidence, avoiding a significant work of calibration for standard measurements (±3 dB). Nevertheless these calibrations must be performed if accurate measurements of field strength are necessary (less than ±1 dB).
GLOSSARY OF SYMBOLS

\( a \)  
Monopole or dipole antenna physical radius [m]

\( A \)  
Vector Potential

\( A_z \)  
The z Component of the Vector Potential

\( A_{eR} \)  
Receiving Antenna Effective Area \([m^2]\)

\( A_{eRM} \)  
Maximum Receiving Antenna Effective Area \([m^2]\)

\( A_{eT} \)  
Transmitting Antenna Effective Area \([m^2]\)

\( A_{eTM} \)  
Maximum Transmitting Antenna Effective Area \([m^2]\)

\( A_f \)  
Antenna Factor \([1/m]\)

\( A_{fR} \)  
Receiving Antenna Factor \([1/m]\)

\( A_{fT} \)  
Transmitting Antenna Factor \([1/m]\)

\( A_{fRM} \)  
Minimum Receiving Antenna Factor \([1/m]\)

\( A_F \)  
Antenna Factor \([dB/m]\)

\( A_{FR} \)  
Receiving Antenna Factor \([dB/m]\)

\( A_{FT} \)  
Transmitting Antenna Factor \([dB/m]\)

\( A_{FRM} \)  
Minimum Receiving Antenna Factor \([dB/m]\)

\( A \)  
Site Attenuation

\( A_c \)  
Current Site Attenuation

\( A_s \)  
Free Space Attenuation

\( A_v \)  
Voltage Site Attenuation

\( A_w \)  
Power Site Attenuation

\( \beta \)  
Space Phase Constant or Wave Number \((\beta = 2\pi/\lambda)\) [rad/m]

\( E \)  
Electric Field Strength \([V/m]\)

\( E_i \)  
Incoming Wave Electric Field Strength \([V/m]\)

\( E_s \)  
Scattered or Reradiated Wave Electric Field Strength \([V/m]\)

\( D \)  
Antenna Directivity

\( D_R \)  
Receiving Antenna Directivity

\( D_{RM} \)  
Maximum Receiving Antenna Directivity

\( D_T \)  
Transmitting Antenna Directivity

\( D_{TM} \)  
Maximum Transmitting Antenna Directivity

\( \epsilon \)  
Soil Permittivity \([F/m]\)

\( \epsilon_0 \)  
Free Space Permittivity \([F/m]\)

\( \epsilon_r \)  
Soil Relative Permittivity \([\epsilon/\epsilon_0]\)

\( f \)  
Operating Frequency \([Hz]\)

\( G \)  
Antenna Gain \((G = \eta D); G = D\) for no losses\)

\( G_R \)  
Receiving Antenna Gain

\( G_{RM} \)  
Maximum Receiving Antenna Gain

\( G_{T} \)  
Transmitting Antenna Gain

\( G_{TM} \)  
Maximum Transmitting Antenna Gain

\( H \)  
Magnetic Field Strength \([A/m]\)

\( H_i \)  
Incoming Wave Magnetic Field Strength \([A/m]\)

\( H_s \)  
Scattered or Reradiated Wave Magnetic Field Strength \([A/m]\)

\( H \)  
Monopole Physical Height \([m]\)

\( H_e \)  
Monopole Effective Height \([m]\)

\( I \)  
Conduction Current \([A]\)

\( I_{aR} \)  
Receiving Antenna Conduction Current \([A]\)

\( I_c \)  
Loading Resistance \(R_c\) Current \([A]\)

\( I_L \)  
Conduction Current at the Input Resistance \(R_L\) \([A]\)

\( I_M \)  
Maximum Antenna Conduction Current \([A]\)

\( I_O \)  
Initial Conduction Current at \(z = 0\) \([A]\)

\( I_T \)  
Transmitting Antenna Conduction Current \([A]\)

\( I(z) \)  
Conduction Current at any point \(z\) along the Antenna \([A]\)

\( j \)  
sqrt{-1} imaginary unit

\( K \)  
Factor for Antenna Current Distribution \([34]\)

\( L \)  
Dipole Antenna Physical Length \([m]\)

\( L_e \)  
Dipole Antenna Effective Length \([m]\)

\( \lambda \)  
Free Space Wavelength \([m]\)

\( \mu_0 \)  
Free Space Permeability \([H/m]\)

\( \eta \)  
Antenna Efficiency \((\eta = 1\) for zero losses\)

\( P \)  
Poynting Vector \([W/m^2]\)

\( P_c \)  
Complex Poynting Vector \([W/m^2]\)

\( P_i \)  
Incoming Wave Power Density in Space \([W/m^2]\)

\( P_s \)  
Scattered Wave Power Density in Space \([W/m^2]\)

\( \Omega \)  
Volume between the surface \(\Sigma\) and the antenna conductor

\( \rho \)  
Radial distance from the antenna base \([m]\)

\( R_a \)  
Antenna Resistance \([\Omega]\)

\( R_{aR} \)  
Receiving Antenna Resistance \((R_{aR} = R_{rad})\) for \(\eta = 0\) \([\Omega]\)

\( R_{aT} \)  
Transmitting Antenna Resistance \((R_{aT} = R_{rad})\) for \(\eta = 0\) \([\Omega]\)

\( R_C \)  
Load Resistance \([\Omega]\)

\( R_{grp} \)  
Ground Plane Equivalent Loss Resistance \([\Omega]\)

\( R_L \)  
Field Strength Meter Input Resistance \([\Omega]\)

\( R_p \)  
Wire Loss Resistance \([\Omega]\)

\( R_{rad} \)  
Antenna Radiation Resistance \([\Omega]\)

\( R_s \)  
Soil Surface Resistance \([\Omega]\)

\( \sigma \)  
Soil Conductivity \([S/m]\)

\( \Sigma \)  
Cylindrical Surface around the Antenna \([m^2]\)

\( U \)  
Radiation Intensity \([W/rad]\)

\( U_0 \)  
Average Radiation Intensity \([W/rad]\)

\( U_{max} \)  
Maximum Radiation Intensity \([W/rad]\)

\( V_i \)  
Receiving Antenna Open Circuit Voltage \([V]\)

\( V_L \)  
Input Voltage at the Field Strength Meter \([V]\)

\( V_T \)  
Transmitting Antenna Input Voltage \([V]\)

\( W_e \)  
Electric Reactive Stored Power

\( W_m \)  
Magnetic Reactive Stored Power

\( W_R \)  
Received Power \([W]\)
\[ W_T \text{ Transmitted Power [W]} \]
\[ \omega \text{ Time Phase Constant or Radian Frequency} \]
\[ (\omega = 2\pi f) \text{ [rad/s]} \]
\[ X_A \text{ Antenna Reactance [\Omega]} \]
\[ X_{CS} \text{ Resonance Capacitor Reactance [\Omega]} \]
\[ X_{LS} \text{ Resonance Coil Reactance [\Omega]} \]
\[ X_s \text{ Soil Surface Reactance [\Omega]} \]
\[ Z_{00} \text{ Intrinsic Free Space Impedance [\Omega]} \]
\[ Z_s \text{ Soil Surface Impedance [\Omega]} \]

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**REFERENCES**

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