TOA Estimation with Pulses of Unknown Shape

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Abstract—This paper investigates time of arrival estimation via impulse radio ultra-wideband technology. A dense multipath channel is assumed and frequency-selective propagation effects are taken into account that make the individual received pulses different in shape from the transmitted monocycles. Assuming only an approximate knowledge of the received pulses’ duration, a time of arrival estimator is derived making use of least mean square techniques. The algorithm operates on energy measurements made on a folded version of the received signal. The estimator performance is investigated by simulation under implementation conditions of different complexity. Comparisons are made with other available schemes.

I. INTRODUCTION

In the last few years an intense research activity on ultrawideband (UWB) systems has led to the conviction that UWB technology is a viable solution for high-rate short-range wireless communications, as well as for low-rate moderate-range communications with ranging capabilities. In particular, the impulse radio version of UWB (IR-UWB) can provide centimeter accuracy in ranging and can be exploited for localization purposes in hospitals, industrial areas, airports and so on [1].

The key toward this goal are the ultrashort pulses (monocycles) employed in IR-UWB radio. On an AWGN channel their time of arrival (TOA) can be measured in a simple way with a correlation device [2]. In a typical indoor environment, instead, the TOA estimation problem is much more demanding for the following reasons:

(i) Propagation takes place with hundreds of distinct trajectories [5] and the goal is the location of the earliest pulse, not of an isolated pulse. To see the difference, suppose that there are no distortions and the correlator’s template is the transmitted monocycle. Then, the TOA of the direct-path pulse (DPP) is found as the instant where the correlation overcomes for the first time a given threshold. With a single-path channel, vice versa, the TOA of the (unique) pulse is found as the position of the highest correlation peak, wherever it happens to occur over the observation interval.

(ii) The received pulses may be severely distorted because of frequency-selective effects due to scattering [6] and frequency variations of the antenna pattern [7]. Since the estimation of their shape is computationally intensive [8], the implementation of a correlation-based TOA estimator is practically impossible.

(iii) Pulse overlaps are likely to occur. If the DPP is partially overlapped with the next pulse, a correlation-based method gives wrong results even with perfect knowledge of the pulse shapes.

TOA estimation is currently the focus of intense research activity. The following is a representative sample of the work published in this area.

Reference [9] approaches TOA estimation adopting a simplified version of the generalized maximum likelihood criterion. The received pulses are undistorted and their common shape is used for correlation with the incoming signal. The estimation process is split in two parts. The TOA of the strongest path is found first, looking for the highest peak in the correlator output. If such a pulse is not the earliest that arrives at the receiver, then TOAs and amplitudes of the prior pulses are computed in succession, proceeding backward in time until the last pulse (the one from the direct path) is reached.

A two-part strategy is also adopted in [10] where TOA estimate is derived from signal energy measurements. As in [9], pulse distortions are ignored.

The approach in [1], [11]-[12] is based on the idea of a “noisy template” that is derived from delay-and-average operations on the received signal. No assumption is made on the pulse shapes and distortions are implicitly taken into account.

TOA estimation based on signal energy measurements is pursued in [13]. The incoming signal is squared and integrated over intervals of duration comparable with the pulse width. The location of the DPP is computed as the index of the first interval where the energy overcomes a suitable threshold.

In this paper we provide a structured approach to TOA estimation based on least squares (LS) techniques. Similar to many previous works, we adopt a two-step procedure which first leads to a coarse estimate and then to a finer result. To this end a monocyte with pseudo-random polarity is transmitted every $T_f$ seconds, so generating a periodic sequence of channel responses (CRs) at the receiver. If each CR is shorter than $T_f$, the incoming signal is constituted of separate CRs and the aim of the coarse estimator is to compute their positions relative to a local clock running with period $T_f$. The fine estimator looks for the position of DPP within each CR.

As we shall see, in both cases the solution involves a pre-processing of the received signal, followed by suitable energy measurements. This result gives mathematical foundation to energy-based methods proposed in literature but, at the same time, it points out the importance of the pre-processing phase to enhance the estimation accuracy.

The estimation scheme is digital and operates on signal
samples taken at rate $1/T_x$, lower than or equal to the Nyquist rate. The value of $1/T_x$ is a critical parameter in view of the power consumption. We show that a good accuracy is achieved with sampling rates of 2 GHz or less. Performance comparisons are made with some TOA estimators proposed in literature.

The rest of the paper is organized as follows. The signal model is described in the next section while the coarse estimator is illustrated in section III. Section IV deals with the fine estimator and section V with implementation issues and approximations that must be made to limit the complexity of the algorithm. In section VI simulation results are provided to assess the estimator performance and make comparisons with other schemes. Conclusions are drawn in section VII.

II. SIGNAL MODEL

The proposed TOA estimator is based on the transmission of a training sequence of the type

$$s_T(t) = \sum_{i=-\infty}^{\infty} \sum_{k=0}^{N_f-1} a_k w(t - iT_b - kT_f)$$

where $a_k = \pm 1$ is a maximal-length pseudo-noise sequence [14] of length $N_f$, $w(t)$ is a pulse of duration $T_p^{\text{true}}$, $T_f$ is the frame period (much longer than $T_p^{\text{true}}$) and $T_b = N_f T_f$ is the symbol period. The propagation occurs on an $L$-path channel whose response to $w(t)$ is modelled as

$$c(t) = \sum_{\ell=0}^{L-1} p_{\ell}(t - \tau_{\ell})$$

$p_{\ell}(t)$ being the pulse from the $\ell$-th path and $\tau_{\ell}$ the associated delay. Note that the waveforms $p_{\ell}(t)$ have different shapes in general. With no loss of generality we assume $\tau_0 \leq \tau_1 \leq \ldots \leq \tau_{L-1}$ so that $p_0(t)$ represents the DPP and $0$ the TOA we want to estimate. For simplicity the abbreviated notation $\tau = \tau_0$ is used henceforth.

Replacing $w(t)$ by $c(t)$ in (1) and setting $c(t) = h(t - \tau)$ yields the noise-free received signal

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{k=0}^{N_f-1} a_k h(t - iT_b - kT_f - \tau)$$

(3)

i.e., a train of amplitude modulated CRs. For the time being they are assumed to be separated but later, in the simulations, this constraint is dropped. Their shape is unknown while their duration is approximately equal to the channel delay spread

$^{2}$The duration of $h(t)$ depends on the positions of the transmit and receive antennas and it is difficult to measure as the operation requires estimating $p_{\ell}(t)$ and $\tau_{\ell}$ in (2).

Our goal is the estimation of $\tau$ based on the observation of $s(t)$ in the presence on noise.

As $s(t)$ is periodic of period $N_f T_f$, $\tau$ can only be measured within multiples of that period. In other words, we must limit $\tau$ within the range $0 \leq \tau < N_f T_f$. Thus, letting $[x]$ the integer part of $x$ and writing $\tau$ as a multiple of $T_f$ plus a fraction thereof, i.e.,

$$\tau = mT_f + \varepsilon$$

(4)

with $m =: [\tau/T_f]$, we have $0 \leq m \leq N_f - 1$ and $0 \leq \varepsilon < T_f$.

In the next section we propose a TOA estimator based on a presumed duration $T_b$ of $h(t)$. In practice $T_b$ will not coincide with the true duration and, as we shall see, the discrepancy will deteriorate the estimation accuracy. In a typical residential environment errors on the order of 20-30 ns are incurred. On the other hand, as the symbol period may be as large as 1000 ns or more [15], the algorithm still provides a good reduction of the original TOA uncertainty. A further reduction is achieved with the fine estimator described in section IV.

III. COARSE ESTIMATOR

To begin, we rewrite $s(t)$ in a more convenient form. Setting $T_b = N_f T_f$ in (3) and combining the result with (4) after straightforward algebraic manipulations yields

$$s(t) = \sum_{j=-\infty}^{\infty} a_{j-m|N_f} h(t - jT_f - \varepsilon)$$

(5)

and we consider its Euclidean distance (over $N$ symbols) from the received signal component plus noise, say $r(t) = s(t) + n(t)$. We have

$$J = \int_{0}^{NT_b} [r(t) - \tilde{s}(t)]^2 dt$$

(7)

Minimizing $J$ with respect to $\hat{\tau}$, $\hat{m}$ and $\hat{\varepsilon}$ and calling $(\hat{m}, \hat{\varepsilon})$ the pair $(\hat{m}, \hat{\varepsilon})$ corresponding to the minimum gives the coarse TOA estimate

$$\hat{\tau}_c = \hat{m}T_f + \hat{\varepsilon}_c$$

(8)

To proceed we substitute (6) in (7). Discarding a factor $NN_f$ and an additive constant independent of $\hat{h}(t)$, $\hat{m}$, $\hat{\varepsilon}$ and, finally, assuming $N >> 1$, we get

$$J = \int_{0}^{T_b} [\hat{h}(t) - r_{\text{fold}}(t + \hat{\varepsilon}, \hat{m})]^2 dt - \int_{0}^{T_b} r_{\text{fold}}^2(t + \hat{\varepsilon}, \hat{m}) dt$$

(9)

with

$$r_{\text{fold}}(t, \hat{m}) =: \frac{1}{NN_f} \sum_{i=0}^{NN_f-1} a_{i-m|N_f} r(t + iT_f)$$

(10)

As $\hat{h}(t)$ is varied, from (9) we see that $J$ achieves a minimum (equal to the last term) for

$$\hat{h}(t) = r_{\text{old}}(t + \hat{\varepsilon}, \hat{m})$$

(11)
Thus, the pair $(\hat{m}_c, \hat{\epsilon}_c)$ we look for is computed as

$$\hat{m}_c, \hat{\epsilon}_c = \arg \max_{\hat{m}, \hat{\epsilon}} \int_{\hat{\epsilon}}^{\hat{\epsilon}+T_h} r_{fold}(t, \hat{m})dt$$

(12)

Figure 1 illustrates $r_{fold}^2(t, \hat{m})$ for $|m - \hat{m}|_{N_f} = 2$ in the absence of noise.

The following comments are in order:

(i) The estimator (12) exploits energy measurements on $r_{fold}(t, \hat{m})$, a folded version of $r(t)$. This is in contrast with the scheme in [13] where the measurements are directly made on $r(t)$. The advantage of operating on $r_{fold}(t, \hat{m})$ rather than $r(t)$ is that the noise component is reduced while the signal component keeps the same timing information.

(ii) A physical interpretation of (12) is useful to explain where the limitations of the TOA estimator come from. Assume for simplicity that the noise is negligible and call $T_h \triangleq T_h^{(true)}$ the true duration of $h(t)$, as distinguished from the assumed duration $T_h$. For $T_h = T_h^{(true)}$ it is clear from Fig. 1 that the the integral in (12) achieves a maximum for $\hat{m}_c = m$ and $\hat{\epsilon}_c = \epsilon$, which is what we expected. Vice versa, for $T_h < T_h^{(true)}$ we have $\hat{\epsilon}_c > \epsilon$ in general, as illustrated in Fig. 2a (an enlargement of Fig. 1 in the interval $2T_f \leq t \leq 3T_f$). The only chance for $\hat{\epsilon}_c$ to equal $\epsilon$ is that the strongest multipath components are packed in the first $T_h$ seconds of the CR, which is not necessarily true. Finally, for $T_h > T_h^{(true)}$ it is seen from Fig. 2b that the integral in (12) exhibits a plateau of length $T_h - T_h^{(true)}$ as $\hat{\epsilon}_c$ is varied. In the presence of noise any point within the plateau is likely to correspond to a maximum of the integral. In conclusion, the accuracy of the estimation algorithm depends on our guess about the CR duration. Errors on the order of 20-30 ns are incurred with channel model CM1 [16] and $T_h$ equal to the channel delay spread.

(iii) The maximization in (12) must be carried out numerically, letting $\hat{m}$ vary in the interval $[0, N_f - 1]$ and changing $\hat{\epsilon}$ by multiples of some step-size $\Delta_\epsilon$. As the estimation accuracy is necessarily rough, it would be useless to take $\Delta_\epsilon$ too small as it would only increase the computational load. In the simulations described later we set $\Delta_\epsilon = T_h$.

IV. FINE ESTIMATOR

In this section we discuss a TOA estimator with improved performance relative to the previous section. It performs a search over a tentative delay letting it vary in a small interval around $\tilde{\tau}_c = \hat{m}_cT_f + \hat{\epsilon}_c$. In this way the information from the coarse estimator is exploited to restrict the range of the fine search. As before, we adopt an LS approach in which the Euclidean distance of the received signal $r(t)$ to a hypothetical realization of $s(t)$ is minimized.

To begin, we use (2) into (3) and after straightforward manipulations we rewrite $s(t)$ as follows

$$s(t) = \sum_{\ell=0}^{L-1} \sum_{j=-\infty}^{\infty} a_{|j-m_{\ell}|_{N_f}} \hat{p}_\ell(t - jT_f - \epsilon_\ell)$$

(13)

where $m_{\ell} = \lfloor \tau_\ell/T_f \rfloor$ and $\epsilon_\ell = \tau_\ell - m_{\ell}T_f$.

In view of (13) we model $\tilde{s}(t)$ as

$$\tilde{s}(t) = \sum_{\ell=0}^{L-1} \sum_{j=-\infty}^{\infty} a_{|j-m_{\ell}|_{N_f}} \hat{p}_\ell(t - jT_f - \epsilon_\ell)$$

(14)

where, as usual, $\hat{x}$ denotes a tentative value of $x$. For simplicity we take the waveforms $\hat{p}_\ell(t)$ with a common duration $T_p$.

In the absence of channel distortions we would set $T_p$ equal to $T_p^{(true)}$, the monocyte duration. Next we compute the Euclidean distance of $\tilde{s}(t)$ to $r(t)$ as given in (7). In doing so we make the simplifying assumption that the multipath components in (2) are separated, which allows us to write

$$\tilde{p}_{\ell_1}(t - \tilde{\tau}_{\ell_1})\tilde{p}_{\ell_2}(t - \tilde{\tau}_{\ell_2}) = 0 \quad \text{for} \quad \ell_1 \neq \ell_2$$

(15)

Condition (15) may not be valid in general and serves only to facilitate the derivation. The performance of the resulting estimator in realistic circumstances is assessed later by simulation. Under the above hypotheses we get after some manipulations (in which a factor $NN_f$ and an irrelevant additive constant are dropped)

$$J = \sum_{\ell=0}^{L-1} \sum_{j=-\infty}^{T_p} \left[ r_{fold}(t + \tilde{\epsilon}_\ell, \hat{m}_\ell) - \tilde{p}_\ell(t) \right]^2 dt$$

(16)

$$- \sum_{\ell=0}^{L-1} \sum_{j=-\infty}^{T_p} r_{fold}^2(t + \tilde{\epsilon}_\ell, \hat{m}_\ell) dt$$

Clearly, the minimum of $J$ as a function of $\tilde{p}_\ell(t)$ is obtained for

$$\tilde{p}_\ell(t) = r_{fold}(t + \tilde{\epsilon}_\ell, \hat{m}_\ell) \quad 0 \leq t \leq T_p \quad 0 \leq \ell \leq L - 1$$

(17)

and its further minimum as a function of $\tilde{\epsilon}_\ell$ and $\hat{m}_\ell$ occurs for

$$\hat{m}_\ell, \tilde{\epsilon}_\ell = \arg \max_{\hat{m}_\ell, \tilde{\epsilon}_\ell} \int_{\tilde{\epsilon}_\ell}^{\tilde{\epsilon}_\ell+T_p} r_{fold}^2(t, \hat{m}_\ell)dt \quad 0 \leq \ell \leq L - 1$$

(18)

The following remarks are useful:

(i) Equation (18) indicates that $L$ separate searches are needed to compute the pairs $(\hat{m}_\ell, \tilde{\epsilon}_\ell)$ and, ultimately, the TOAs $\tilde{\tau}_\ell = \hat{m}_\ellT_f + \tilde{\epsilon}_\ell$ of all the arriving pulses. The operations can be carried out sequentially by repeating $L$ times the basic algorithm

$$\hat{m}, \tilde{\epsilon} = \arg \max_{\hat{m}, \tilde{\epsilon}} \int_{\tilde{\epsilon}}^{\tilde{\epsilon}+T_p} r_{fold}^2(t, \hat{m})dt$$

(19)

In other words, letting $\hat{\tau} = \hat{m}T_f + \hat{\epsilon}$, the $L$ highest peaks of the integral

$$I(\hat{\tau}) = \int_{\hat{\epsilon}}^{\hat{\epsilon}+T_p} r_{fold}^2(t, \hat{m})dt$$

(20)

provide the TOAs of all the multipath components while the earliest peak gives the TOA of the DPP. It is worth noting...
that (19) coincides with (12) except for the replacement of $T_h$ with $T_p$.

(ii) Unfortunately, as $L$ is unknown, the above procedure cannot be applied. Indeed, if we assume $L$ paths instead of $L$, two adverse situations occur. For $L < L$, the set of the highest peaks may not include the DPP peak, meaning that a later pulse is mistaken for the DPP. Vice versa, for $L > L$, the set of the highest peaks include noise artifacts and, again, an error occurs if an artifact precedes the DPP peak. Note that the highest peak need not be due to the DPP as the latter may be attenuated by obstructions.

(iii) A way to overcome this problem is to adopt a threshold-based approach in which the DPP TOA is computed as the first time $I(\hat{\tau})$ overcomes a suitable threshold $\lambda$. Intuitively, the threshold value is an important design parameter. If it is too low, i.e., comparable with the peaks $I(\hat{\tau})$ exhibits in the noise-only region (NOR) (i.e., where no signal pulses are present, as shown in Fig. 1), the threshold overcoming is likely to be a noise artifact (false alarm). Vice versa, if it is too high, it may not be reached by the DPP peak (missed detection). The problem is further exacerbated by the strength of the DPP, which varies in a random way with the channel realizations. This makes it difficult to design $\lambda$ so as to neglect missed detections. The choice of $\lambda$ is discussed in the next section.

(iv) Another important issue is the search region. Its size should be as small as possible to limit the computational complexity. Although the coarse search indicates $\hat{\tau}_c$ as the arrival time of the strongest multipath components, the DPP might be in advance on them. In particular, with the channel models CM1 (residential LOS) described in [16], the DPP may arrive up to 40 ns earlier [13]. Thus, bearing in mind that $\hat{\tau}_c$ may be in error up to 30 ns or so, in the following the search region is set as $\hat{\tau}_c - 70 \leq \tau \leq \hat{\tau}_c + 30$ ns.

Henceforth the algorithm described above is referred to as energy-based estimator (EBE). Some issues about its implementation are discussed in the next section.

V. IMPLEMENTATION ISSUES

We start with the folding operation in (10) which involves delayed versions of $r(t)$. It is easily recognized that the implementation of the delays must be very accurate, say within a fraction of the monocycle width, otherwise pulses from adjacent frames would not add coherently. As this accuracy cannot be achieved with analog delay lines [17], we must proceed digitally. Thus, we compute the samples of $r_{fold}(t, \tilde{m})$ from those of $r(t)$. Calling $1/T_f$ the sampling rate, assuming $T_f/T_h$ an integer $M_f$, and letting $r[k] = r(kT_h)$ and $r_{fold}[k, \tilde{m}] = r_{fold}(kT_h, \tilde{m})$, from (10) we have

$$r_{fold}[k, \tilde{m}] = \frac{1}{NN_f} \sum_{i=0}^{NN_f} a_{i-\tilde{m}} r[k + iM_f].$$

Next we concentrate on the coarse search (12). As is pointed out in section III, the variable $\tilde{\varepsilon}$ is varied by multiples of $T_h$.

Thus, assuming $T_h/T_s$ an integer $M_h$, letting $\tilde{\varepsilon} = nM_hT_s$ ($n = 0, 1, 2, \ldots$), and approximating the integral by a sum yields

$$\hat{m}_c, \tilde{\varepsilon} = \arg\max_{\tilde{m}, \tilde{\varepsilon}} \sum_{k=\tilde{m},\tilde{\varepsilon}}^{\tilde{m}+\tilde{h}-1} r_{fold}^2[k, \tilde{m}]$$

Note that $\tilde{\varepsilon}$ in (12) is computed as $\tilde{\varepsilon} = nM_hT_s$.

A similar approach is adopted with the integral in (20). Here we let $\tilde{\varepsilon}$ vary by multiples of $T_s$, i.e., we set $\tilde{\varepsilon} = nT_s$ ($\tilde{n} = 0, 1, 2, \ldots$). Also, we assume $T_p/T_s$ an integer $M_p$. Finally, we approximate the integral by a sum. Dropping an irrelevant factor $T_s$ we get

$$I(\tilde{\tau}) = \sum_{k=\tilde{n}}^{\tilde{n}+M_p-1} r_{fold}^2[k, \tilde{m}]$$

This is recognized bearing in mind that the channel responses have a duration $T_h^{(true)}$ and, as indicated in (5), they begin at $iT_f + \varepsilon$ ($i = 0, 1, 2, \ldots$). In practice neither $T_h^{(true)}$ nor $\varepsilon$ are exactly known and we must content ourselves with an approximate delimitation of NOR obtained from (24) by replacing $T_h^{(true)}$ and $\varepsilon$ with $T_h$ and $\tilde{\varepsilon}$, respectively. It is worth noting that in NOR the samples $r[k]$ are independent zero mean Gaussian random variables (RVs) whose variance $\sigma^2$ can be easily measured. Thus, $P_{FA}$ can be expressed as a function of $\sigma^2$ as follows. From (21) and (23) it can be shown that $I(\tilde{\tau})$ is a chi-square RV with $M_p$ degrees of freedom, mean value $M_p\sigma^2/(NN_f)$ and variance $2M_p(\sigma^2/(NN_f))^2$. Thus, for $M_p$ even we have [18]

$$P_{FA} = \exp \left( -\frac{\lambda NN_f}{2\sigma^2} \right) \sum_{m=0}^{M_p/2-1} \frac{1}{m!} \left( \frac{\lambda NN_f}{2\sigma^2} \right)^m$$

from which $\lambda$ is computed setting the right hand side equal to $10^{-8}$. In practice the map $\sigma^2 \rightarrow \lambda$ can be stored in a read-only memory and the threshold value can be picked up as a function of the noise level.

VI. SIMULATION RESULTS

Computer simulations have been run to assess EBE performance and make comparisons with other algorithms available in literature. The following setting has been chosen.

The channel model is CM1 [16] and corresponds to line-of-sight propagation in residential environments. The duration $T_h$ is 20 ns, about the channel delay spread. The frame period $T_f$ equals 200 ns, the length of the pseudo-noise sequence is $N_f = 15$ and the symbol period is $T_s = N_fT_f = 3000$ ns. The TOA estimate is derived from observation intervals of $N = 100$ symbols.
The monocycle is shaped as the second derivative of a Gaussian function and its duration is either 4 ns (type-1 monocycle) or 1 ns (type-2 monocycle).

The EBE performance is compared with that of the schemes discussed in [9], [1] and [13]. The accuracy of these schemes depends on some design parameters, that have been experimentally chosen so as to get the best results. In our simulations the received pulses are undistorted.

Figure 3 illustrates the EBE performance and make comparisons with [9], [1] and [13]. The ordinates represent the TOA root-mean-square error (RMSE) in meters (i.e., the RMSE in seconds multiplied by the speed of light) while the abcissas give the ratio of the received signal energy per bit, $E_b$, to the noise spectral density $N_0$. A type-1 monocycle is adopted and the assumed DPP duration is $T_p = 4$ ns. The sampling rate $f_s$ varies from 0.5 GHz to 2 GHz. The sampling rate with [9] and [1] is the Nyquist rate, i.e. 2 GHz, while the I&D circuit in [13] is clocked at $1/T_p = 250$ MHz. It is seen that EBE outperforms the algorithms in [1] and [13] at any $E_b/N_0$. The method in [9] is a little better at low SNRs but its advantage is questionable in view of the optimistic assumptions made on the template.

Figure 4 shows analogous results with a type-2 monocycle. As expected, all the algorithms have improved performance but the ranking remains the same.

Degradations due to mismatch between true and assumed pulse duration are discussed in Fig. 5. It is seen that, while taking $T_p < T_p^{(true)}$ has only marginal consequences, the opposite is true with $T_p > T_p^{(true)}$, especially at high SNR. This suggests a strict attitude when guessing the pulse duration.

VII. CONCLUSIONS

A new TOA estimation algorithm has been proposed that operates on energy measurements made on a folded version of the received signal. Its motivation is based on the idea that, because of frequency-selective propagation effects, the shape of the received pulses is not known at the receiver and, in consequence, a correlation-based estimator is not feasible. We have revisited the TOA estimation problem assuming only an approximate knowledge of the pulse duration. Making use of LS techniques we have come up with an algorithm that incorporates the pulse duration information in the window size of the energy measurements.

Several approximations to the original scheme have been made to facilitate implementation. In particular the effects of sub-Nyquist sampling have been explored and it has been shown that estimation accuracies of a meter are achievable even with sampling rates of 500 MHz.

Comparisons with other estimation methods available in literature show that, with comparable implementation complexity, our scheme exhibits much better performance.

REFERENCES


Fig. 1. Waveform $r_{old}^2(t, \bar{n})$ for $|m - \bar{n}|N_0 > 2$ in the absence of noise.
Fig. 2. Explaining the estimation errors.

Fig. 3. Comparisons between EBE and the algorithms in [9], [1] and [13] with type-1 monocycle.

Fig. 4. Comparisons between EBE and the algorithms in [9], [1] and [13] with type-2 monocycle.

Fig. 5. Effects of mismatch between true and assumed pulse duration.
Sampling rate $f_s = 2$ GHz.