Ranking of units by positive ideal DMU with common weights

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A R T I C L E   I N F O

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Common weights analysis (CWA)
Ranking
The ideal line
The special line

A B S T R A C T

Conventional data envelopment analysis (DEA) assists decision makers in distinguishing between efficient and inefficient decision making units (DMUs) in a homogeneous group. However, DEA does not provide more information about the efficient DMUs. In this research, the researchers will be proposed two ranking methods. In the first method, an ideal line will be defined and determined a common set of weights for efficient DMUs then a new efficiency score will be obtained and ranked them with it. In the second method, a special line will be defined then compared all efficient DMUs with it and ranked them. An example 20 branch banks of Iran illustrates two ranking methods.

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1. Introduction

Charnes, Cooper, and Rhodes (1978) introduce data envelopment analysis (DEA) to assess the relative efficiency of a homogeneous group of decision-making units (DMUs), such as schools, hospitals, or sales outlets. DEA successfully divides them into two categories; efficient DMUs and inefficient DMUs. A ranking for inefficient DMUs is given, however, efficient DMUs cannot be ranked. Some of the methods which are proposed for ranking efficient DMUs are mentioned here. Anderson and Petersen (1993) evaluate that a DMUs efficiency possibly exceeds the conventional score 1.0, by comparing the DMU being evaluated with a linear combination of other DMUs, while excluding the observations of the DMU being evaluated. They try to discriminate between these efficient DMUs, by using different efficiency scores larger than 1.0. Cook, Kress, and Seiford (1992) developed prioritization models to rank only the efficient units in DEA. They divide those with equal scores, on the boundary, by imposing the restrictions on the multiplies (weights) in a DEA analysis. Mehrabian, Alirezaee, and Jahanshahloo (1999) (MAJ) presented of the popular of these methods. These methods would have some deficiencies if data have certain structures. There are some methods based on norms. Jahanshahloo, Junior, Hossein- zadeh Lotfi, and Akbarian (2007) introduced an l1–norm approach that removes some deficiencies arising from AP and MAJ, but that cannot rank non-extreme DMUs. Liu and Peng (2006) searched a common set weights to create the best efficiency score of one group composed of efficient DMUs. Then they use this common set of weights to evaluate the absolute efficiency of each efficient DMUs in order to rank them. The methods that we propose in this paper rank efficiency DMUs by comparing with an ideal line and the special line and obtain the common set of weights to evaluate the absolute efficient DMUs. In this paper, a common set of weights will be obtained by comparing with the positive ideal line in Section 2. Also in Section 3, a common set of weights will be obtained by comparing with the special DMUs. In Section 4, they will rank DMUs by CWA-efficiency. In Section 5, they will bring an empirical example. Finally, Section 6 will give their conclusions.

2. Common set of weights by comparing with the positive ideal line

DEA was initially developed as a methodology for assessing the comparative efficiencies of organized units. The initial problem is usually expressed as: n DMUs to be assessed with m inputs and s outputs indices. For each DMU, say DMUj, the given values of indices are denoted as X = (x11, x12, ..., xmn) and Y = (y11, y12, ..., ysn) respectively, such that X ≥ 0 and Y ≥ 0. In conventional DEA models each DMU in turn maximizes the efficiency score, under the constraint that none of DMUs efficiency scores is allowed to exceed 1.0. Decision maker always intuitively takes the maximal efficiency score 1.0 as the common benchmark level for DMUs. Liu and Peng (2006), have taken advantage of this benchmark level to help them describe concretely the concept about the generation of common weights here. By the definition of the efficiency score. The common benchmark level is one straight line that passes through the origin, with slope 1.0 in the coordinate. In this paper, we want to rank efficiency DMUs with common weights by ideal line.

Definition 1. The virtual positive ideal DMU is a DMU with minimize inputs of all of DMUs as its input and maximize outputs of all of DMUs as its output. That is if we show positive ideal DMU with DMU = (X, Y) then xi = min(xij | j = 1, ..., n), (i = 1, ..., m) and yr = max(yij | j = 1, ..., n), (r = 1, ..., s).

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\textbf{Definition 2.} An ideal level is one straight line that passes through the origin and positive ideal DMU with slope 1.0.

In Fig. 1 the vertical and horizontal axes are set to be the virtual output (weighted sum of \( s \) outputs) and the virtual input (weighted sum of \( m \) inputs), respectively and \( o \) is an ideal line and \( DMU = (\sum_{i=1}^{m} x_i v_i', \sum_{r=1}^{s} y_r u_r') \) is an ideal DMU. The notation of a decision variable with superscript symbols “\(^*\)” represents an arbitrary assigned value. For any \( DMU_{in}, DMU_{ob} \) if given one set of weights \( u_i^r \) (\( r = 1, \ldots, s \)) and \( v_i^r \) (\( i = 1, \ldots, m \)) then the coordinate of points \( M \) and \( N \) in Fig. 1 are \( (\sum_{i=1}^{m} x_i v_i', \sum_{r=1}^{s} y_r u_r') \) and \( (\sum_{i=1}^{m} x_i v_i', \sum_{r=1}^{s} y_r u_r') \). The vertical gaps, between points \( M \) and \( M' \) on the horizontal axes and vertical axes, are denoted as \( A_M^0 \) and \( A_M^0 \), respectively. Similarly, for points \( N \) and \( N' \), the gaps are \( A_N^0 \) and \( A_N^0 \). We observe that there exists a total virtual gap \( A_M^0 + A_M^0 + A_N^0 + A_N^0 \) to the ideal line. Let the notation of a decision variable with superscript “\(^*\)” represents the optimal value of the variable. We want to determine an optimal set of weights \( u_i^r \) (\( r = 1, \ldots, s \)) and \( v_i^r \) (\( i = 1, \ldots, m \)) such that both points \( M \) and \( N \) below the ideal line could be as close to their projection points, \( M'' \) and \( N'' \) on the ideal line, as possible. In other words, by adopting the optimal weights, the total virtual gaps \( A_M^0 + A_M^0 + A_N^0 + A_N^0 \) to the ideal line is the shortest to both DMUs.

As for the constraint, the numerator is the weighted sum of outputs plus the virtual gap \( A_N^0 \) and the denominator is the weighted sum of inputs minus the horizontal virtual gap \( A_M^0 \). The constraint implies that the direction closest to the ideal line is upwards and leftwards at the same time. The ratio of the numerator to the denominator equals 1.0, which means that the projection point on the ideal line is reached. Therefore we have following model:

\[
\begin{align*}
A^* &= \min \left\{ \sum_{j=1}^{n} (A_j^0 + A_j^0) \right\} \\
\text{s.t.} \quad \sum_{i=1}^{m} u_i y_i &= 1, \\
\sum_{i=1}^{m} v_i x_i + A_j^0 &= 1, \quad j \in E, \\
A_j^0, A_j^0 &\geq 0, \quad j \in E, \\
u_r &\geq \epsilon > 0, \quad r = 1, \ldots, s, \\
v_i &\geq \epsilon > 0, \quad i = 1, \ldots, m.
\end{align*}
\]

If a DMU was on positive ideal then we use definition of the CWA-efficiency score of DMU \( j \) that Liu and Peng (2006) was defined as following equation:

\[
\bar{c}_j = \frac{\sum_{i=1}^{m} u_i y_i v_i'}{\sum_{i=1}^{m} x_i v_i'}, \quad j \in E.
\]

Therefore the CWA-efficiency score of it is 1.0. So that constrain (*) in (3) become redundant and this model become same the CWA model in paper of Liu and Peng (2006). On the other hand, the ideal line is the benchmark line. We result CWA model is special case of (3) in this paper. Therefore DMU \( j \) is CWA-efficient if \( A_j^0 = 0 \) or \( \bar{c}_j = 1.0 \) otherwise, DMU \( j \) is CWA-inefficient.

\textbf{Definition 3.} The performance of DMU \( j \) is better than DMU \( i \) if \( A_j^0 < A_i^0 \).

To elaborate, we apply our proposed model in the following example.

\textbf{Example 1.} We consider the following example from Sexton, Silkman, and Hogan (1986). There are six DMUs, each using two inputs to produce two outputs. The data and CCR efficiency of them are shown in Table 1. According to Table 1, DMUs A, B, C and D are CCR efficient and DMUs E and F are inefficient. In order to rank the four DMUs by the proposed model, we solve model (3). The results of the model and ranking of them can be seen in the two last columns of Table 1.

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{DMU} & \textbf{Input} & \textbf{Output} & \textbf{CCR Efficiency} \\
\hline
A & 10 & 12 & 0.85 \\
B & 12 & 14 & 0.90 \\
C & 14 & 16 & 0.95 \\
D & 16 & 18 & 1.00 \\
E & 24 & 26 & 1.00 \\
F & 26 & 28 & 1.00 \\
\hline
\end{tabular}
\caption{Data table for examples of Sexton, Silkman, and Hogan (1986).}
\end{table}

\textbf{3. Common set of weights by comparing with the special DMU}

For instance sometimes a general manager of a bank desires to compare the performance of all branches of the bank with a special bank and he ranks their ratio with it. Therefore we choose that
Table 1
Data, $\theta^{\text{ccx}}$ and results of model (3) in Example 1.

<table>
<thead>
<tr>
<th>DMU</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$\theta^{\text{ccx}}$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>150</td>
<td>0.2</td>
<td>14,000</td>
<td>3500</td>
<td>66.500</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>400</td>
<td>0.7</td>
<td>14,000</td>
<td>21,000</td>
<td>189,000</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>320</td>
<td>1.2</td>
<td>42,000</td>
<td>10,500</td>
<td>126,700</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>520</td>
<td>2.0</td>
<td>28,000</td>
<td>42,000</td>
<td>221,201</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>350</td>
<td>1.2</td>
<td>19,000</td>
<td>25,000</td>
<td>0.978</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td>320</td>
<td>0.7</td>
<td>14,000</td>
<td>15,000</td>
<td>0.885</td>
<td>6</td>
</tr>
</tbody>
</table>

$A^r = \min A^r_i + A^r_j + A^r_k + A^r_l - (A^r_m + A^r_o + A^r_t)$

s.t. $\sum_{r=1}^{s} y_{rg} u_r + A^r_j = 1$, $j = M, N,$

$\sum_{r=1}^{s} y_{rg} u_r - A^r_j = 1$, $j = L, G,$

$\sum_{r=1}^{s} y_{rg} u_r = 1$, $j = M, N, L, G,$

$u_r \geq \epsilon > 0$, $r = 1, \ldots, s,$

$v_i \geq \epsilon > 0$, $i = 1, \ldots, m.$

This model has a problem, that we hardly know that which DMU is below the ideal line and which DMU is above the ideal line, we only know ideal DMU. So that we can solve (5) model instead (4) model.

$A^r = \min \sum_{j=1}^{n} (A^r_j + A^r_j^o) - \sum_{j=1}^{n} (A^r_j + A^r_j^o)$

s.t. $\sum_{r=1}^{s} y_{rg} u_r + A^r_j - A^r_j^o = 1$, $j = 1, \ldots, n,$

$\sum_{r=1}^{s} y_{rg} u_r - A^r_j = 1$, $j = 1, \ldots, n,$

$\sum_{r=1}^{s} y_{rg} u_r = 1$, $j = 1, \ldots, n,$

$u_r \geq \epsilon > 0$, $r = 1, \ldots, s,$

$v_i \geq \epsilon > 0$, $i = 1, \ldots, m.$

Here, the constraint $\sum_{r=1}^{s} u_r + \sum_{r=1}^{s} v_i = 1$ is added for normalization purpose. The ratio form of constrains in (5) can be rewritten in a linear form, and formulated in the constrains (6).

$A^r = \min \sum_{j=1}^{n} (A^r_j + A^r_j^o) - \sum_{j=1}^{n} (A^r_j + A^r_j^o)$

s.t. $\sum_{r=1}^{s} y_{rg} u_r - \sum_{r=1}^{s} y_{rg} v_i = (A^r_j + A^r_j^o) - (A^r_j + A^r_j^o) = 0$, $j = 1, \ldots, n,$

$\sum_{r=1}^{s} y_{rg} u_r - \sum_{r=1}^{s} y_{rg} v_i = 0$, $j = 1, \ldots, n,$

$\sum_{r=1}^{s} y_{rg} u_r - \sum_{r=1}^{s} y_{rg} v_i = 1$, $j = 1, \ldots, n,$

$u_r \geq \epsilon > 0$, $r = 1, \ldots, s,$

$v_i \geq \epsilon > 0$, $i = 1, \ldots, m.$

(6)
Then, if we let $\Delta^0_j + \Delta^j_j$ be $A_j$ and $A^0_j + A^j_j$ be $A_j'$ then (9) is simplified to the following linear:

$$
A^* = \min \sum_{j=1}^{n} (A_j - A'_j)
$$

s.t. \[ \sum_{s=1}^{s} y_{ij} u_t - \sum_{t=1}^{m} x_{ij} v_t + A_j - A'_j = 0, \quad j = 1, \ldots, n, \]

\[ \sum_{s=1}^{s} y_{ij} u_t - \sum_{t=1}^{m} x_{ij} v_t = 0, \quad A_j, A'_j = 0, \quad j \in E, \quad (***) \]

\[ A_j, A'_j \geq 0, \quad j \in E, \]

\[ u_r \geq \varepsilon > 0, \quad r = 1, \ldots, s, \]

\[ v_t \geq \varepsilon > 0, \quad i = 1, \ldots, m. \]

If we solve this model with simplex method then equation (***) is redundant and we can ignore it. Therefore if we let $A_j - A'_j = A_j$ then model (7) could be rewritten to equivalent linear programming (8).

$$
A^* = \min \sum_{j=1}^{n} A_j
$$

s.t. \[ \sum_{s=1}^{s} y_{ij} u_t - \sum_{t=1}^{m} x_{ij} v_t + A_j = 0, \quad j \in E, \]

\[ \sum_{s=1}^{s} y_{ij} u_t - \sum_{t=1}^{m} x_{ij} v_t = 0, \quad i = 1, \ldots, m. \]

4. Ranking DMUs by CWA-efficiency

For ranking efficiency DMUs, we solve the model (8), DMUs are divided into the three groups:

- $J_1 = \{DMU | A_j > 0\}$
- $J_2 = \{DMU | A_j < 0\}$
- $J_3 = \{DMU | A_j = 0\}$

DMUs that belong to $J_1$ are below the special line, DMUs that belong to $J_2$ are above the special line and DMUs that belong to $J_3$ are on the special line. It is clear that DMU$_1$ belongs to $J_3$.

Definition 4. DMU$_i$ is CWA-efficient if $A^i_j = 0$ or $j \in J_2$, DMU$_i$ is super CWA-efficient if $A^i_j < 0$ or $j \in J_2$ and DMU$_i$ is CWA-inefficient if $A^i_j > 0$ or $j \in J_1$.

Definition 5. If the efficient DMUs $i$ and $j$ belong to super CWA-efficient ($i, j \in J_2$) then the performance of DMU$_i$ is better then DMU$_j$ if $A^i_j > A^j_j$. If they belong to CWA-inefficient ($i, j \in J_1$) then the performance of DMU$_i$ is better than DMU$_j$ if $A^i_j < A^j_j$.

If $A^i_j = A^j_j = 0$ i.e. they are both CWA-efficient then we have to obtain dual model (8) for ranking them. (8) could be rewritten to the equivalent linear programming (9) by taking out the slack variable $A_j$. 

### Table 2: $A_j$, rank of DMUs in Example 1 in compare with DMU$_D$

<table>
<thead>
<tr>
<th>DMU</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$A_j^*$</td>
<td>-106.23907</td>
<td>137.44050</td>
<td>-446.3405</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

In order to obtain more information, we write dual of (9).

$$
- A^* = \max \sum_{j=1}^{n} \sum_{r=1}^{s} y_{ij} u_t - \sum_{t=1}^{m} v_t x_{ij} + \beta \left( \sum_{j=1}^{n} \left( \Pi_{j} x_{ij} - \Pi_{j}^* x_{ij} + Q_i \right) + \beta \right)
$$

s.t. \[ \sum_{j=1}^{n} \Pi_{j} x_{ij} - \sum_{j=1}^{n} \Pi_{j}^* x_{ij} + Q_i + \beta = \sum_{j=1}^{n} y_{ij}, \quad i = 1, \ldots, m, \]

\[ \sum_{j=1}^{n} \Pi_{j}^* x_{ij} < 0, \quad j \in J_1, \]

\[ \sum_{j=1}^{n} \Pi_{j} x_{ij} + \Pi_{j}^* x_{ij} > 0, \quad j \in J_2, \]

\[ \sum_{j=1}^{n} \Pi_{j} x_{ij} = 0, \quad j \in J_3, \]

\[ u_r \geq \varepsilon > 0, \quad r = 1, \ldots, s, \]

\[ v_t \geq \varepsilon > 0, \quad i = 1, \ldots, m. \]

Here, $\Pi_{j}^* \geq 0$ $j \in J_1$ is the jth dual variable related to the first group of constraints in (9). The jth dual variables $\Pi_{j}$, $j \in J_2$ and $\Pi_{j}^*$, $j \in J_3$ are related to the second and third groups of constraints in (9), respectively. The dual variable $\beta$ regards the normalization constraint in (9). The dual variables $P_i$ and $Q_i$ are derived from the fifth and sixth constraints, respectively, in (9).

Definition 6. If $A^j_j = A^i_j = 0$ i.e. they are both CWA-efficient, then the performance of DMU$_i$ is better than DMU$_j$ if $\Pi_{ij}^* > \Pi_{ij}^*$. We explain the whole idea with a small example here:

**Example 2.** We run proposed ranking model with data of Example 1 i.e. we solve model with six DMUs that use two inputs to produce two outputs. We know that DMUs A, B, C and D are efficient. For ranking we compare them with DMU$_D$ and results of model (8) are shown in Table 2. The results show that DMU$_A$ and DMU$_B$ belong to $J_1$, DMU$_C$ belongs to $J_2$, and DMU$_D$ belongs to $J_3$. Also $A^i_j > A^j_j$ so rank of DMU$_A$ is more than the rank of DMU$_B$, and the rank of them is better than rank of DMU$_D$, and rank of DMU$_C$ is more than rank of DMU$_D$. The results of ranking are shown in Table 2. In this example, DMUs compare with DMU$_D$, a DMU has a rank closer to the rank of DMU$_D$ if its performance is closer than to the performance of DMU$_D$.

5. Empirical example

Let us rank 20 branches of bank in Iran by our proposed method. This data was previously analyzed by Amirteimoori and Kordro-
Table 3
Data of DMUs and their CCR efficiency.

<table>
<thead>
<tr>
<th>Branch</th>
<th>$I_1$ (Staff)</th>
<th>$I_2$ (Computer)</th>
<th>$I_3$ (Space)</th>
<th>$O_1$ (Deposits)</th>
<th>$O_2$ (Loans)</th>
<th>$O_3$ (Change)</th>
<th>$\theta_{CCR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.950</td>
<td>0.700</td>
<td>0.155</td>
<td>0.190</td>
<td>0.521</td>
<td>0.293</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.796</td>
<td>0.600</td>
<td>1.000</td>
<td>0.227</td>
<td>0.627</td>
<td>0.462</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>0.798</td>
<td>0.750</td>
<td>0.513</td>
<td>0.228</td>
<td>0.970</td>
<td>0.261</td>
<td>0.991</td>
</tr>
<tr>
<td>4</td>
<td>0.865</td>
<td>0.550</td>
<td>0.210</td>
<td>0.193</td>
<td>0.632</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>5</td>
<td>0.815</td>
<td>0.850</td>
<td>0.268</td>
<td>0.233</td>
<td>0.722</td>
<td>0.426</td>
<td>0.899</td>
</tr>
<tr>
<td>6</td>
<td>0.842</td>
<td>0.650</td>
<td>0.500</td>
<td>0.207</td>
<td>0.603</td>
<td>0.569</td>
<td>0.748</td>
</tr>
<tr>
<td>7</td>
<td>0.719</td>
<td>0.600</td>
<td>0.350</td>
<td>0.182</td>
<td>0.900</td>
<td>0.716</td>
<td>1.000</td>
</tr>
<tr>
<td>8</td>
<td>0.785</td>
<td>0.750</td>
<td>0.120</td>
<td>0.125</td>
<td>0.234</td>
<td>0.298</td>
<td>0.798</td>
</tr>
<tr>
<td>9</td>
<td>0.476</td>
<td>0.600</td>
<td>0.135</td>
<td>0.080</td>
<td>0.364</td>
<td>0.244</td>
<td>0.789</td>
</tr>
<tr>
<td>10</td>
<td>0.678</td>
<td>0.550</td>
<td>0.510</td>
<td>0.082</td>
<td>0.184</td>
<td>0.049</td>
<td>0.289</td>
</tr>
<tr>
<td>11</td>
<td>0.711</td>
<td>1.000</td>
<td>0.305</td>
<td>0.212</td>
<td>0.318</td>
<td>0.403</td>
<td>1.000</td>
</tr>
<tr>
<td>12</td>
<td>0.811</td>
<td>0.650</td>
<td>0.255</td>
<td>0.123</td>
<td>0.923</td>
<td>0.628</td>
<td>1.000</td>
</tr>
<tr>
<td>13</td>
<td>0.659</td>
<td>0.850</td>
<td>0.340</td>
<td>0.176</td>
<td>0.645</td>
<td>0.261</td>
<td>0.817</td>
</tr>
<tr>
<td>14</td>
<td>0.976</td>
<td>0.800</td>
<td>0.540</td>
<td>0.144</td>
<td>0.514</td>
<td>0.243</td>
<td>0.470</td>
</tr>
<tr>
<td>15</td>
<td>0.685</td>
<td>0.950</td>
<td>0.450</td>
<td>1.000</td>
<td>0.262</td>
<td>0.098</td>
<td>1.000</td>
</tr>
<tr>
<td>16</td>
<td>0.613</td>
<td>0.900</td>
<td>0.525</td>
<td>0.115</td>
<td>0.402</td>
<td>0.464</td>
<td>0.639</td>
</tr>
<tr>
<td>17</td>
<td>1.000</td>
<td>0.600</td>
<td>0.205</td>
<td>0.090</td>
<td>1.000</td>
<td>0.161</td>
<td>1.000</td>
</tr>
<tr>
<td>18</td>
<td>0.634</td>
<td>0.650</td>
<td>0.235</td>
<td>0.059</td>
<td>0.349</td>
<td>0.068</td>
<td>0.473</td>
</tr>
<tr>
<td>19</td>
<td>0.372</td>
<td>0.700</td>
<td>0.238</td>
<td>0.039</td>
<td>0.190</td>
<td>0.111</td>
<td>1.000</td>
</tr>
<tr>
<td>20</td>
<td>0.583</td>
<td>0.550</td>
<td>0.500</td>
<td>0.110</td>
<td>0.615</td>
<td>0.764</td>
<td>1.000</td>
</tr>
</tbody>
</table>

As a result the distance of the performance of the bank 17 is far from the performance of the bank 7 to the performance of the ideal bank. Thus a performance of the bank 7 is better than the bank 17, hence, we can rank these seven banks by this proposed method. Results of ranking of 20 banks are shown in Table 6 by this method.

Now, we want to rank these seven banks by comparing them with a special bank. We suppose that a performance of the bank 4 is acceptable for economic policies of management, so, they compare a performance of these six bank with it. For doing it, we run the model (8) then they show the results of the model in Table 5.

Table 4 shows: $\Delta_7' = 0.00201 < \Delta_{17}' = 0.00269$.

Table 5 shows: $\Delta_7 = 0.001763 > \Delta_{15} = -0.6921$.

Thus, the distance of the performance bank 7 is far from the distance of the performance of the bank 15 to the bank 4, therefore the performance of the bank 7 is better than bank 15. Also,
Thus, the distance of the performance bank 17 is far from the distance of the performance bank 1 to the bank 4, therefore the performance of bank 1 is better than bank 17. We can rank all of DMUs with this method, the results of the ranking of the 20 banks are shown in Table 6 by this method.

6. Conclusion

Ranking of DMUs in DEA is an important phase for efficiency evaluation of DMUs. DEA techniques generally do not rank the efficient DMUs. This paper researched a common set of weights that was the most favorable for determining the absolute efficiency for all of DMUs. The application practical of this methodology was aimed at the ranking of a group of DMUs by comparing them by ideal line or comparing them with spacial DMU. We used the CWA methodology that Liu and Peng (2006) stated. In the first method, we defined an ideal line and determined a common set of weights for efficient DMUs then we obtained a new efficiency score and ranked them with it. In the second method, we defined a special line then we compared all efficient DMUs with it and ranked them.

References