Role of Random Capacity Risk and the Retailer in Decentralized Supply Chains with Competing Suppliers

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ABSTRACT

This research considers a supply chain under the following conditions: (i) two heterogeneous suppliers are in competition, (ii) supply capacity is random and pricing is endogenous, (iii) consumer demand, with and without an intermediate retailer, is price dependent. Specifically, we examine how uncertainty in supply capacity affects optimal ordering and pricing decisions, supplier and retailer profits, and the incentives to reduce such uncertainty. When two suppliers sell through a monopolistic retailer, supply uncertainty not only affects the retailer’s diversification strategy for replenishment, but also changes the suppliers’ wholesale price competition and the incentive for reducing capacity uncertainty. In this dual-sourcing model, we show that the benefit of reducing capacity uncertainty depends on the cost heterogeneity between the suppliers. In addition, we show that a supplier does not necessarily benefit from capacity variability reduction. We contrast this incentive misalignment with findings from the single-supplier case and a supplier-duopoly case where both suppliers sell directly to market without the monopolistic retailer. In the latter single-supplier and duopoly cases, we prove that the unreliable supplier always benefits from reducing capacity variability. These results highlight the role of the retailer’s diversification strategy in distorting a supplier’s incentive for reducing capacity uncertainty under supplier price competition.

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INTRODUCTION

Numerous examples from the marketplace illustrate the impact of supply uncertainty on a firm’s sourcing strategy and supply chain (SC) performance. For instance, consider the long production delays faced by the new fuel-efficient Boeing 787 Dreamliner jet. Many reasons have been cited for these delays, such as the degree of innovation and increased global outsourcing of aircraft modules resulting in heightened complexity and uncertainty (Kesmodel & Michaels, 2011). One step that Boeing has taken in response to the production delays is to “in-source” some of the work that it had previously shifted to a supplier (Cohan, 2011). Boeing and its suppliers have also reduced sourcing risk by replacing lean, single-source parts’ contracts with contracts from two different vendors. In-house production and dual sourcing are common operational actions that global firms use to mitigate supply risk while increasing efficiency and lowering costs. In this context, we seek to understand firms’ strategies and incentives to handle uncertain supply capacity in a two-echelon (supplier–retailer) chain.

There is a growing body of analytical and empirical research that examines the impact of supply risk (Hendricks & Singhal, 2005; Dada, Petruzzi, & Schwarz, 2007; Burke, Carrillo, & Vakharia, 2009; Hult, Craighead, & Ketchen Jr., 2010). Previous models have generated valuable insight on firms’ sourcing strategies under supply uncertainty. However, these models often treat suppliers’ decisions as exogenous and focus mainly on the retailer’s optimal newsvendor procurement policies (Anupindi & Akella, 1993; Dada et al., 2007).

The purpose of this research is to study the impact of random supply capacity on the sequential, dynamic supplier–retailer SC game with competing suppliers. Our decentralized SC gaming model not only produces retailer and supplier decisions under equilibrium with corresponding profits, but it also captures the economics that governs firms’ incentives and interactions under supply risk. Recent literature has shown that the buying firm may offer incentives or work directly with its suppliers to influence supply reliability (Wang, Gilland, & Tomlin, 2010; Tang, Gurnani, & Gupta, 2011). To better explore this topic, suppliers’ decisions—specifically wholesale prices—are made endogenous. We assume that the retailer faces a market demand that is downward sloping in retail price (Liu, Fry, & Raturi, 2006; Anand, Anupindi, & Bassok, 2008; Tang & Kouvelis, 2011). Further, we assume that a supplier’s actual delivery is subject to random capacity, as it can only deliver the lesser of its capacity or the retailer’s order size. This type of operational risk is independent of wholesale pricing, contract type, and the retailer’s order quantity. This assumption is justified if supply risk is attributed to exogenous events such as limits in production capacity, labor strikes, unforeseen production difficulties, loss of products in shipping, material shortages, or infrastructure disruptions.

This research first considers single sourcing where a monopolistic retailer orders from an unreliable supplier. Next, we consider an SC where the retailer uses dual sourcing to diversify the orders between two competing suppliers that are heterogeneous in their unit costs and supply uncertainty. We argue that supply uncertainty will not only impact supplier–retailer gaming, but also shape suppliers’ price competition. In our models, we consider a common, price-only contract and
a Stackelberg game where the supplier, as leader, determines a wholesale price and the retailer then decides order quantity. We explore how supply risk influences players’ strategy preferences under these different SC structures. Note that we use the term supply capacity “risk” to denote both limited and uncertain capacity, because in both cases the supply amount received may be less than the desired target amount.

In our characterization of equilibrium outcomes, we also establish a number of new results and insights. Our single sourcing model shows that, even in a decentralized SC where agents’ incentives are normally not aligned, synergies among the supplier, retailer, and consumer can effectively reduce capacity uncertainty. This phenomenon has been observed in practice as buyers like Honda and Toyota work relentlessly with their independent suppliers to improve sourcing reliability (Liker & Choi, 2004; Sheffi, 2005). Our dual sourcing model not only includes vertical competition between the supplier and retailers, but also horizontal competition between suppliers. The retailer can now diversify its supply base by splitting its replenishment order between suppliers and enjoy the benefits of diversification. However, the previous alignment of players’ interests in improving reliability no longer holds. As such, the supplier may not always benefit from reduction in its capacity variability, although the monopolistic retailer always benefits from both fierce supplier price competition and reliability improvements in its supply base. Studies from empirical research have recognized that building long-term relationships with suppliers to enable the supplier improvement process is important to the retailer, as is ensuring price competition within a diversified supplier base (Wu & Choi, 2005).

To highlight the idea that an unreliable supplier could have an incentive to become more unreliable in the dual sourcing model, we consider a duopoly model where both supplying firms sell to the market directly under random capacity risk. In contrast to dual sourcing, the new SC setup removes the monopolistic retailer from the SC, with both suppliers first engaging in a Cournot production quantity competition and then a Bertrand pricing game after deliveries are realized. Without the monopolistic retailer, the duopoly model reverses our conclusions in dual sourcing, as the unreliable firm always has an incentive to improve its reliability. By contrasting the results from competing suppliers with and without the monopolistic buyer, we demonstrate that the retailer’s supply-base diversification in the decentralized, dual-sourcing SC affects the impact of random capacity, causing a distortion of the suppliers’ incentives for capacity risk reduction.

The remainder of this article is organized as follows. First, we review related literature on SC risks and contracting. We then describe a single sourcing model that serves as a template for our supplier competition models. Next, we model dual sourcing and focus on diversification and competition effects: first for the case with one reliable and one unreliable supplier, and next for the case when both suppliers are unreliable. We then present a supplier duopoly model without the retailer. We compare equilibrium results from these models computationally, focusing on the impact of random capacity upon SC decisions and incentives. Finally, we discuss conclusions and future research. All omitted proofs are provided as an electronic companion available from the authors.
LITERATURE REVIEW

SC coordination with contracts and newsvendor sourcing problems under supply uncertainty has drawn a lot of attention in the recent literature. However, classical SC models have not considered the connection between suppliers’ pricing decisions and their heterogeneous (and uncertain) capabilities of delivering quality goods, and have not analyzed the impact of supply risk upon channel performance.

Supply uncertainties can be modeled differently based on whether the cause and impact are attributed to random capacity (Ciarallo, Akella, & Morton, 1994; Kouvelis & Milner, 2002; Bollapragada, Rao, & Zhang, 2004) or random yield (Henig & Gerchak, 1990; Yano & Lee, 1995; Gurnani, Akella, & Lehoczky, 2000; Tomlin & Wang, 2005; Burke et al., 2009; Chaturvedi & Martinez-de-Albeniz, 2011). With random capacity, an uncertain exogenous upper bound on actual delivery is independent of order size. Under random yield, an uncertain production loss makes actual delivery a random fraction of order size. The supply disruption model with random on/off periods is a special case of random yield or random capacity, with actual delivery limited by an all-or-nothing Bernoulli trial. However, disruption models tend to be studied in a multiperiod setting. The existing supply uncertainty literature has focused mainly on optimal production and inventory policies to meet demand.

Recent SC literature has studied the optimal procurement policy or contingency strategy to mitigate risk associated with uncertainties, and provided insights on optimal newsvendor procurement strategies for supply risk mitigation. Tomlin (2006) considers dual sourcing from one reliable and another unreliable supplier under disruption risks. Dada et al. (2007) present a newsvendor procurement problem with customer service level requirements under sourcing from a supplier base comprised of reliable and unreliable suppliers to develop insights on supplier selection and lot size allocation decisions.

Another central theme in the supply risk literature is incentives and strategies for supply reliability improvement. Liu, So, and Zhang (2009) demonstrate that a retailer may be willing to pay a price premium for improved reliability. Wang et al. (2010) develop a mitigation strategy for a buying firm to expend reliability improvements on its supply base by comparing results from process improvement and dual sourcing under both random capacity and random yield. Finally, Tang et al. (2011) consider process improvement from the supplier’s side rather than from the retailer’s side. Their model allows the retailer to influence the supplier’s decision through a subsidy contract.

Most of the existing literature does not account for the gaming effects among SC agents and takes supplier pricing as exogenous. Thus, their insights are limited to diversification effects of the retailer’s decisions in the presence of supply uncertainty. An early exception is Babich, Burnetas, and Ritchken (2007), which explicitly considers the interaction of competition and diversification in studying the effects of supplier default risk with binary outcomes. They show that low supplier default correlations dampen suppliers’ competition and increase the equilibrium wholesale price. Tang and Kouvelis (2011) also include competition in their study of supplier diversification strategies under proportional random yield.
but their focus is on the retailers’ Cournot competition and not the suppliers’ Bertrand competition.

Although recent research offers valuable insights on suppliers’ diversification with competition, this research offers significant, new contributions to the SC literature. First, we focus on the impact of random supply capacity on SC performance under wholesale price contracts with retailer-specified endogenous retail price. We also demonstrate the existence of optimal polices under different system structures, including single sourcing, dual sourcing, and duopoly. Most important, we show the differences in the optimal strategies and incentives between these structures, which has been underexplored in previous random capacity studies.

Because our work investigates incentives and profits under a wholesale price contract, we also note the extensive coordination literature on the ability of vertical contracts to achieve first-best channel profits in a single-period game (for reviews, see Cachon, 2003; Li & Wang, 2006). This literature has generally ignored the impact from competition as well as diversification under supply uncertainties, and some scholars (Cachon, 2003) have noted an existing research gap related to how multiple suppliers compete for the affection of multiple retailers, and how scarce capacity is allocated and influences behavior in the SC. This study helps to fill this gap by investigating the impact of random supply capacity on SC gaming with competing suppliers.

MODELING FRAMEWORK

We first define the notation, assumptions, and sequence of events for a wholesale price contract between the supplier and retailer. Supply risk stems from exogenous random delivery capacity (Wang et al., 2010), which is beyond the control of the contract inputs. All agents are assumed to be rational, risk-neutral decision makers that possess complete and symmetric information. All production and shipment lead times, as well as salvage values, are assumed to be zero. We use backward induction to compute the subgame perfect equilibrium in a sequential game between supplier and retailer.

Model Features and Notation

Consider the two-echelon supplier–retailer chain illustrated in Figure 1. Supplier $i$ has unit production cost $c_i$ and a stochastic delivery capacity $K_i$. If the

Figure 1: Supplier–retailer–market model.
supplier is perfectly reliable, $K_i$ is constant and, to avoid trivial cases, is larger than the retailer’s optimal order quantity. In general, the sequence of events is as follows:

Stage 1. Supplier $i$ sets its unit wholesale price $w_i$.
Stage 2. Retailer then sets its order quantity $q_i$.
Stage 3. Supplier $i$ plans to produce quantity $q_i$. Supply capacity $K_i$ is realized at value $k_i$ and the supplier produces and ships $z_i = \min(k_i, q_i)$ to the retailer.
Stage 4. Retailer receives shipments and sets retail price $p$. Demand materializes; revenues and costs are incurred.

We use the following notation. Index $i$ is omitted for single sourcing:

- $K_i = \text{random capacity of supplier } i$, with realized capacity $k_i \leq k_{\text{max}}$.
- $f(k_i) = \text{probability distribution function (PDF) of capacity } K_i$.
- $F(k_i) = \text{cumulative distribution function (CDF) of capacity } K_i$, and $F(k) = 1 - F(k)$.
- $c_i = \text{unit production cost of supplier } i$.
- $w_i = \text{wholesale price from supplier } i$.
- $q_i = \text{order quantity placed with supplier } i \text{ by the retailer}$.
- $z_i = \text{actual quantity delivered by supplier } i$ with $z_i = \min(k_i, q_i)$. 
- $p = \text{unit retail price and to avoid trivial cases, } p > w > c$.
- $s = \text{retail sales quantity}$.
- $Re = \text{revenue from the consumer market}$.
- $D = \text{linear price dependent demand: } D(p) = \alpha - \beta p$ where $\alpha$ is the market size and $\beta$ is the slope of the demand curve.
- $\Pi_j = \text{profit of agent } j, j \in \{S_i, \text{supplier } i; B, \text{retailer}; C, \text{whole SC, that is, channel}\}$.

The analysis below specifies a single-period model and provides expressions for the corresponding optimal decisions. We assume that initial inventory for the retailer is zero. A linear deterministic demand model, $D(p) = \alpha - \beta p$, enables us to focus on supply uncertainty while allowing the retailer to influence market demand by setting the retail price. Under this assumption, we can infer the impact of supply capacity risk on the consumer market by checking consumer welfare, measured by the change in consumer surplus.

**Single Supplier and Retailer: Single Sourcing Case**

In the decentralized SC, we solve the subgame Nash equilibrium using backward induction. We start our analyses reverse-chronologically with stage 4, when the retailer sets retail price $p$ with complete information regarding results from previous stages. The retailer maximizes its expected revenue as the procurement cost is
sunk.

\[
\max_p \Re = E(p_s), \text{ s.t. } s = \min(D(p), z). \tag{1}
\]

At stage 3, the supplier’s optimal production quantity that will maximize profits equals the retailer’s order quantity \(q\), assuming that the supplier incurs costs only for items that are actually delivered (when the ordered quantity \(q\) is larger than the realized capacity \(k\)). At stage 2, the retailer decides on the order quantity \(q\) to maximize its profit:

\[
\max_q \prod_B \equiv E(\Re^* - wz), \text{ s.t. } z = \min(k, q). \tag{2}
\]

Solving the optimizations yields the following lemma regarding the retailer’s best strategy:

**Lemma 1:** In a decentralized SC, the optimal retail pricing and ordering strategy is given by \(p^* = \max(\frac{\alpha - z}{\beta}, \frac{\alpha}{2\beta})\) and \(q^* = \min(\frac{\alpha - \beta w}{2}, k_{max})\), and revenue \(\Re^*(z) = (\frac{\alpha - z}{\beta})z\).

In the first stage, we solve for the supplier’s wholesale price that maximizes her profit:

\[
\max_w \prod_s \equiv E((w - c)z), \text{ s.t. } z = \min(k, q^*), w > c. \tag{3}
\]

The result is given in the following lemma:

**Lemma 2:** The unique optimal wholesale price \(w^*\) that maximizes supplier profit is \(w^* = c + \frac{2}{\beta} \int_0^{q^*} \frac{F(k)dk}{F(q^*)}\) when \(q^* < k_{max}\) and \(w^* = \frac{\alpha - 2k_{max}}{\beta}, \) when \(q^* = k_{max}\).

Combining Lemmas 1 and 2, we obtain the unique sub-game perfect equilibrium order quantity \(q^*\) and wholesale price \(w^*\), specified below in Theorem 1.

**Theorem 1:** In equilibrium, the optimal order quantity \(q^* = \frac{\alpha - \beta c}{2} - \int_0^{q^*} \frac{F(k)dk}{F(q^*)}\) and the optimal wholesale price \(w^* = \frac{\alpha - 2q^*}{\beta}\).

It is apparent that in an SC without capacity risk (\(K = \infty\)), the optimal wholesale price \(w' = \frac{c}{2} + \frac{\alpha}{2\beta}\), and the corresponding order size \(q' = \frac{\alpha - \beta w'}{2} = \frac{\alpha - \beta c}{4}\). Comparing the two cases with and without capacity risk, we observe that although the retailer does not structurally alter order size, it ends up ordering less under random capacity because of the higher wholesale price set by the supplier. In addition to the equilibrium outcomes, Table 1 provides price and profits for individual players as well as the whole chain, from which we obtain the following proposition.

**Proposition 1:**

(a) The introduction of risk to a decentralized SC does not alter the relationship between a retailer’s order size and wholesale price, but leads to a supplier charging a higher wholesale price, resulting in a decrease in the retailer’s order quantity.
Table 1: Decentralized SC with and without random capacity (RC) constraints.

<table>
<thead>
<tr>
<th>Optimal Solutions</th>
<th>Without RC Constraints</th>
<th>With RC Constraints</th>
<th>∆</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order quantity $q$</td>
<td>$\frac{\alpha - \beta w}{2} = \frac{\alpha - \beta w}{4}$</td>
<td>$\frac{\alpha - \beta w}{2} - \int_0^q \frac{F(k)dk}{F(q)}$</td>
<td>↓</td>
</tr>
<tr>
<td>Wholesale price $w$</td>
<td>$\frac{c}{2} + \frac{\alpha}{2\beta}$</td>
<td>$\frac{c}{2} + \frac{2}{\beta} \cdot \int_0^q \frac{F(k)dk}{F(q)}$</td>
<td>↑</td>
</tr>
<tr>
<td>Retail price $p$ or $E(p)$</td>
<td>$\frac{3\alpha + \beta c}{4\beta}$</td>
<td>$\frac{\alpha - q}{\beta} + \frac{1}{\beta} \int_0^q F(k)dk$</td>
<td>↑</td>
</tr>
<tr>
<td>Retailer profit $\Pi_B$</td>
<td>$(\alpha - \beta c)^2$</td>
<td>$(\alpha - \beta w)^2 - \int_0^q F(k)d(Re(k) - wk)$</td>
<td>↓</td>
</tr>
<tr>
<td>Supplier profit $\Pi_S$</td>
<td>$\frac{(\alpha - \beta c)^2}{8\beta}$</td>
<td>$(w - c)q - (w - c) \int_0^q F(k)dk$</td>
<td>↓</td>
</tr>
<tr>
<td>Supply chain profit $\Pi_C$</td>
<td>$\frac{3(\alpha - \beta c)^2}{16\beta}$</td>
<td>$(\frac{\alpha - q}{\beta} - c)q - \int_0^q F(k)d(Re(k) - ck)$</td>
<td>↓</td>
</tr>
</tbody>
</table>

(b) Both the supplier and the retailer suffer lower profits under supply capacity risk. Consumer surplus and welfare are also lower because of an increase in retail price. However, under the same capacity risk, a reduction in the supplier’s unit cost increases its profit.

In this decentralized chain, the supplier setting the wholesale price is the leader of the Stackelberg game. Because of the first mover advantage, the supplier can effectively shift a portion of the supply risk downstream by raising wholesale prices, to which the retailer can only respond by ordering a reduced quantity. The retailer may have to decide whether to pick a high cost, reliable supplier or a low cost, unreliable supplier. Proposition 1(a) indicates that the potential savings for the retailer in switching to a low cost, unreliable supplier is limited as the sole supplier always increases the wholesale price under unreliable capacity. From Proposition 1(b), both the supplier and the retailer benefit from reducing capacity uncertainty under single sourcing. Because the retailer also benefits from the supplier’s risk mitigation, it is justifiable for the retailer to share the cost of such an effort. This finding corresponds with prior research in supplier development under various scenarios (Krause, Handfield, & Tyler 2007; Liu et al., 2009; Wang et al., 2010; Tang et al., 2011).

Typically, the supplier can achieve increased reliability (lower capacity uncertainty) at a higher unit cost. The supplier may have a choice of what reliability level to offer if the relationship between unit cost and capacity uncertainty is known. From the last statement of Proposition 1(b), we see that the supplier may be better off with uncertain capacity when such uncertainty permits a significant reduction in its unit production cost (an exogenously specified parameter in the model). The supplier may then compare the following two options: (i) providing
ample capacity to fulfill the retailer’s order without uncertainty or, (ii) making order deliveries subject to random capacity. Based on Proposition 1(b), the supplier will always choose option (i) when unit production cost is the same. However, when flexible delivery capacity drives unit costs down because of lower costs of equipment maintenance and staff scheduling, the supplier can choose option (ii) and achieve higher profit. Effectively, the capacity risk reduces the supplier’s profit, whereas the lower unit cost increases it. The supplier has to weigh the trade-offs between providing reliable capacity at a premium and allowing capacity uncertainty at a reduced cost.

Finally, Proposition 1(b) shows that consumer welfare measured by the change in consumer surplus is also less under capacity risk as the supplier and retailer raise prices, which moves some supply risk further downstream. To summarize, both supplier and retailer observe a decrease in profits because of random (and limited) supply capacity: the SC with uncertainty is Pareto-dominated by the SC without uncertainty. Furthermore, the supplier, retailer, and consumer share supply risk. Therefore, the reduction in capacity risk benefits all of these stakeholders.

Given the above findings, the next question is whether the impact of supply risk is a consequence of the absence of competition in single sourcing. We next consider capacity risk along with supplier competition, specifically in the cases of dual sourcing and duopoly.

**SUPPLIER COMPETITION: DUAL SOURCING CASE**

The research question examined here is motivated by the observation of multinational corporations adopting different pricing structures when sourcing from different suppliers for goods of similar quality (Allon & Van Mieghem, 2010; Chung, Talluri, & Narasimhan, 2010). Specifically, we consider whether a supplier (and the retailer) benefits from a reduction in capacity risk, and under what conditions.

We first consider dual sourcing where the retailer can source from two suppliers (one reliable and one unreliable) engaging in wholesale price competition. We then relax the reliability assumptions to consider dual sourcing with random supply capacities for both suppliers and characterize the equilibrium solution.

**Reliable Supplier versus Unreliable Supplier**

Under the previous framework, we formulate the SC model of two suppliers with heterogeneity in their delivery capabilities. One supplier (denoted as $S_1$) is reliable and committed to deliver the order size $q_1$ at unit cost $c_1$, whereas another supplier (denoted as $S_2$) is unreliable and subject to a random delivery capacity $K$ at unit cost $c_2$. Reliability and pricing are two important factors that lead to dual sourcing, which is one of the most common SC contracting practices and offers risk hedging against potential supply and pricing uncertainties.

Following the same assumptions and event sequence as in the single sourcing case, we study the impact of reliability on the Nash equilibrium of this multistage game in which both the reliable and unreliable suppliers compete on price and
reliability, and the retailer’s order allocation depends on the outcome of the suppliers’ price competition. In stage 4, the retailer sets retail price $p$ with complete information from previous stages and seeks to maximize its revenue, given the actual shipments received. This process is identical to the revenue maximization model described in Equation (1). Thus, the optimal retail pricing structure from Lemma 1 still holds in the dual sourcing case with total deliveries $z = z_1 + z_2$.

The random capacity is realized in stage 3. In stage 2, the retailer makes ordering decisions to maximize the following expected profit function:

$$\text{Max} \quad q_1, q_2 \prod_B \equiv E(Re^* - w_1z_1 - w_2z_2), \text{ s.t. } z_1 = q_1, z_2 = \min(k, q_2).$$  \quad (4)

The retailer chooses $q = (q_1, q_2)$ to maximize expected profit given pricing structure $w = (w_1, w_2)$ from the suppliers, which is an outcome of the preceding first-stage supplier wholesale pricing game. Given the expression for optimal revenue $Re^*$ with total delivery $z = z_1 + z_2$, we rewrite the retailer’s procurement problem as

$$\text{Max} \quad q_1, q_2 \prod_B (q_1, q_2|w_1, w_2) = [1 - F(q_2)] \left[ \frac{\alpha - q_1 - q_2}{\beta} (q_1 + q_2) - w_1q_1 - w_2q_2 \right]$$

$$+ \int_0^{q_2} \left[ \frac{\alpha - q_1 - k}{\beta} (q_1 + k) - w_1q_1 - w_2k \right] f(k) dk \quad \text{s.t. } q_1 \geq 0, q_2 \geq 0.$$  \quad (5)

The first order condition yields $q_1 + q_2 = \frac{1}{2} (\alpha - \beta w_2)$ and $q_2^* = \frac{1}{\frac{1}{2} \beta (w_1 - w_2)}$. Thus, for the retailer’s procurement problem, there is always a unique optimal procurement quantity vector $q$ that optimizes its profit, given the suppliers’ wholesale price.

Returning to the first stage, we solve the problem of each individual supplier searching for a wholesale price to maximize its expected profit.

$$\text{Max} \quad w_i \prod_{S_i} \equiv E[(w_i - c_i) z_i], \text{ s.t. } \begin{cases} z_1 = q_1^* \quad \\ z_2 = \min(k, q_2^*) \end{cases}.$$  \quad (6)

The unique Nash equilibrium of the suppliers’ game is achieved when each supplier chooses a wholesale price anticipating the best ordering strategy from the retailer and its competitor’s optimal pricing strategy. We suppress the dependence of $q$ on $w$ in the following problems for simplicity, knowing that $(q_1^*, q_2^*)$ is dependent on $(w_1^*, w_2^*)$:

$$\text{Max} \quad \Pi_{S_1} \left( w_1 | q_1^*, q_2^*, w_2^* \right) = (w_1 - c_1) q_1^* \text{ s.t. } w_1 > c_1,$$  \quad (7)

$$\text{Max} \quad \Pi_{S_2} \left( w_2 | q_1^*, q_2^*, w_2^* \right) = (w_2 - c_2) \left[ q_2^* - \int F(k) dk \right] \text{ s.t. } w_2 > c_2.$$  \quad (8)
The Nash equilibrium of this first-stage supplier game \((w_1^*, w_2^*)\) must satisfy Equations (7) and (8) simultaneously, whose existence and uniqueness are stated below in Lemma 3.

**Lemma 3:** There exists a unique Bertrand Nash equilibrium when suppliers optimize their utilities by individually choosing their prices \((w_1, w_2)\). The equilibrium solution satisfies \[ w_1 = c_1 + \frac{2F(q_2)}{\beta} q_1 \] and \[ w_2 = c_2 + \frac{2F(q_2)}{\beta F(q_2)} \left[ \int_0^{q_2} F(k)dk \right] \beta F(q_2). \]

Although the suppliers’ profit functions are generally not concave, they are unimodal so that we can still solve for the optimal pricing vector \(w^*\), which is characterized below in Theorem 2.

**Theorem 2:** There is a unique optimal solution \(\{q_1^*, q_2^*; w_1^*, w_2^*\}\) for the SC game under capacity uncertainty that is based on the nontrivial solution to the following:

\[
\begin{align*}
    q_1 + q_2 &= \frac{1}{2}(\alpha - \beta w_2) \\
    \int_0^{q_2} F(k)dk &= \frac{1}{2}\beta(w_1 - w_2) \\
    w_1 &= c_1 + \frac{2F(q_2)}{\beta} q_1 \\
    w_2 &= c_2 + \frac{2F(q_2)}{\beta F(q_2)} \left[ \int_0^{q_2} F(k)dk \right].
\end{align*}
\]  

(9)

Theorem 2 characterizes the equilibrium in dual sourcing, and the optimal profits of individual players are readily obtained by incorporating the solutions into the corresponding profit functions. By nontrivial solution in the theorem, we mean to exclude trivial cases corresponding to \(q_1 \) or \(q_2 \leq 0\); for example, if \(\alpha - \beta w_2 < 0\) then \(q_1 + q_2 = 0\); if \(w_1 - w_2 < 0\) then \(q_2 = 0\). The general solution to (9) is largely dependent upon the exact form of CDF \(F(k)\) and there are limited cases such as the uniform or triangular distributions that allow us to achieve closed-form solutions.

In certain instances, the equilibrium dual-sourcing solution will lead to a sole-sourcing outcome. From (9), a necessary and sufficient condition for \(q_2^* > 0\) is that the wholesale price difference, \(w_1^* - w_2^* > 0\), with the unreliable supplier offering a wholesale price lower than the reliable supplier’s. In fact, from the second equation of (9), noting that \(F(k) \geq 0\), we see that \(q_2^*\) is strictly increasing in the wholesale price difference between the two suppliers, with \(q_2^* = 0\), when \(w_1^* = w_2^*\). To compete for a share of the retailer’s order and to ensure \(w_2^* < w_1^*\), the unreliable supplier has to maintain a lower production cost (i.e., \(c_2 \leq c_1\)). This price discrimination by the retailer toward the unreliable supplier could also be understood as a risk premium (i.e., the additional discount required by the retailer to place an order with the unreliable supplier). Because the more reliable supplier enjoys a price premium in the competition, she is always the price leader; but not always the market leader in terms of the order size.
Both Suppliers with Random Capacity

For completeness, we now consider the general case of both suppliers $S_1$ and $S_2$ being unreliable with random capacities, $K_1$ and $K_2$ having CDFs of $F(k_1)$ and $G(k_2)$, respectively. When the causes for such uncertainty are unrelated, we may assume that the delivery capacities of these two stand-alone suppliers are also independent of each other. For circumstances when suppliers’ capacities are dependent or correlated, our modeling framework still holds using the joint distribution $F(k_1, k_2)$ of the bivariate random capacity.

For the retailer’s procurement problem, we can show that the optimal order quantity vector $q$ for retailer’s profit optimizing problem simultaneously satisfies $q_1 + q_2 = \frac{1}{2}(\alpha - \beta w_2) + \int_0^{q_1} F(k_1)dk_1$ and $q_1 + q_2 = \frac{1}{2}(\alpha - \beta w_1) + \int_0^{q_2} G(k_2)dk_2$. As expected, the expressions here and later in this section are symmetric in the suppliers’ capacity distributions $F(\cdot)$ and $G(\cdot)$. Given the outcome of the retailer’s optimal decision, we are able to obtain the equilibrium of the supplier pricing game and the following key result:

**Theorem 3:** There is a unique optimal solution $\{q_1^*, q_2^*; w_1^*, w_2^*\}$ for dual sourcing from two suppliers with random capacities. It is the nontrivial solution to the following:

$$
\begin{align*}
q_1 + q_2 &= \frac{1}{2}(\alpha - \beta w_2) + \int_0^{q_1} F(k_1)dk_1 \\
\quad &= \frac{1}{2}(\alpha - \beta w_1) + \int_0^{q_2} G(k_2)dk_2 \\
\frac{1}{2} \int_0^{q_1} F(k_1)dk_1 &= \frac{1}{2} \int_0^{q_2} G(k_2)dk_2 \\
\frac{\beta F(q_1)}{2} &= \frac{\beta G(q_2)}{2}
\end{align*}
$$

(10)

In dual sourcing with supply uncertainties, the wholesale price and the retailer’s order quantities at equilibrium are driven by the competition between suppliers as well as the gaming between the retailer and suppliers. For the suppliers, supply reliability offers a new dimension of competition so that they don’t have to compete solely on price. Therefore, heterogeneity in supply reliability offers a service level differentiation in addition to price competition. Note that in the suppliers’ Bertrand oligopoly, both firms are able to charge a wholesale price higher than their marginal costs and from Theorem 3, the wholesale price difference between the two unreliable suppliers satisfies $\frac{1}{2} \beta (w_1 - w_2) = \int_0^{q_1} G(k_2)dk_2 - \int_0^{q_2} F(k_1)dk_1$.

To summarize, in the dual sourcing case, random capacity risk affects wholesale pricing differently than in single sourcing because of the suppliers’ competition for the retailer’s order. Later, we use an illustrative numerical example (Example 1) to study SC performance at different levels of supply capacity uncertainty. From Example 1 we note that, for dual sourcing, a supplier competing for a monopolistic retailer’s orders may not always benefit from reducing its capacity risk. Believing that this result stems from the decentralized SC wholesale-pricing Bertrand game
with a monopolistic retailer, we next consider a different supplier duopoly model without the retailer.

**SUPPLIER COMPETITION: DUOPOLY CASE**

**Both Suppliers Selling to Market Directly**

In this section, we consider a supplier duopoly model where the two suppliers sell directly to a consumer market with a market demand characterized by \( D(p_1, p_2) \). This duopoly model serves to identify the critical role of the retailer; it also switches the first-stage decision from a Bertrand wholesale-pricing competition to a Cournot production-quantity competition. In particular, we check whether our previous dual sourcing results may be reversed (i.e., both firms could enjoy positive profit when they are equally reliable and the unreliable firm always has an incentive to reduce its variability). With this motivation, we study the effect of random capacity on the supplier duopoly and compare resulting insights with those from the previous dual sourcing model.

In this duopoly model, the sequence of events is similar to the previous analysis: First, supplier \( i \) plans to produce quantity \( q_i \). Next, supply capacity \( K_i \) is realized at value \( k_i \) and the supplier \( i \) produces \( z_i = \min(k_i, q_i) \) units. Then supplier \( i \) sets its retail price \( p_i \), market demand occurs, and profits accrue. We restrict attention to the case where supplier 1 is perfectly reliable (\( K_1 = \infty \)) and supplier 2 has random supply capacity.

From economic theory under duopoly, we assume that the demand function for supplier \( i \) is given by \( D_i(p_i) = \alpha - \beta p_i + \gamma p_j \), with \( \beta > \gamma \) for the demand model to be stable. Using backward induction, we investigate the last-stage retail pricing game first: both suppliers set their retail price \( p_i \) to maximize their individual revenue after the random capacity is realized.

\[
\text{Max } Re_i = E(p_i | s_i), \quad s.t. \quad s_i = \min(D_i(p_i, p_j), z_i), \quad (i, j) = (1, 2) \text{ or } (2, 1).
\]  

The optimal equilibrium price \( p_i \) now depends explicitly on the other retailer’s price \( p_j \). Given that the model is symmetric, we derive the Nash equilibrium as in dual-sourcing and obtain the following result.

**Lemma 4:** Supplier \( i \)'s optimal retail price is \( p_i^* = \max\left(\frac{\alpha - z_i + \gamma p_j}{\beta}, \frac{\alpha + \gamma p_j}{2\beta}\right) \) and the corresponding optimal revenue is

\[
Re_i^*(z_i) = \begin{cases} 
\left(\frac{\alpha - z_i + \gamma p_j}{\beta}\right) z_i, & \text{if } z_i \leq \frac{\alpha + \gamma p_j}{2\beta} \\
\left(\frac{\alpha + \gamma p_j}{2\beta}\right)^2, & \text{if } z_i > \frac{\alpha + \gamma p_j}{2\beta}
\end{cases}
\]

Next we determine the optimal production quantity \( q_i \) that maximizes supplier \( i \)'s profit in the production stage. Like all previous models, we assume costs are incurred only for items actually produced (even when the planned quantity \( q \) is
larger than the actual output $z$):

$$\text{Max} \prod_{S_i} (q_i) \equiv E(Re_i^c - c_i z_i), \quad s.t. \ z_i = \min (k_i, q_i). \quad (12)$$

From Lemma 4 and Equation (12) with $c_i > 0$, we see that it is never optimal for the supplier $i$ to have $z_i > \frac{\alpha + \gamma p_j}{\beta}$. Solving the simultaneous equations for $p_i$ and $p_j$ from Lemma 4, we write supplier $i$’s optimal retail pricing in terms of $z_i$ and $z_j$:

$$p_i^* = \frac{\alpha}{\beta - \gamma} - \frac{\beta z_i + \gamma z_j}{\beta^2 - \gamma^2}, \quad \text{for } (i, j) = (1, 2) \text{ or } (2, 1). \quad (13)$$

For the reliable supplier 1, $z_1 = q_1$, and for the unreliable supplier 2, $z_2 = \min(q_2, k)$, where we drop the subscript on realized capacity $k_2$. From Lemma 4 and the relationship between $q$ and $z$, we compute the suppliers’ optimal first-stage production decisions under random capacity.

**Theorem 4:** The Cournot duopoly in the production stage between the reliable supplier 1 and the unreliable supplier 2 yields a unique equilibrium for production quantity $q$, which satisfies the following equations:

$$2\beta q_1 + \gamma E (z_2) = \alpha (\beta + \gamma) - c_1 (\beta^2 - \gamma^2), \quad (14)$$

$$\gamma q_1 + 2 \beta q_2 = \alpha (\beta + \gamma) - c_2 (\beta^2 - \gamma^2). \quad (15)$$

where $E (z_2) = q_2 - \int_0^{q_2} F (k) \, dk.$

From Theorem 4, we first note that as expected, $q_1 \geq q_2$ when $c_2 \geq c_1$, indicating that the reliable supplier will produce and sell more products when its cost is equal or lower than the unreliable supplier’s. We also note that production quantities translate into inventory availability $z_1$ and $z_2$, which then yields market share capture, revenue, and profit for each retailer. For the previous dual sourcing model, we show in the next section that the unreliable supplier’s profit could increase or decrease with an improvement in supply capacity reliability. However, in the duopoly model, we prove below that the unreliable supplier always benefits from improvements in supply capacity. This result requires an understanding of how the equilibrium production quantities in Theorem 4 change with supplier 2’s random capacity, $K$.

Because supply capacity is random, improving (increasing) the capacity random variable can be defined using different stochastic orders. To be consistent with the numerical example used in dual sourcing—where we keep the mean fixed and increase the variance—we use a variability ordering called convex (CX) ordering (Shaked & Shanthikumar, 2007). Our key results may also be proven for other stochastic orderings in which the mean capacity is allowed to improve. We consider two capacity random variables $K^{lv}$ and $K^{hv}$ with $K^{lv} \leq_{CX} K^{hv}$. By definition, $K^{lv} \leq_{CX} K^{hv}$ iff $E(\phi(K^{lv})) \leq E(\phi(K^{hv}))$ for all real-valued convex functions, $\phi(\cdot)$; in particular, using $\phi(k) = k, \phi(k) = -k$ and $\phi(k) = [k - E(K)]^2$, the numerical example used in dual sourcing—where we keep the mean fixed and increase the variance—we use a variability ordering called convex (CX) ordering (Shaked & Shanthikumar, 2007). Our key results may also be proven for other stochastic orderings in which the mean capacity is allowed to improve. We consider two capacity random variables $K^{lv}$ and $K^{hv}$ with $K^{lv} \leq_{CX} K^{hv}$.
this implies \(E(K^{lv}) = E(K^{hv})\) and \(E[Var(K^{lv})] \leq E(Var(K^{hv}))\) so \(K^{lv}\) has lower variability, hence the \(lv\) superscript. As shown in Appendix S1, if unit cost \(c_2\) does not change when supplier reliability changes, we obtain the following result, where we use superscripts \(lv\) and \(hv\) on the equilibrium decisions to distinguish the cases with low and high variability capacity, \(K^{lv}\) and \(K^{hv}\), respectively.

**Lemma 5:** For the duopoly model with \(K^{lv} \leq_{CX} K^{hv}\) and \(c_2^{lv} = c_2^{hv}\), supplier 2’s corresponding equilibrium production quantity satisfies \(q_2^{lv} \geq q_2^{hv}\).

By Lemma 5, if supplier 2 improves capacity (reduces variability), it will pick a higher optimal production quantity. This higher \(q_2\) translates into a higher value of \(E(z_2) = E[\min(q_2, k)]\). In fact, by the proof of Lemma 5, we see that \(E(z_2^{lv} - z_2^{hv}) = \frac{4\beta}{\alpha}(q_2^{lv} - q_2^{hv})\). Thus \(q_2^{lv} \geq q_2^{hv}\) implies \(E(z_2^{lv}) \geq E(z_2^{hv})\). Hence, by Equation (14) in Theorem 4, when supplier 2 improves its capacity, supplier 1’s order (and expected sales) quantity decreases (i.e., \(q_1^{lv} \leq q_1^{hv}\)). Further, each supplier’s expected retail price decreases with improved capacity (reduced variability) at the unreliable supplier 2. This may be seen for \(E(p_1)\) by noting that, by Equations (13) and (14), \(E(p_1)\) depends solely on \(-E(\beta z_1 + \gamma z_2) = -\alpha(\beta + \gamma) + c_1(\beta^2 - \gamma^2) + \beta q_1\), which increases with \(Var(K)\) because \(q_1^{lv} \leq q_1^{hv}\).

A similar argument applies to \(E(p_2)\) for which the monotonicity result is more intuitive because the expected supply \(E(z_2)\) increases with \(Var(K)\), and this increased supply corresponds to a lower retail price. These ideas are summarized below in Theorem 5.

**Theorem 5:** For the duopoly model with \(K^{lv} \leq_{CX} K^{hv}\) and \(c_2^{lv} = c_2^{hv}\), in addition to \(q_2^{lv} \geq q_2^{hv}\), we have: (i) \(E[z_2^{lv}] \geq E[z_2^{hv}]\), (ii) \(q_1^{lv} \leq q_1^{hv}\), (iii) \(E[p_1^{lv}] \leq E[p_1^{hv}]\), and \(E[p_2^{hv}] \geq E[p_2^{hv}]\).

Because supplier 1’s order quantity and retail price both decrease when supplier 2 improves capacity (moving from \(K^{hv}\) to \(K^{lv}\)), their expected profits will also decrease. However, because supplier 2’s expected retail price decreases and order quantity increases with improvement in supply capacity, the net effect on supplier 2’s expected profit and its share of the chain’s profit is less apparent. As stated below, in the duopoly model, supplier 2’s expected profit behaves in an intuitive manner, increasing with supply capacity improvement (variability reduction). One way to see this result is that when supplier 2 improves its capacity, the system consisting of both suppliers is better off (has higher total SC profits). Supplier 1 is worse off, therefore supplier 2 must be better off.

**Proposition 2:** Supplier 2’s equilibrium expected profit and profit share are non-decreasing when its supply capacity, \(K\), improves in the convex ordering sense (reducing variability).

This analytical result for the Cournot quantity competition in the duopoly model should be contrasted against the Bertrand competition examples below for dual sourcing, where we find that Supplier 2’s equilibrium expected profit could either increase or decrease when its supply capacity improves (in convex order corresponding to lower variability with the same mean). The presence of the monopolistic retailer in dual sourcing sometimes allows this retailer to extract all the benefits of supply capacity improvement, necessitating some transfer of these
benefits to the supplier implementing capacity improvement. In the next section, we use numerical examples to study SC performance at different levels of supply capacity uncertainty.

**ILLUSTRATIVE NUMERICAL EXPERIMENTS**

In this section, for uniformly distributed supply capacity, we first study the effects of supplier competition in the dual sourcing case where the retailer allocates its order between the two retailers. We find that depending on the cost heterogeneity in dual sourcing, the suppliers may not benefit from a reduction in capacity risk. We then contrast this result with the duopoly case where, consistent with Proposition 2, the unreliable supplier always profits from its supply capacity improvement.

**Dual Sourcing Example: Competition vs. Diversification**

We now illustrate that under heterogeneous supplier capacities, an increase in the unreliable supplier’s capacity CV (coefficient of variation) reduces direct supplier price competition and therefore results in an increase in both suppliers’ wholesale price. As reliability heterogeneity increases and suppliers become more divergent, more orders go to the reliable supplier. However, both suppliers’ profits are higher. Nevertheless, the retailer’s profit decreases as wholesale price increases and the total order quantity decreases. When the unreliable supplier’s reliability improves, the retailer rewards this supplier with a larger order quantity. However, this reliability improvement reduces reliability heterogeneity between suppliers, which leads to more direct competition in price, thus reducing wholesale price. Therefore it is the retailer, and not the unreliable supplier, who is able to capture the benefit of reliability improvements as competition becomes fiercer and both suppliers’ profits decrease. We also examine unit cost heterogeneity under dual sourcing and illustrate that the unreliable supplier’s incentive to improve is much greater when the reliable supplier’s unit production cost is high.

The above results are from the trade-offs between order diversification and price competition effects. From the retailer’s perspective, if the unreliable supplier has high uncertainty and low unit cost, this supplier is perceived as risky. Therefore, the benefits of diversification are high. However, because the two suppliers differ along two dimensions (price and reliability) and are less substitutable, price competition is reduced and both suppliers can increase wholesale prices, diminishing the benefits of competition for the retailer. In contrast, as the supplier’s capacity uncertainty is reduced, the benefit of diversification decreases. Meanwhile, the unreliable supplier’s uncertainty improvement decreases the suppliers’ reliability heterogeneity and the two suppliers become more substitutable. Therefore, fierce price competition between suppliers occurs, and the overall wholesale price charged to the retailer decreases. This increased competition benefits the retailer’s bottom line, but could hurt the unreliable supplier, providing it with a disincentive to reduce risk.

We present these ideas through example 1 below. When the unreliable supplier 2 has a Uniform\([a, b]\) capacity distribution, the equilibrium equations for
Table 2: Equilibrium solutions for dual sourcing under homogenous unit costs ($c = 10$).

<table>
<thead>
<tr>
<th>Equilibrium Solutions</th>
<th>$w_1^*$</th>
<th>$w_2^*$</th>
<th>$q_1^*$</th>
<th>$q_2^*$</th>
<th>$\Pi_{S1}$</th>
<th>$\Pi_{S2}$</th>
<th>$\Pi_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>10</td>
<td>10</td>
<td>200</td>
<td>200</td>
<td>0</td>
<td>0</td>
<td>8000</td>
</tr>
<tr>
<td>Case A: Low variability</td>
<td>11.8</td>
<td>11.7</td>
<td>206.1</td>
<td>176.7</td>
<td>371.0</td>
<td>298.4</td>
<td>7312.8</td>
</tr>
<tr>
<td>Unif (150, 450)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case B: Medium variability</td>
<td>12.6</td>
<td>12.3</td>
<td>209.4</td>
<td>167.7</td>
<td>544.4</td>
<td>379.7</td>
<td>7039.9</td>
</tr>
<tr>
<td>Unif (125, 475)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case C: High variability</td>
<td>13.2</td>
<td>12.7</td>
<td>212.7</td>
<td>159.9</td>
<td>680.6</td>
<td>419.6</td>
<td>6841.0</td>
</tr>
<tr>
<td>Unif (100, 500)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: 1: reliable supplier, 2: unreliable supplier. * denotes optional value of parameter.

The unique wholesale prices $\{w_1^*, w_2^*\}$ and retailer’s order quantities $\{q_1^*, q_2^*\}$ from Theorem 2 simplify to

\[
\begin{align*}
q_1 + q_2 &= \frac{1}{2} (\alpha - \beta w_2) \\
\frac{(q_2 - a)^2}{b - a} &= \frac{1}{2} \beta (w_1 - w_2) \\
w_1 &= c_1 + \frac{2(q_2 - a)}{\beta (b - a)} q_1 \\
w_2 &= c_2 + \frac{2(q_2 - a)(a^2 - q_2^2 + 2bq_2)}{\beta (b - a)(b - q_2)}
\end{align*}
\]

(16)

**Example 1**: Consider the following dual sourcing cases. Market size $\alpha = 1,000$ and price elasticity $\beta = 20$, thus demand $D = 1,000 - 20p$. For both suppliers, unit production cost $c = 10$. Supplier 1 is reliable and supplier 2 is unreliable with uniform capacity distribution. For cases A through C in Table 2, the random capacities have the same mean equal to 300, but with increasing standard deviation (std). In other words, the random capacity’s std increases and mean capacity remains constant, hence the coefficient of variation ($CV = \frac{\text{std}}{\text{mean}}$) increases. In term of reliability, case C is riskier because $K_C$ is larger than $K_B$ and $K_A$ in convex order (Shaked & Shanthikumar, 2007).

We also consider a benchmark base case where both suppliers are reliable. This base case will yield less interesting results on wholesale pricing because of Bertrand competition, as the retailer extracts all SC profits, enjoying maximum competition benefits.

Table 2 shows the optimal solutions as well as players’ profits under cases with heterogeneous capacities. As noted earlier, an increase in the unreliable supplier’s $CV$ reduces direct supplier price competition and therefore results in an increase in both suppliers’ wholesale price. As reliability heterogeneity increases and suppliers become more divergent, more orders go to the reliable supplier. However, both suppliers’ profits are higher. Nevertheless, the retailer’s profit decreases as wholesale price increases and the total order quantity decreases.
Table 3: Equilibrium solutions for dual sourcing under heterogenous unit costs.

<table>
<thead>
<tr>
<th>Optimal Solutions</th>
<th>$w_1^*$</th>
<th>$w_2^*$</th>
<th>$q_1^*$</th>
<th>$q_2^*$</th>
<th>$\Pi_{S1}$</th>
<th>$\Pi_{S2}$</th>
<th>$\Pi_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case D: $c_1 = 15$, $c_2 = 10$ Medium production cost Unif (100, 500)</td>
<td>18.3</td>
<td>16.9</td>
<td>125.0</td>
<td>205.9</td>
<td>412.5</td>
<td>1324.0</td>
<td>5253.5</td>
</tr>
<tr>
<td>Case E: $c_1 = 20$, $c_2 = 10$ High production cost Unif (100, 500)</td>
<td>22.2</td>
<td>20.1</td>
<td>68.5</td>
<td>230.4</td>
<td>150.7</td>
<td>2112.4</td>
<td>4233.6</td>
</tr>
<tr>
<td>Case F: $c_1 = 20$, $c_2 = 10$ High cost and low variability Unif (150, 450)</td>
<td>21.7</td>
<td>20.3</td>
<td>54.6</td>
<td>242.9</td>
<td>92.8</td>
<td>2353.7</td>
<td>4290.2</td>
</tr>
</tbody>
</table>

Note: 1: reliable supplier, 2: unreliable supplier. * denotes optional value of parameter.

Another important insight from Table 2 relates to the incentives for reducing uncertain capacity risk. Among cases A, B, and C, when reliability improves, the unreliable supplier can expect a larger order quantity from the retailer. At the same time, this reliability improvement reduces reliability heterogeneity, which leads to a more direct competition in price. Our computations confirm this reduced level of wholesale price. Thus, it is the retailer, and not the unreliable supplier, who is able to capture the benefit of reliability improvement effort as competition becomes fiercer and both suppliers’ profits decrease. On the other hand, we can explain the result from the reliable supplier’s perspective. The reliable supplier in this competitive SC may effectively deter its competitor’s motivation for reliability improvement by lowering price. This action may be seen as a barrier erected by the reliable supplier to hinder reliability improvement in a dual sourcing environment.

As shown in Theorem 2, the riskier supplier would never win any orders unless it maintains a lower unit cost than its competitor. However, the reliable supplier can still compete with a higher unit cost. Next, we relax the cost homogeneity assumption in the previous cases, starting from the low production cost case C with a random capacity of $\text{Uniform}(100, 500)$. Holding $c_2 = 10$ and other parameters constant, we consider unit cost of the reliable supplier $c_1$, as 15, and then 20 to obtain cases D and E. From case E, we change the capacity to $\text{Uniform}(150, 450)$ to obtain case F.

The impact of supplier cost on the optimal sourcing strategy is illustrated in Table 3. In general, the reliable supplier can still compete even when its production cost is significantly higher. When reliability heterogeneity remains the same among cases C, D, and E, the unreliable supplier’s order share increases significantly with $c_1$, as expected. This is because of the retailer’s diversification strategy of resorting more to the low-cost supplier when the reliable supplier’s cost (and wholesale price) increases.

Interestingly, in comparing cases E and F, the unreliable supplier 2 benefits from the capacity improvement, with profit increasing by $2353.7 - 2112.4 = 241.3$. This profit increase is contrary to the results in Table 2, where there was no production cost heterogeneity between the two suppliers. This
example illustrates that the unreliable supplier’s incentive to improve is much greater when the reliable supplier’s unit production cost is high. The greater benefit from supplier 2’s reliability improvement effort can be explained by the fact that supplier 1’s ability to deter such an initiative is hindered by its relatively high unit cost.

In summary, we demonstrate that depending on the cost heterogeneity between suppliers, the suppliers may not benefit from a reduction in capacity risk. It is the retailer that always benefits from price competition as it allocates order quantities between the suppliers, enjoying both order diversification and supplier competition benefits. Any effort by the suppliers to reduce their capacity variability also affects their heterogeneity in reliability. With decreasing reliability heterogeneity, the two suppliers become more substitutive, and more intense price competition follows. The supplier has to carefully weigh the trade-offs between receiving a larger order from the retailer versus increased price competition when reducing its capacity risk.

An important managerial insight from these results relates to the need for the monopolistic buyer to work with the unreliable supplier to design a payment scheme so that the unreliable supplier can also obtain some benefit from the supply variability reduction. Although the design of that coordination scheme is beyond the scope of our work, our findings illustrate the dramatic difference between single sourcing and dual sourcing: under dual sourcing there may be a misalignment in the supplier’s incentive for improvement, as the reduction in capacity risk may not be a win-win situation for both the supplier and the retailer.

Next, we illustrate the system behavior in the duopoly model for the relevant numerical instances that we have considered in the dual sourcing case.

**Duopoly Example**

**Example 2:** Consider the duopoly model with market demand for supplier $i$: $D_i = 500 - 20p_i + 10p_j$. For both suppliers, unit production cost $c = 10$. The unreliable supplier 2 has a stochastic capacity following the same uniform distributions as those in Table 2. In other words, we obtain cases A’, B’, and C’ here with the same random capacities as in cases A, B, and C. We maintain the base case where both suppliers are reliable.

Table 4 shows the duopoly model’s equilibrium results for the same instances that were presented in Table 2 for dual sourcing. In the duopoly model, we first note that for the base case (both suppliers are reliable with sufficient capacity and zero variability), both suppliers make positive expected profit, unlike in the dual sourcing case where they made zero profit. Further, the system’s total profit of 7,680 is smaller than the corresponding profit of 8,000 in the dual sourcing model. The result in Lemma 5 can be verified by noting that the low and high variability cases in Table 4 correspond to $K^{lv}$ and $K^{hv}$, respectively. Consistent with Theorem 5, we see that supplier 1’s production quantity and both retail prices increase with $\text{Var}(K)$. Consistent with Proposition 2, supplier 2 always profits from its supply capacity improvement.

This result is contrary to our findings on competition versus diversification in dual sourcing, where both suppliers sell through a monopolistic retailer. Through
Table 4: Equilibrium solutions for duopoly under homogenous unit costs \((c = 10)\)

<table>
<thead>
<tr>
<th>Equilibrium Solutions</th>
<th>(q_1^*)</th>
<th>(q_2^*)</th>
<th>(p_1^*)</th>
<th>(p_2^*)</th>
<th>(\Pi_{11})</th>
<th>(\Pi_{12})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>240</td>
<td>240</td>
<td>26</td>
<td>26</td>
<td>3840</td>
<td>3840</td>
</tr>
<tr>
<td>Case A': Low variability</td>
<td>243.53</td>
<td>239.12</td>
<td>26.24</td>
<td>26.82</td>
<td>3953.78</td>
<td>3391.57</td>
</tr>
<tr>
<td>Unif (150, 450)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case B': Medium variability</td>
<td>244.93</td>
<td>238.77</td>
<td>26.33</td>
<td>27.15</td>
<td>3999.40</td>
<td>3214.48</td>
</tr>
<tr>
<td>Unif (125, 475)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case C': High variability</td>
<td>246.39</td>
<td>238.40</td>
<td>26.43</td>
<td>27.49</td>
<td>4047.04</td>
<td>3031.15</td>
</tr>
<tr>
<td>Unif (100, 500)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note 1:* reliable supplier, 2: unreliable supplier. * denotes optional value of parameter.

dual-sourcing results in Table 2 of Example 1, we have shown that with a monopolistic retailer, it may be the retailer and not the unreliable supplier that benefits from a decrease in capacity risk. This may be especially true when the unreliable supplier’s unit production cost is not significantly lower than the reliable supplier’s. The different results from the dual-sourcing and duopoly models confirm that the retailer diversification strategy in the decentralized chain can significantly shape how random capacity affects supplier competition under capacity risks.

CONCLUSIONS AND EXTENSIONS
Managing SCs in today’s competitive economy is increasingly challenging because of greater uncertainty and complexity (Lee, 2002). Thus, the reliability of a product or service provider plays an important role in procurement and contracting decisions. In our study of random capacity risk in decentralized SCs with competing suppliers, we obtain the following main result: Under specified supply capacity risk, if a single supplier sells its product indirectly to the market through a retailer, or if two suppliers sell their identical products directly to the market without the retailer, then capacity risk reduction translates into increased profits for the supplier that reduces its risk (as might be expected intuitively). However, if two suppliers sell their identical products indirectly to the market through a retailer, then capacity risk reduction does not necessarily translate into increased profits for the supplier that reduces its risk. We also provide the reasoning and economic insights for such differences.

This research is among the first to demonstrate optimal quantities and wholesale prices in both dual sourcing and duopoly problems with supplier competition under random delivery capacity. After identifying equilibrium solutions for each SC structure, we illustrate optimal strategies for the supplier and retailer. In single sourcing, we confirm the alignment of incentives for reliability improvement even in the decentralized environment. In dual sourcing with supplier competition, we consider uniformly distributed random capacity to illustrate that the benefit from reliability improvement is also dependent on the suppliers’ cost heterogeneity. We then investigate the impact of random capacity in a supplier duopoly where both firms compete by selling to the market directly without the monopolistic retailer. We prove that the unreliable supplier always benefits from reliability improvement.
in the duopoly model when such an improvement reduces capacity variability. This finding drastically contradicts our conclusion in dual sourcing, revealing that retailer diversification changes the supplier’s incentive for capacity risk mitigation.

One possible extension of this work is to consider the case where two suppliers’ random capacities are correlated, as is the case when events such as inclement weather, unionized labor strikes, and infrastructure disruptions affect both suppliers’ capacity (Babich et al., 2007). Another possible extension is to consider a random yield model of supply uncertainty. This would require stipulation of how a supplier’s production quantity is related to a retailer’s order quantity. Another extension would be to investigate alternate demand functions, such as an exponential multiplicative demand form (Petruzzi & Dada, 1999). Other extensions include the incorporation of models for the cost of reliability improvement and specific coordination mechanisms, along with multi-period models including inventory and information updates. An empirical study of wholesale pricing under dual sourcing and duopoly also could be conducted.

SUPPORTING INFORMATION

Additional supporting information may be found in the online version of this article at the publisher’s website:

APPENDIX S: PROOFS

REFERENCES


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