Cooperative Spectrum Estimation using Kalman Filter based Adaptive Bayesian System for Cognitive Radio

R. Vadivelu
Assistant Professor, Department of Electronics and Communication Engineering
Sri Krishna College of Technology, Kovaipudur, Coimbatore 641 042, India
Tel: 91-97874 12317 E-mail: vadiveluce@gmail.com

K. Sankaranarayanan
Dean, Easa College of Engineering and Technology, Coimbatore 641 105, India
Tel: 91-94431 26363 E-mail: kkd_sankar@yahoo.com

Abstract
Efficient usage of a Cognitive Radio depends upon the fundamental aspect of Spectrum utilization. The Dynamic spectrum access belongs to spectrum holes estimation to utilize the natural resource in an effective way. Former works on spectrum approximation that predicted a perfect knowledge of the Signal to Noise Ratio of the received signal from the licensed users or primary users. In this paper we propose a open situation where the SNR of the primary user's signal is unknown to both Cognitive User terminals as well as the Fusion Center. A Kalman Filter based Adaptive Bayesian system is well-thought-out to make the global spectrum sensing decision based on the observed energies from the Cognitive Users. With the availability of the regulating system parameters, the fusion center can make a global sensing resolution reliably without any additional requirement of channel state information, prior information and prior prospects of the primary user's signal. Numerical result shows that the sensing performance of the proposed Bayesian system scheme outperforms the performance of the adaptive Takagi and Sugeno’s fuzzy system model.

Keywords: Cognitive user, primary user, fusion center, fuzzy system, kalman filter, Bayesian system.

1. Introduction
Cognitive radio (CR) has been identified as a new design technique which wishes to enhance the efficient utilization of scarce electromagnetic radio spectrum by enabling dynamic spectrum access (DSA) for the next-generation wireless Communication. The inspiration for the design of CR communication systems comes from the fact that voluminous portions of the licensed spectrum are underutilized by the primary users or licensed users. As per Federal communications Committee (FCC) around 15-85 % of the spectrum is estimated as underutilized [1]. This lays the foundation for a secondary user or unlicensed user or CR user (CU) permitted to admittance a spectrum band unoccupied by the primary user at a specific time and or geographic location [2]. The spectrum hole or white space is the frequency band that has been allocated to a PU who is not using at the allotted time [3]. Opportunistic spectrum access (OSA) by CU depends on how efficiently and reliably the spectrum is sensed. Furthermore, periodic spectrum sensing is the basic requirement of CU data transmission, to overcome the interference with the PU [4].

The optimal number of CU’s involved in spectrum sensing under cooperative sensing. An energy efficient setup defined for minimizing the number of CU’s subject to a constraint on the global probability of false alarm (P_{fa}) and detection (P_d). Data throughput optimization scenario in which the throughput of the CU’s network is maximized subject to a constraint on the global probability of detection (P_d) in order to determine the optimal number of CU’s are dealt in [5]. To avoid interference to the licensed user by the CU,
spectrum sensing process or algorithms must be more accurate and should be highly reliable. Various techniques are used for spectrum sensing, generally energy detection, Cyclostationary feature detection and Hidden Markov Model (HMM) are used to detect the presence of the PU signal in the channel considered. Of the listed technique Energy Detection has more advantage over other techniques as the primary user signal is estimated without any prior knowledge of the PU since it has very low operation cost and creditable sensing performance [6][7][8]. Reliable identification of PU and the spectrum hole are more tedious task in the faded environment [9]. To evade this disadvantage, cooperative spectrum sensing has been proposed [10][11][12].

Cooperation between CUs established to sense the PU’s presence or absence, fusion center (FC) is used to take the overall resolution about the PU’s. Here the sensing precision of the PU’s is improved by the spatial diversity gain. An analytical framework for the analysis and design of cooperative spectrum sensing method over correlated shadow–fading environment, when each cooperative user is equipped with a simple energy–based detector was derived in [13]. In [14], Quan et al., proposed an optimal linear cooperation framework for spectrum sensing in order to accurately detect the weak primary user signal in the spectrum band or sub band considered. However, the drawback of algorithm implementation in [14] is the overall knowledge of signal to noise ratio (SNR) and the noise variance of the PU signal should be known at the FC. In [15] a fuzzy inference system was proposed by Kieu-Xuan et al., assuming the SNR of the PU is known to the CU which provides an advantage of local soft spectrum sensing decision made at CUs terminal. Results in [15] shows that the sensing performance of the proposed scheme is comparable with the sensing performance of the maximal-ratio combination (MRC) based scheme which does not require SNR of the PU signal from CUs to the FC.

In practice it is very difficult for a CU to accurately estimate SNR of the PU signal in a given spectrum band or sub band since there is no cooperation between the CU and the PU. It is observed that most of the existing cooperative spectrum sensing schemes makes an assumption that the SNR of the PU signal at the CU is perfectly known. Furthermore, even though the CUs can estimate these parameters well, it is very difficult to communicate them along with local observations to the FC. In [16] Thuc Kieu-Xuan et al. assumed each CU in the CR network estimated the energy of the received signal in the given band or sub band of interest and then transmitted the observed parameters to the FC. Data fusion at the FC is accomplished by using an adaptive Takagi and Sugeno’s fuzzy system where fuzzification parameters are adapted from received data through a Kalman filter. In this paper we first estimated the energy of each CU in the CR network and then it is transmitted along with the parameters of the received signal in the given band or sub band of interest to the FC. Data fusion is performed at the FC by an adaptive Bayesian system where SNR are adapted from received data through a Kalman filter. It means that the detection problem and the estimation problem are solved at the FC concurrently and cooperatively. Therefore, the FC can make a global decision based on local observed energies without the knowledge of the SNR of the PU signal at CUs.

This paper is organized as follows: Section 2 describes the system model for adaptive cooperative spectrum sensing problem. An energy detection overview is given in Section 3. Section 4 derives the Kalman filter for estimating the mean under hypothesis of the $H_0$ observed energy from the local observation. An adaptive data synthesis algorithm using Kalman filter and adaptive Bayesian system are dealt in Section 5. Numerical analysis and its results are presented in Section 6. Finally Section 7 concludes the paper.

2. System Model

The spectrum sensing problem can be formulated based upon the presence or absence of PU in the concerned band or sub band based on binary hypothesis testing model [17] as

$$H_0 \text{ PrimaryUserAbsent}$$
$$H_1 \text{ PrimaryUserPresent}$$
Considering N number of CUs scattered across a given CR network with a single FC. The received signal at each CU based on the presence or absence of PU is given by

\[
\begin{align*}
    y_i(t) &= H_0 : n_i(t) \\
    y_i(t) &= H_1 : h_i(t)x_i(t) + n_i(t) \\
\end{align*}
\]

Where the received signal at the \( i^{th} \) CU is represented by \( y_i(t) \) and the channel gain of the channel between the PU and the \( i^{th} \) CU represented by \( h_i(t) \). The signal transmitted by the PU represented by \( x_i(t) \) and the additive white gaussian noise (AWGN) at the \( i^{th} \) CU represented by \( n_i(t) \). In addition to above considerations we assume that the channels corresponding to different CUs are assumed to be identically independent, and the CUs and the PU share a common spectrum of concerned band or sub band.

![Cooperative Spectrum Sensing Environment](image)

Figure 1. Cooperative Spectrum Sensing Environment

Figure. 1 show the spectrum sensing process considered in a cooperative environment to identify the occupancy of concerned band or sub band by PU. For a given sequence of sensing, the CU estimates the energy of its received signal in the concerned spectrum band or sub band. The energies observed from the \( H_1 \) cognitive users are then communicated to the overall FC through the control channel for final decision. Finally, the FC coordinates with the observations of all the CUs and their observed energies to make a final decision on the presence or absence of the PU signal by an adaptive data fusion algorithm.

The main idea of this paper is to design an algorithm for adaptive data fusion at the FC based on the prior knowledge of the PU signal, the prior probability of the PU activity, and SNRs of the PU signal at CUs are unknown.

3. Energy Detection Technique

In cognitive wireless applications, it is of prodigious interest to check the presence or absence of an active PU in the communication link when the transmitted signal is unknown. In such situations, one apposite
The confined test statistic of the $i^{th}$ CU using energy detection technique is given by

$$\Omega = \sum_{j=1}^{N} |y_i(j)|^2$$  \hspace{1cm} (3.1)

Where $y_i(j)$ is the $j^{th}$ sample of the received signal at the $i^{th}$ CU and $N$ is the number of samples, $N = T_j W$ where $T_j$ the detection time and $W$ is the signal bandwidth.

Here we assume that the noise at each given sample is an additive white gaussian random noise with unit variance and zero mean. If the PU signal is absent, $\Omega$ follows a central chi-square distribution with $N$ degree of freedom; otherwise, $\Omega$ follows a non-central chi-square distribution with $N$ degree of freedom and a non-centrality parameter $N\delta$ [19] defined as

$$\Omega \approx H_0 : \chi^2_N$$
$$\Omega \approx H_1 : \chi^2_N(N\delta)$$  \hspace{1cm} (3.2)

Where

$$\delta_i = \frac{E_i |h_i|^2}{N}$$  \hspace{1cm} (3.3)

$\delta_i$ is the SNR of the PU signal at the $i^{th}$ CU and the parameter

$$E_s = \sum_{m=1}^{N} |s(m)|^2$$  \hspace{1cm} (3.4)

Where $E_s$ is the transmitted energy of the signal over a given sequence of $N$ samples during each detection interval.

When $N$ is comparatively large $\Omega$ can be well approximated as a gaussian random noise based on hypothesis $H_0$ and $H_1$ with a mean of $m_{0_i}$ and $m_{1_i}$, and a variance of $\nu_{0_i}$ and $\nu_{1_i}$ respectively, which are given as follows:

$$H_0 : m_{0_i} = N, \nu_{0_i} = 2N$$
$$H_1 : m_{1_i} = N(1+\delta_i), \nu_{1_i} = 2N(1+2\delta_i)$$  \hspace{1cm} (3.5)
4. Kalman Filter Technique

Kalman filter can be characterized as an algorithm for computing the conditional mean and covariance of the probability distribution of the linear stochastic system with uncorrelated Gaussian process and measurement noise. The conditional mean is the unique unbiased estimate. Assuming the transmission of PU signal with noise changing at each CU linearly over a given time period, the mean under the binary hypothesis $H_0$ of the observed energy at the arbitrary $i^{th}$ CU is nearly unchanged between two adjacent sensing cycles of assumed time [20]:

$$m_i(k+1) = m_i(k)$$  \hspace{1cm} 4.1

The observed output energy $\Omega$ can be considered as a noisy measurement of the mean $m_i$ as

$$\Omega(k) = m_i(k) + n_i(k)$$  \hspace{1cm} 4.2

Where $n_i$ is the received noise and it follows a Gaussian distribution with zero mean and variance $R_i(k) = v_i(k)$

We assume that the initial estimate of mean $m_i$ estimated at a given time $k$. This priori estimation represented as $\hat{m}_i(k)$ and the estimation error is defined as

$$err_i^-(k) = m_i(k) - \hat{m}_i(k)$$  \hspace{1cm} 4.3

And the corresponding variance error is given as

$$P_i^-(k) = E[(m_i(k) - \hat{m}_i(k))^2]$$  \hspace{1cm} 4.4

A posterior estimation $m_i(k)$ is obtained by linearly combining the noise measurement $\Omega$ with the prior estimate $\hat{m}_i(k)$ as

$$m_i(k) = K_i(k)\Omega(k) + (1 - K_i(k))\hat{m}_i(k)$$  \hspace{1cm} 4.5

Where $K_i(k)$ is the combination factor and $K_i(k) > 0$.

The variance error related with this final estimate as

$$P_i^+(k) = E[(m_i(k) - \hat{m}_i(k))^2]$$  \hspace{1cm} 4.6

Substituting 4.5 in 4.6 and shortening we get

$$P_i^+(k) = (1 - K_i(k))^2 P_i^-(k) + K_i(k)^2 R_i(k)$$  \hspace{1cm} 4.7

To minimize the mean-square estimation error, differentiate $P_i^+(k)$ with respect to $K_i(k)$ that results in
From equation 4.8 we get the Kalman gain by equating the derivative to zero

\[ K_i(k) = \frac{P^-_i(k)}{R_i(k) + P^-_i(k)} \]  

4.9

Now substituting the Kalman Gain in equation 4.7 we get

\[ P^+_i(k) = (1 - K_i(k))P^-_i(k) \]  

4.10

The updated estimate of \( m_i \) is anticipated based on the equation 4.1. Thus, we have

\[ m_p(k+1) = m_i(k) \]  

4.11

The prior error variance related with the prior estimate as

\[ P^-_i(k+1) = P_i(k) \]  

4.12

The above equations derived to obtain the Kalman filter recursive function for estimating the mean under the hypothesis \( H_1 \) of the observed energy at each CU.

5. Adaptive Bayesian System at Fusion Centre

The global comprehensive decision making spectrum sensing choice based on a Kalman filter, adaptive Bayesian system are made by local observations received by the Fusion Centre and the final decision is based on the results of the cognitive users. Figure. 3 show the proposed system for Comprehensive Decision Making at the FC.

5.1 Adaptive Bayesian System based global decision

The Bayesian decision is a probabilistic approach used to estimate the usage of the channel. A stochastic process is used as a base for channel estimation theory and is used to the estimate the channel with a set of
classes. The probability \( P(\omega_k) \) of a class \( \omega_k \) is called the prior probability, represents the knowledge about the channel before the measurement. Since the number of possible classes is \( K \) we have:

\[
\sum_{k=1}^{K} P(\omega_k) = 1 \quad 5.1
\]

Here we assume the channel estimation problem as a classification problem with two possible classes i.e., \( K = 2 \). In this case, the Bayes decision rule can be made into a simple assuming a uniform cost function and the maximum a posteriori (MAP) classifier expressed as

\[
p(z | \omega_1)P(\omega_1) > p(z | \omega_2)P(\omega_2) \quad 5.2
\]

We test if the test fails, it is identified as \( \omega_2 \) otherwise \( \omega_1 \) symbolically we write this as

\[
\omega_1
\]

\[
p(z | \omega_1)P(\omega_1) > p(z | \omega_2)P(\omega_2) \quad 5.3
\]

By rearranging the above equation we get

\[
\frac{p(z | \omega_1)}{p(z | \omega_2)} > \frac{P(\omega_1)}{P(\omega_2)} \quad 5.4
\]

The conditional probability density function \( p(z | \omega_k) \) is called the likelihood function of \( \omega_k \). Therefore, the Likelihood ratio arrived as

\[
L(z) = \frac{p(z | \omega_1)}{p(z | \omega_2)} \quad 5.5
\]

The types of errors involved in the channel detection system are two. Suppose that \( \hat{\omega}(z) \) is the result of a decision based on the measurement \( z \). The true value \( \omega \) of an object is either \( \omega_1 \) or \( \omega_2 \). Then based on above analysis four states may occur as shown in table. 1

<table>
<thead>
<tr>
<th>( \omega(z) )</th>
<th>( \omega = \omega_1 )</th>
<th>( \omega = \omega_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega(z) = \omega_1 )</td>
<td>True Negative</td>
<td>Probability of Miss Detection (( P_{\text{md}} ))</td>
</tr>
<tr>
<td>( \omega(z) = \omega_2 )</td>
<td>Probability of False Alarm (( P_{\text{fa}} ))</td>
<td>Probability of Detection (( P_{\text{d}} ))</td>
</tr>
</tbody>
</table>

The decision is associated with a device that decides whether the primary users channel is occupied (\( \omega = \omega_2 \)) or vacant (\( \omega = \omega_1 \)). Our interest to identify the probabilities of the two types of errors (i)
Probability of false alarm $P_{fa} = P(\lambda(z) < T \mid \omega_j) = \sum_{m=1}^{M} p(\lambda \mid \omega_j) \lambda$ 5.6

(ii) Probability of miss detection $P_{md} = P(\lambda(z) > T \mid \omega_j) = \sum_{m=1}^{M} p(\lambda \mid \omega_j) \lambda$ 5.7

$P_d(T) = 1 - P_{md}(T)$ 5.8

The posterior distribution of system $\lambda(z)$ is fully specified by its conditional expectations and its variance, as $\lambda(z)$ varies linearly with $P_{fa}$ and the expected value obtained as

$$E[\lambda(z) \mid \omega_j] = \frac{1}{2} (\eta_1 - \eta_2) C^{-1} (\eta_1 - \eta_2)$$ 5.9

$$E[\lambda(z) \mid \omega_j] = -\frac{1}{2} (\eta_1 - \eta_2) C^{-1} (\eta_1 - \eta_2)$$ 5.10

And the corresponding variance obtained as

$$V[\lambda(z) \mid \omega_j] = (\eta_1 - \eta_2)^T C^{-1} (\eta_1 - \eta_2) = V[\lambda(z) \mid \omega_j]$$ 5.11

With that the signal to noise ratio (SNR) obtained as

$$SNR = \frac{(E[\lambda \mid \omega_1] - E[\lambda \mid \omega_2])^2}{V(\lambda \mid \omega_2)} = (\eta_1 - \eta_2)^T C^{-1} (\eta_1 - \eta_2)$$ 5.12

The required SNR for the probability of detection $P_d$ estimated based on equation 5.12 with reference to probability of false alarm $P_{fa}$ and SNR.

6. Result and Discussion

Firstly, the sensing performance of the proposed scheme, in terms of its threshold detection and the probability of false alarm $P_{fa}$ is evaluated under Rayleigh fading condition. Rayleigh fading occurs when the PU signal experiences a Non-Line-of-Sight multi-path channel. The ratio of the two probability density functions (PDFs), each estimated for the specific experimental data, should be compared with the threshold. If the estimated data that is the “likelihood ratio” exceeds the threshold, we have chosen hypothesis $H_1$, i.e., affirm a PU is to be present. If it does not exceed the threshold value then choose $H_0$ and declare that a PU is not present or absent. Under the Neyman-Pearson optimization criterion, the probability of a false alarm ($P_{fa}$) cannot exceed the original design value. The models of $P_{fa}$ and $P_d$ are essential to carry out the LRT. Finally, we realize that in computing the LRT, the data processing operations to be carried out on the observed data with specification. The essential operations depend on the specific PDFs. Figure. 4 shows the graph between probability of false alarm $P_{fa}$ versus the detection threshold $T_d$. The modulation
schemes used for simulation is BPSK. In this simulation, we assume that all CUs suffer independent and identically distribution Rayleigh/Log-normal shadow fading channel with mean SNRs of PU signal at CUs.

![Detection Threshold vs Probability of False Alarm](image1)

**Figure 4** Detection Threshold versus probability of false alarm

![Probability of False Alarm (P_{fa}) vs Signal to Noise Ratio](image2)

**Figure 5** Probability of False Alarm (P_{fa}) versus signal to noise ratio

Figure 5 shows the graph between probability of false alarm versus the SNR with required SNR of probability of detection assumed at 0.9 to detect the presence of the primary user in the noisy environment.
The comparison graph shows the SNR reduces in the proposed model of Bayesian system model compared to the adaptive fuzzy based system model with required SNR of probability of detection at 0.9 for number of CUs assumed to be 10.

7. Conclusions

To detect spectrum holes reliably and efficiently, in this paper we proposed an adaptive Bayesian system based data fusion algorithm for cooperative spectrum sensing in CR networks. The advantage of the proposed scheme comes from the fact that it can work without any requirements about the knowledge of the PU signal, the prior probability of the PU activity, and SNRs of the PU signal at cognitive radio terminals. Simulation results showed that the sensing performance of the proposed scheme outperforms the performance of equal gain combination based scheme, and matches the performance of fuzzy based adaptive system.

References


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