A Distributed Scheduling Algorithm for Multiuser MIMO Systems with 1-bit SINR Feedback

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Abstract—Opportunistic scheduling in random beamforming maximizes the sum-rate by allocating resources to the users with the best channel condition, thus leveraging on multiuser diversity/multiplexing gain. Even if the scheduler assigns resources according to the SINR, the main drawback to the practical deployment of these schemes is the intense feedback to be provided by every user. In this paper we address the sum-rate maximization of downlink multiuser MIMO Space Division Multiple Access systems based on 1-bit quantization of SINR feedback where each available spatial beam is assigned to a different user. To minimize signalling, here it is proposed a distributed method to locally estimate the optimal quantization thresholds among the terminals that guarantees a fair allocation of the available resources. Analytical results prove that the distributed scheme is asymptotically (when increasing the number of users) optimal in terms of sum-rate scaling law. Numerical simulations validate the analysis and highlight the capability of the distributed scheme to guarantee fairness constraints.

Keywords – Space Division Multiple Access (SDMA), Feedback Reduction, Multiuser Diversity, Opportunistic Communication

I. INTRODUCTION

Large interests are focused on the improvement of throughput performances of downlink (DL) broadcast channels in multiuser cellular systems where the base stations (BSs), and possibly the mobile stations (MSSs), are provided with multiple antennas. The demand for computationally-effective transmission schemes that avoid the need for the full channel state information (CSI) at the BS led to several nearly-optimal algorithms. For example, an opportunistic space division multiple access (O-SDMA) scheme that adopts parallel data transmission from the BS to different users using $M$ orthogonal randomly generated beams was proposed in [1]. In [2-5] a 1-bit per user feedback O-SDMA scheme is considered with a simplified system model where all the users receive the signal with the same average power. In 1-bit O-SDMA every MS (say the $k$-th) compares the signal to interference plus noise ratio (SINR) experienced on each transmit beam with a system-defined threshold $\alpha_k$, and it reports a single bit feedback indicating whether the SINR exceeds the threshold. A similar 1-bit feedback scheme is provided in [4] for single antenna systems (the extension to MIMO systems is conjectured in that paper). Although in [2-3] the system is proved to asymptotically (for a number of users $K \rightarrow \infty$) reach the optimal sum-rate scaling law, a valid threshold value is only provided in for an asymptotically large number of users with the same channel distributions and number of antennas $N$.

In this paper we propose a method to evaluate the set of user-dependent thresholds $\alpha_k$ in a distributed way (i.e., each user evaluates the threshold independently from the others) so as to maximize the sum-rate and to guarantee the fairness constraints.

In the proposed protocol, each user solves (part of) the non-linear optimization problem that is conventionally carried out by the BS. More specifically, each MS (say the $k$-th) is only aware of i) the average channel power $\rho_k$, ii) the instantaneous SINR $\gamma_{k,m}$ for each transmission beamforming $m$ and iii) the number of the active users in the system $K$ as obtained from a broadcast signalling channel having negligible rate. On a timeslot basis, every MS feeds back the BS of the index of the beam with the highest SINR and the 1-bit feedback that indicates whether the corresponding SINR $\gamma_{k,m}$ exceeds the threshold $\alpha_k$. For every beam the BS schedules one user among those that have reported a positive feedback, according to a given scheduling policy.

The basic idea of the Equal Probability (EP) method proposed in this paper is to let each of the MSSs evaluate the quantization threshold $\alpha_k$ according to the same feedback load probability $\overline{F}$ (i.e., the probability that the SINR for the $k$-th MS exceeds the threshold $\alpha_k$) taken as a system-defined parameter shared by all the MSSs (Section II). It is shown here that this protocol based on the EP algorithm allows for the asymptotically (for $K \rightarrow \infty$) optimal resource allocation when the feedback load probability $\overline{F}$ and the number of active users $K$ satisfy a simple closed-form relationship. The main practical benefit of the EP algorithm is that it does not require any dedicated signaling because both the user-dependent thresholds $\alpha_k$ (Section III) and the feedback load $\overline{F}$ (Section IV) can be locally and independently evaluated by both the MSSs and the BS.

The use of a random scheduler when more than one user reports a positive feedback provides a fair resource assignment in the sense that each user is allocated an equal fraction of the total scheduling time when evaluated on a long term average. A Proportional Rate Scheduler (PRS) which allocates...
resources in order to assign a given fraction of the sum-rate to each user is proposed in Section VI. It is finally stressed that this paper is focused on a realistic system where the users are provided with arbitrary channel power $\rho_k$ and number of antennas $N_k$.

$$y_k = b_k^H \left( \sum_{m=1}^{M} q_{k,m} d_m + w_k \right).$$  \hspace{1cm} (1)

$d_m \sim CN(0,1)$ are the uncorrelated symbols transmitted on beam $m$ as taken from a Gaussian codebook as this allows the use of closed-form channel capacity relationship, $w_k \sim CN(0_{N_k \times 1}, I_{N_k \times N_k})$ is the Gaussian noise, $b_{kj}$ is the unitary norm combining vector employed when receiving from beam $j$, $q_{k,m} = H_j s_m$ is the equivalent $N_k \times 1$ SIMO channel when the BS transmits on beam $m$ over the $H_k = \sqrt{\rho_k} Z_k$ $N_k \times M$ channel matrix between the BS and the $k$-th MS. To simplify, we assume to have independent Rayleigh fading so that each entry of $Z_k$ is i.i.d. $Z_{k,i,j} \sim CN(0,1)$. The average channel power $\rho_k$ depends on path loss and shadowing and is considered as constant over many time slots while $Z_k$ is considered as constant within a single time slot but varying from slot to slot (block fading assumption) with arbitrary correlation among slots. The time fluctuations for the equivalent channel $q_{k,m}$ are indeed maximized by the use of random transmit beams in each slot (possibly varying according to some pre-defined cyclic rules) [8]. The SINR on beam $m$ when the BS multiplexes $M$ independent streams is

$$\gamma_{k,m} = \frac{\left| b_{k,m}^H q_{k,m} \right|^2}{1/\rho_k + \sum_{j=1,j \neq m}^{M} \left| b_{k,m}^H q_{k,j} \right|^2}.$$  \hspace{1cm} (2)

In the model exploited herein, every MS is assumed to have perfect instantaneous knowledge of its SINR $\gamma_{k,m}$ for every beam $m$ as well as the overall number of users $K$ (for the reasons specified in Section III). The BS has only knowledge of the number of receiving antennas $N_k$ for each user and of the set of average channel powers $\{\rho_k\}$ that can be easily provided from a slow-rate feedback channel or exploiting the channel reciprocity for the power only (not the instantaneous channel realizations). We assume for now that the BS schedules the users for the same fraction of time (more realistic fairness criteria will be discussed in Section V-VI).

During the uplink signalling phase needed to optimize the DL scheduling allocations, each of the $K$ users provides the BS with the index of the beam with the highest SINR $\hat{m}_k$ and its SINR value $\gamma_{k,\hat{m}_k}$ quantized with 1-bit, yielding a feedback load of up to $\log_2(M+1)$ bits/slot/user for the simplest protocol (other feedback protocols not taken into account here are also possible, such as pre-assigning a certain beam to each user [3] or allowing for a variable rate feedback channel [6]). The quantization threshold $\alpha_k$ of SINR $\gamma_{k,\hat{m}_k}$ is obtained independently by each MS so as to support heterogeneous terminals with unbalanced channel powers and receiver architectures. The scheduler allocates the users on a slot-by-slot basis according to their instantaneous channel quality quantized over 1-bit and to the fairness requirements. In every time slot, every beam is assigned to one user among those that reported a positive feedback on that beam. If no user reported a positive feedback on a certain beam, the conservative strategy adopted here is to leave the beam unused as result of scheduling outage (of course, other solutions not covered here are equally possible as sharing the increased available power on the scheduled beams or scheduling a random user on the beams that lack of any positive feedback). Once the user $k$ is scheduled on the $m$-th beam, a conservative solution adopted in this paper is to let the BS transmit with a data rate $\log_2(1+\alpha_k)$ b/s/Hz.

Our goal here is to optimize both the distributed estimation algorithm of the SINR quantization thresholds $\{\alpha_k\}$ and the scheduling strategy, in order to maximize the cell throughput $R_{SUM}$ and to share available resources among the users according to the equal scheduling probability fairness requirements.

III. EQUAL PROBABILITY (EP) ALGORITHM

Assume that the BS schedules in each slot on each beam one user randomly chosen among those that reported a positive feedback for that beam. Notice that channel uncorrelation
makes feedbacks from different users to be independent. According to the conservative rate assignment described in Section II, the average sum-rate
\[ R_{\text{SUM}} = \frac{K}{m} \sum_{k=1}^{K} R_k = \frac{K}{m} \sum_{k=1}^{M} P^{S}(k,m) \log_2(1 + \alpha_k) \]  
(3)
depends on the probability of scheduling the user \( k \) on beam \( m \), here rewritten as
\[ P^{S}(k,m) = \left( \sum_{j=1}^{K} P_{m}(j | k) \frac{1}{j} \right) p(\gamma_{k,m} \geq \alpha_k), \]  
(4)
where \( P_{m}(j | k) \) is the probability that \( j \) users (out of \( K \)) exceed their thresholds on beam \( m \) conditioned to the event that the user \( k \) exceeds its threshold on the same beam \( m \), and \( p(\gamma_{k,m} \geq \alpha_k) \) is the probability that the SINR of user \( k \) on beam \( m \) exceeds the threshold \( \alpha_k \). The beamforming randomization of opportunistic strategy implies that the SINR probability density function experienced by each user depends only on the channel power \( \rho_k \) and on the receiver characteristics (namely the number of antennas \( N_k \) and the receiver’s combining scheme), but it is independent of the beam index \( m \), so that \( p(\gamma_{k,m} \geq \alpha_k) = p(\gamma_{k,j} \geq \alpha_k) \). Hence, all the beams have the same (average) performance and thus the beam index \( m \) will be omitted. The sum-rate (4) reduces to:
\[ R_{\text{SUM}} = \frac{K}{m} \sum_{j=1}^{K} P^{S}(j) \frac{1}{j} p(\gamma_{k,j} \geq \alpha_k) \log_2(1 + \alpha_k). \]  
(5)

The drawback to sum-rate (5) maximization is the need to set up a centralized optimization problem to evaluate all the optimal thresholds \( \alpha_k \) in addition to the lack of a manageable expression for \( P(j | k) \). The nearly-optimal distributed threshold estimation algorithm described below guarantees the evaluation of a set of thresholds \( \alpha_k \) for the asymptotical maximization of the sum-rate based on local estimates carried out by each MS independently.

The EP threshold setting algorithm forces all the K users to set their SINR threshold on any of the beams according to the system-defined feedback loading probability
\[ \overline{F} = p(\gamma_{k} \geq \alpha_k). \]  
(6)
Once every MS is aware of the system-defined feedback probability value \( \overline{F} \), it can locally evaluate its SINR threshold by solving (6) for \( \alpha_k \). From a practical point of view, it is convenient to redefine (6) according to the scheduling outage probability [6] (i.e., the probability that no user exceeds its threshold on a given beam and thus the beam is not used):
\[ P_{\text{out}} = (1 - \overline{F})^K. \]  
(7)
Sum-rate (5) and thresholds \( \{\alpha_k\} \) can thus be rewritten in terms of outage probability \( P_{\text{out}} \) once it is known the number of users \( K \). Channels independence implies that the scheduling probability is the same for all the users so that
\[ P^{S}(k) = P^{S}(1 - P_{\text{out}})^K. \]  
(8)
Therefore, when constraint (6) holds the resources are shared in a fair way among all the \( K \) MSs with respect to the scheduling probabilities. The achievable sum-rate (5) under EP scheduling constraint (6) reduces to
\[ R_{\text{SUM}} = M \frac{1 - P_{\text{out}}}{K} \sum_{k=1}^{K} \log_2(1 + \alpha_k(P_{\text{out}})), \]  
(9)
where we highlighted the dependence of \( \alpha_k \) from \( P_{\text{out}} \). The sum-rate maximization problem is thus reduced to optimizing the system-defined outage probability value \( P_{\text{out}} \) in (9). As it will be shown in the next paragraph, the nearly optimal value of \( P_{\text{out}} \) (or equivalently \( \overline{F} \) according to (7)) can be locally evaluated by the BS and all the MSs without any need for dedicated signalling.

A. Analytical Threshold Evaluation for MIMO receivers

Here the analytical solution of (6) is provided for both single and multiple antennas MSs in order to allow every user to locally evaluate its threshold \( \alpha_k \) according to the feedback load probability \( \overline{F} = 1 - P_{\text{out}}^{1/K} \). For the sake of simplicity, we consider receivers where each receiving antenna is processed independently without any adaptive combining at the receiver side. Let \( \gamma_{k}^n \) be the SINR experienced on the \( n \)-th receiving antenna of the \( k \)-th MS on a given transmit beam, and let \( \gamma_{k} = \max_n \{ \gamma_{k}^n \} \) be the largest value, the complementary distribution function (CDF) of \( \gamma_{k}^n \) for Rayleigh channels is
\[ p(\gamma_{k}^n \geq \beta) = \frac{e^{-\beta \rho_k}}{(1 + \beta)^{N_k-1}}. \]  
(10)
From the statistical independence of \( \gamma_{k}^n \) with respect to \( n \), we have
\[ p(\gamma_{k} < \beta) = p(\gamma_{k}^n < \beta)^{N_k} \]  
so that (6) reduces to
\[ -\frac{\alpha_k}{\rho_k} \frac{e^{-\beta \rho_k}}{(1 + \beta)^{N_k-1}} = 1 - (P_{\text{out}}^{1/K})^N_k. \]  
(11)

The BS and all the MSs have to jointly (but independently to avoid signalling) solve (11) (or it is pre-solved in a look-up table) with respect to the set of thresholds \( \alpha_k \). Since (11) holds for any arbitrary number of receiving antennas \( N_k \), for single-antenna MSs it is enough to set \( N_k = 1 \).
IV. Evaluation of the Outage Probability

The main focus here is to have the value of $P_{\text{out}}$ which maximizes $R_{\text{SUM}}$ (9) as a function of $K$ only as this allows a fully distributed algorithm. Herein we show that the choice $P_{\text{out}} = 1/K^\phi$ is a sufficient condition for the sum-rate in (9) to reach the asymptotically (for $K \to \infty$) optimal scaling law of $M \log \log(NK)$ [7], thus justifying the asymptotical efficiency of the EP algorithm. This is based on the following property.

Property 1: The average sum-rate $R_{\text{SUM}}$ of the EP algorithm with a random scheduler satisfies

$$\lim_{K \to \infty} \frac{R_{\text{SUM}}}{\sum_{k=1}^{K} \log \log(N_k K)} = 1 \text{ if } P_{\text{out}} = \frac{1}{K^\phi} \quad \phi > 0$$

and it is therefore asymptotically optimal with respect to the sum-rate scaling law for any set $\{N_k\}$ of number of antennas per MS.

Proof: The threshold $\alpha_k$ given by (11) cannot be expressed in closed form. Since $P(\gamma_k \geq \alpha_k) \geq e^{(1-M-1/\rho_k)\log x}$, it follows the lower bound $\bar{\alpha}_k = -\log \left(1 - \frac{P_{\text{out}}}{1-M-1/\rho_k}\right) < \alpha_k$ on the $k$-th user threshold. The sum-rate (9) based on this lower bound is certainly conservative and thus

$$R_{\text{SUM}} \geq \sum_{k=1}^{K} M \frac{1-P_{\text{out}}}{K} \log(1+\bar{\alpha}_k(P_{\text{out}}))$$

$$= \sum_{k=1}^{K} M \frac{1-P_{\text{out}}}{K} \log \left(1 - \frac{\log \left(1 - \frac{P_{\text{out}}}{1-M-1/\rho_k}\right)}{1-M-1/\rho_k}\right).$$

(13)

The selection $P_{\text{out}} = 1/K^\phi$ for any arbitrary real value $\phi > 0$ lets the term $P_{\text{out}}/KN_k$ in (13) reduce to (after some algebra)

$$P_{\text{out}} / KN_k = \frac{\phi}{N_k K} \log(K).$$

(14)

The asymptotic ratio between the sum-rate lower-bound (13) and the scaling law of optimal sum-capacity with full CSI at the transmitter is

$$\lim_{K \to \infty} \frac{R_{\text{SUM}}^L}{\frac{K}{M} \sum_{k=1}^{K} \log \log(N_k K)}$$

$$= \sum_{k=1}^{K} M \frac{1-P_{\text{out}}}{K} \log \left(1 - \frac{\log \left(1 - \frac{P_{\text{out}}}{1-M+1/\rho_k}\right)}{M+1/\rho_k - 1}\right)$$

$$= \lim_{K \to \infty} \frac{K}{M} \sum_{k=1}^{K} \log \log(N_k K)$$

$$= \lim_{K \to \infty} \frac{K}{M} \sum_{k=1}^{K} \log \log(N_k K) - \log(M + 1/\rho_k - 1)$$

(15)

This proves that the choice $P_{\text{out}} = 1/K^\phi$ is a sufficient condition to enable the same asymptotical scaling law as optimal precoding, and concludes the proof of Property 1.

A. Optimal Outage Probability

From a practical point of view we are interested in finding the value of $P_{\text{out}}$ that maximizes the lower-bound on sum-rate (13) when the number of users $K$ is large but still finite. To allow for a distributed algorithm, we assume that $K$ is the only system parameter available to the MSs for the optimization. From (13) and (14) and after similar substitutions that yield to (15) we obtain (after some simplifications) the outage as a function of $K$ only:

$$P_{\text{out}} = \arg \max_x \left(1-x\right) \log \log \left(\frac{K}{\log(x)}\right).$$

(16)

Figure 2 shows the comparison of the average value of the thresholds $\alpha_k$ in the optimal case (by globally maximizing (9)) and under the EP condition (i.e. the thresholds are obtained separately for each user by solving (16) and (11)). The system has $M = 4$ antennas and terminals are single antennas ($N_k = 1$), the channel power set is uniformly distributed over $0 - 20 \text{dB} : \rho_k = 10^{x_k/10}$ with $x_k \sim \mathcal{U}(0, 20)$. The EP algorithm is shown to provide a tight approximation of the optimal threshold for a wide range of values of $K$.

We also consider the asymptotically optimal EP algorithm, where the outage probability parameter is defined as

$$P_{\text{out}} = 1/K$$

(17)

resulting from the simple closed-form relationship from Property 1 (with $\phi = 1$). Figure 2 shows that the average threshold obtained from setting (17) is suboptimal with respect to the average threshold given by (16). However, it will be shown in Section VII that the Sum-Rate loss induced by the use of (17) instead of (16) is moderate.
V. FAIRNESS CONSTRAINTS

So far the equal scheduling probability of different users has been considered as the fairness requirement for the EP algorithm. However, the fairness requirements in wireless systems are often related to rate constraints rather than to scheduling probability. The purpose here is to find a scheduling strategy that maximizes the sum-rate still preserving a set of user-rates that are proportional to the sum rate with constraints \( \lambda_k \):

\[
R_k = \hat{R}_k R_{SUM}
\]

with \( \sum_{k=1}^{K} \lambda_k = 1 \). From (9) and (18) there is a set of \( K \) constraints on the user scheduling probability \( P_k^S = \hat{R}_k R_{SUM} / \log_2(1 + \alpha_k) \) and another set of constraints on the scheduling probabilities when the scheduler lacks of positive feedbacks. By adding these constraints, the scheduling problem can be written as

\[
\{ \hat{P}_k^S \} = \arg \max_{\{ P_k^S \}} \left\{ M \sum_{k=1}^{K} \hat{P}_k^S \log_2(1 + \alpha_k) \right\}
\]

\[
\hat{P}_k^S = \frac{\lambda_k M \sum_{k=1}^{K} \hat{P}_k^S \log_2(1 + \alpha_k)}{\log_2(1 + \alpha_k)} \quad \forall k
\]

s.t.

\[
\sum_{k=1}^{K} \hat{P}_k^S = 1 - P_{out}
\]

\[
\hat{P}_k^S \leq \bar{F} \quad \forall k
\]

where \( \{ \hat{P}_k^S \} \) is the set of optimal scheduling probabilities.

The proportional fair scheduler [8] has been modified here in order to assign the resources according to (18). This is referred to as Proportional Rate Scheduler (PRS).

Let \( T \) be the time-index of the current time slot and \( s_k(t) = \{0,1\} \) where 1 (or 0) denotes the case when user \( k \) is scheduled in time slot \( t \). The effective scheduling probability \( \hat{P}_k^S(T) \) of user \( k \) in time slot \( T \) can be iteratively updated as running average

\[
\hat{P}_k^S(T) = \left( 1 - \frac{1}{N_W} \right) \hat{P}_k^S(T-1) + \frac{1}{N_W} s_k(T)
\]

(20)

according to the memory length parameter \( N_W \). The scheduling probabilities \( \{ \hat{P}_k^S(T) \} \) asymptotically (in time) reach the desired scheduling probabilities \( \{ \hat{P}_k^S \} \) when the scheduling is performed according to

\[
k_m^*(T) = \arg \min_{k \in S_m(T)} \hat{P}_k^S(T) - \hat{P}_k^S,
\]

(21)

which is an adaptive way to solve (19). Here \( k_m^*(T) \) represents the user scheduled on beam \( m \) in time slot \( T \) and \( S_m(T) \) is the set of users that give positive feedback on beam \( m \) at time slot \( T \). The only missing element for the evaluation of (21) is the set of optimal scheduling probabilities \( \{ \hat{P}_k^S \} \). Notice that when channel powers \( \{ \rho_k \} \) and rate constraints \( \{ \lambda_k \} \) are moderately unbalanced, the solution of (19) tends to equal values of \( \hat{P}_k^S \) for all the users, and the condition \( \hat{P}_k^S \leq \bar{F} \) in (19) is satisfied for the values of \( K \) and \( P_{out} \) of practical interest. Therefore, we obtain from (19)

\[
\hat{P}_k^S = \frac{\lambda_k}{\log_2(1 + \alpha_k)} \hat{R}_{SUM}
\]

(22)

with \( \hat{P}_k^S \) as the target scheduling probability of user \( k \) and

\[
\hat{R}_{SUM} = \frac{1 - P_{out}}{\sum_{k=1}^{K} \frac{1}{\log_2(1 + \alpha_k)}}
\]

as the effective sum-rate.

VI. NUMERICAL RESULTS

In the following the performance of the EP algorithm is numerically compared to that of other relevant schemes from recent literature. The considered system consists of one BS with \( M \) antennas and \( K \) MSs, each provided with a random i.i.d. number of receiving antennas taken from the limited set \( N_i \in \{1,2,3,4\} \). Every user is assigned an equal share \( \lambda_k = 1/K \) of the sum-rate and it experiences a channel power value which is uniformly distributed among the users as \( \rho_k = 10^{\eta_i} / 10 \) with \( x_i \sim U(0,20) \).

Figure 3 shows the sum-rate performance of the considered algorithms as a function of the number of users \( K \). Although EP...
and O-SDMA leverage the same sum-rate scaling law, as it was analytically proved in Section IV, O-SDMA outperforms EP by an almost constant gap of 3b/s/Hz, which is mainly due to the reduced feedback load and the higher fairness provided by the EP schemes. Figure 3 highlights that the simple closed-form expression (17) is already close in performance to the nearly optimal (as it was shown in Figure 2) 1-bit feedback scheme (16). As the number of the users in the cell grows, EP outperforms the algorithm of [8] and the SISO schemes with unquantized feedback and proportional fair constraints.

The CDF of the average user-rate (i.e., the probability that a random user achieves an average rate which is lower than the abscissa) is plotted in Figure 4 for a system with \( K = 100 \) users. The EP algorithm with the PRS is shown to achieve a fair assignment, as the actual average user-rates span a narrow range of values. About 40\% of the users achieve a higher rate with the EP algorithm than with O-SDMA [1], even if the overall signalling required for the EP protocol is lower. On the other hand, O-SDMA clearly outperforms EP with regards to the average user-rate, by opportunistically privileging the users with the instantaneous highest SINR in the resource allocation.

VII. CONCLUSIONS

An asymptotically optimal protocol for the 1-bit SINR feedback for SDMA MIMO scheduling is proposed in this paper for heterogeneous realistic systems. Since the SINR quantization thresholds are locally estimated by the terminals, no dedicated signalling is needed. The adoption of a simplified scheduler is shown to leverage a fair resource allocation in terms of user-rates.

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