Abstract: To improve downlink packet throughput, the base schedules a mobile when its signal to interference ratio is higher than on average. The mobiles measure downlink pilots from the serving and interfering basestations, calculating the SIR, and feeding back their achievable downlink rates. We examine the prediction of the signal to interference ratio by the mobile. Both signal and interference are modeled as auto-regressive (AR) stochastic processes. We derive the density function of the conditional SIR, in order for the mobile to determine the backoff margin required for a percentage of packets to be decoded successfully.

I. Introduction

Downlink wireless packet systems use channel knowledge to adapt transmission rate on packet by packet basis as the channel fluctuates. A key part of the downlink access scheme is the scheduler, which uses this knowledge to transmit to mobiles when their channel capacity is high, and defers when it is not. The channel is estimated at the mobile from downlink pilots and fed back regularly on the uplink. By the time the information is ready to be utilized at the base station, several milliseconds may have passed since the measurement was taken. This delay is the sum of the measurement interval, uplink MAC, transmission, and internal processing delays. Although we are considering a frequency duplexed system, it may be noted that even in a time duplexed system, there will be a delay between when the uplink pilots are transmitted, and when downlink transmission begins. Depending on the speed of the mobile, the delay, and the channel's spatial/temporal properties any information will be out-of-date or stale to varying degree. Two issues arise: first the scheduler makes 'wrong' decisions, and second the transmission rate must be backed off to ensure that this rate does not exceed the actual channel capacity. The effect of this delay and the margin required (w/o prediction) was studied in [1] by Avidor. Channel prediction has been proposed, to support mobile velocities [2-4]. If a mobile could feed back an accurate prediction of the channel, the gains by the described feedback system could be maintained. For example, in CDMA 2000 EV-DO there is 2.5 slot, or 4.1 ms delay from the measurement to the downlink slot. Using the rule of thumb $f_d=1/(20*\text{delay})$, prediction may be beneficial when the mobile velocity exceeds 4.2 mph. Yet this gain is only partial, as [5] compares performance at 2 mph to 13 mph showing a loss of 67% without HARQ but only 33% loss with HARQ/IR. It may be noted that with on-time channel feedback or prediction, gains are not just maintained, but system throughput increases with mobile velocity. Besides adding to the delay, another reason not to depend only on HARQ/IR is that multiple decodes a single packet is expensive in terms of processing and battery power.

The channel prediction itself will have error, but when the error and associated backoff margin is small enough, prediction will result in overall improvement. The mean prediction error may be easily estimated, since the prediction and the actual value of the channel will both be available, after a short delay. We desire a backoff margin such that most of the packets can be decoded successfully on the first attempt. This requires an estimate of the true density of the SIR, given the best estimate, which is the prediction. That is, given a prior predicted SIR, determine the posterior probability. An alternative approach taken in [6] is to model the effect of prediction error on uncoded BER. Section II discusses the system model. Section III reviews the AR model highlighting some important relationships. Section IV begins by applying Bayes rule to determine the desired posterior probability.

II. System Model

In cellular networks signals from basestations interfere with each other on downlink. In such systems with reuse of one, typically more than 40 % of the cell area faces an signal to interference ratio of 0 dB or lower. In general there are many interferers and their average powers are random. Suppose that there is a dominant interferer. This leads to SIR that is equally "depends on" the signal fading as well as the fading of the interference, as demonstrated in Figure 1. An observation is that there are large up-fades, and ignoring them can result in significant loss in rate. For the situation in Figure 1 it is clear both the signal and the interference powers must be predicted. This is the model taken here – the prediction of the signal and a single dominate co-channel interferer. Unless the feedback channel is somehow “inexpensive”, the prediction may be performed more conveniently at the mobile rather than the basestation. It is assumed that the interfering basestation continuously transmits at full power. This is justified by the fact that the multi-user wireless channel capacity is orders of magnitude less than a single wire, and is thus likely to be on all the time even with few users. Another approach taken in [7], is to postulate high capacity so that the transmit power will be bursty just as the data is.

The narrowband received signal and interference at the mobile may be given as

$$r(t) = h_s(t)D_s(t) + \sum_{m=1}^{M} h_m(t)D_m(t) + n_o(t) \quad (1)$$
where \( h(t) \) is the complex channel gain, \( D(t) \) is the modulated data symbol with unit variance, and \( n(t) \) is thermal noise. As mentioned previously the first dominate interferer is predicted. The thermal noise and the interference from the remaining M-1 basestations is modeled as \( \text{CN}(0, \sigma^2_n) \), where \( \sigma^2_n \) equals the sum of the individual variances, and is assumed to be known. The path gain, which is a factorized in \( h(t) \), is often modeled in two parts, an average distance dependent gain, and random shadowfading \( G = G_{\text{avg}}(d)S_f \) where \( S_f \) is lognormal. Both \( S_f \) and \( G_{\text{avg}} \) are assumed to be varying over a long time scale, compared to the fast fading, so that it is reasonable to assume that they are fixed. The fast fading, which occurs on the scale of wavelengths, may be described as a stationary complex Gaussian process.

**III. Quadratic Predictor**

A prediction of the fading signal or interference may be generated from previous samples of the respective process so

\[
\hat{h}(t) = \sum_{i=1}^{F_c} h(t - i - R + 1)w(i) = c^H w
\]

(2)

describes a \( F_c \)-th order filter predicting \( R \) steps ahead, where \( w \) is a vector of coefficients \( [w_1, w_2, ..., w_{F_c}]^T \) and \( c \) is a vector of previous samples \( [h(t - R), h(t - R - 1), ..., h(t - R - F_c + 1)]^T \).

Multiplying both sides of the equation by \( h(t - \tau) \) gives the Yule-Walker equations. These are written as:

\[
\begin{bmatrix}
  r_{xx}(0) & r_{xx}(1) & \cdots & r_{xx}(F_c-1) & w_1 \\
  r_{xx}^*(1) & r_{xx}(0) & \cdots & r_{xx}(F_c-2) & w_2 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  r_{xx}^*(F_c-1) & r_{xx}^*(F_c-2) & \cdots & r_{xx}(0) & w_{F_c}
\end{bmatrix}
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_{F_c}
\end{bmatrix}
= \begin{bmatrix}
  r_{xx}^*(R) \\
  r_{xx}^*(R+1) \\
  \vdots \\
  r_{xx}^*(R+F_c)
\end{bmatrix}
\]

(3)

The solution \( w \) is \( w = R^{-1}b \). Since the autocorrelation is not known it is replaced by its sample estimate \( \hat{r}_{xx}(\tau) \). The variance of the prediction error of the complex channel, i.e. the mean square error (MSE), using the optimal coefficients is \( \sigma^2_e = \sigma^2_h - w^H R w \). We are interested in the obtaining the signal powers \( |h(t)|^2 \) to feed back to the basestation. A straightforward predictor uses the square of (2).

\[
\hat{P} = |\hat{h}(t)|^2 = |c^H w|^2 \text{Watts.} \tag{4}
\]

Thus the predictor is a quadratic function of previous channel samples. As the prediction error increases, \( h(t) \) tends towards the mean which is zero, and the correspondingly the power also drops to zero. Obviously, we desire the average power when the error is large. To obtain the unbiased predictor first, the expected value of the error is

\[
E\{e(t)\} = E\{|\hat{g}(t)|^2 - |g(t)|^2\} = -\sigma^2_h + w^H R w \tag{5}
\]

The variance of \( \hat{P} \), with the optimal coefficients is

\[
\sigma^2_e = E\{|\hat{h}(t)|^2\} = w^H E\{c c^H\}w = w^H R w \tag{6}
\]

Let \( \sigma^2_{\text{bias}} = -\sigma^2_h + \sigma^2_e \). Thus we obtain the unbiased predictor in [8],

\[
\hat{P}_u = |c^H w|^2 - \sigma^2_{\text{bias}}. \tag{7}
\]

The performance of the predictor for synthetic channels as well as for measured channels taken in rural area, is given in [9].

**IV. Determining the Backoff Factor**

Mathematically the objective is to determine a backoff factor \( \theta \) such that

\[
P_s \{ \theta \delta_{\text{signal}} > y \} < \Omega \tag{8}
\]

where \( \Omega \) is the outage probability, \( y \) is the true signal to interference ratio (SIR), and \( \delta_{\text{signal}} \) is the predicted SIR. When \( y \) is less than the fed back \( \hat{y} \) this is called an outage. It is assumed that actual transmission rate is some function of \( y \), such as \( \log_2(1+y) \). In order to determine \( \theta \) we need to obtain the conditional distribution of \( y \) given \( \hat{y} \). Towards this end, we are interested in the joint density of \( P_s \) the signal power and \( P_i \) the interference power, given their biased estimates \( \hat{P}_s \) and \( \hat{P}_i \) based on (4). By Bayes rule,

\[
f(p_s, p_i \mid \hat{p}_s, \hat{p}_i) = \frac{f(p_s, p_i) f(\hat{p}_s, \hat{p}_i)}{f(\hat{p}_s)} \tag{9}
\]

and using the fact that \( P_s \) and \( P_i \) are uncorrelated and independent random processes. The joint density of the true signal and the prediction, for the case of the linear predictor [10, pg. 349] is a bivariate normal. Consider the correlation of the envelopes of two CN processes, which leads to bivariate Rayleigh. Note, a development of this interesting distribution related to the field correlation is given in [11, pp. 90-92]. The joint PDF for the biased power predictor may be obtained by variable substitution, using the notation in [12, pg. 171, eq 7.42].
The density of \( y \), may be obtained in terms of the PDF of the two random variables. By Papoulis [14, pg. 138] and using again the fact that \( P_s \) and \( P_i \) are independent,
\[
f(Y = y \mid p_s, p_i) = \int_0^\infty p f(yp \mid p_s) f(p \mid p_i) dp \tag{11}
\]
where
\[
f(p_s \mid p_s) = \frac{f(p_s, p_i)}{f(p_i)} \quad \text{and} \quad f(p_i \mid p_i) = f(p_i) . \tag{12}
\]
Using the fact that \( r_{hh} = \sigma^2_h = \sigma^2_s - \sigma^2_e \) when the optimal coefficients are used yields the conditional distribution [12, pg. 172, eq. 7.46]
\[
f(p \mid \hat{p}) = \frac{1}{\sigma_e^2} \exp \left( - \frac{p + \hat{p}}{\sigma_e^2} \right) I_0 \left( \frac{2\sqrt{pp^*}}{\sigma_e^2} \right) . \tag{13}
\]
Eq. (13) with a change of variable may be recognized as a Rice distribution. \( \sigma^2_n \) may be added to \( p_i \) to account for the remaining noise and interference. Using (13) and substituting into (11) gives
\[
f(Y = y \mid p_s, p_i) = \int_0^\infty dp \cdot \frac{p}{\sigma_s^2 \sigma_e^2 \sigma_i^2} \exp \left( - \frac{yp + p_s}{\sigma_s^2 \sigma_i^2} \right) \left( \frac{p + p_i}{\sigma_e^2} \right) I_0 \left( \frac{2\sqrt{pp^*}}{\sigma_e^2} \right) . \tag{14}
\]
Rearranging terms
\[
f(p \mid \hat{p}) = \frac{1}{\sigma_e^2} \exp \left( - \frac{p + \hat{p}}{\sigma_e^2} \right) I_0 \left( \frac{2\sqrt{pp^*}}{\sigma_e^2} \right) . \tag{15}
\]
The above integral may be found in closed form by starting with [13, Pg. 107, 33.401], reprinted below:
\[
\int_0^\infty dx \cdot e^{-mx} I_0 \left( (a - b)\sqrt{2x} \right) I_0 \left( (a + b)\sqrt{2x} \right) = \frac{1}{m} \exp \left( \frac{a^2 + b^2}{m} \right) I_0 \left( \frac{a^2 - b^2}{m} \right) \tag{16}
\]
where \( I_0 \) is the zero-th order modified Bessel function. Interestingly the arguments in the exponential are positive. Taking the derivative with respect to \( m \), on both sides yields,
\[
\int_0^\infty dx \cdot xe^{-mx} I_0 \left( (a - b)\sqrt{2x} \right) I_0 \left( (a + b)\sqrt{2x} \right) = \exp \left( \frac{a^2 + b^2}{m} \right) \left( \frac{m + a^2 + b^2}{m^3} \right) I_0 \left( \frac{a^2 - b^2}{m} \right) \tag{17}
\]
Setting \( a \) and \( b \) as
\[
a = \frac{\sigma_s^2 \sqrt{yp_s} + \sigma_e^2 \sqrt{p_i}}{\sqrt{2\sigma_s^2 \sigma_e^2}} \tag{18}
\]
\[
b = \frac{\sigma_s^2 \sqrt{p_i} - \sigma_e^2 \sqrt{yp_s}}{\sqrt{2\sigma_s^2 \sigma_e^2}} \tag{19}
\]
Setting \( m \) as
\[
m = \left( \frac{y\sigma_s^2 + \sigma_e^2}{\sigma_s^2 \sigma_e^2} \right) .
\]
Finally the PDF of the conditional SIR is
\[
f(Y = y \mid p_S, p_I) = \frac{1}{(y\sigma^2_{\epsilon} + \sigma^2_{\epsilon})^3} \cdot \frac{1}{(y\sigma^2_{\epsilon} + \sigma^2_{\epsilon})} \left(\frac{v\sigma^2_{\epsilon}}{\sigma^2_{\epsilon}} + \frac{p\sigma^2_{\epsilon}}{\sigma^2_{\epsilon}}\right) \cdot \left(\frac{v\sigma^2_{\epsilon} + \sigma^2_{\epsilon}}{\sigma^2_{\epsilon}} + \frac{p\sigma^2_{\epsilon}}{\sigma^2_{\epsilon}}\right) \cdot \left(\frac{v\sigma^2_{\epsilon} + \sigma^2_{\epsilon}}{\sigma^2_{\epsilon}} + \frac{p\sigma^2_{\epsilon}}{\sigma^2_{\epsilon}}\right) + \left(\frac{(y\sigma^2_{\epsilon} + \sigma^2_{\epsilon})^2}{\sigma^2_{\epsilon}} + \frac{v\sigma^2_{\epsilon}}{\sigma^2_{\epsilon}} + \frac{p\sigma^2_{\epsilon}}{\sigma^2_{\epsilon}}\right) \cdot \left(\frac{2\sqrt{yp_{\epsilon}p_I}}{(y\sigma^2_{\epsilon} + \sigma^2_{\epsilon})}\right)
\]

As a numerical example, let \( P_S = 2, \ P_I = 1, \ \sigma^2_{\epsilon} = \sigma^2_{\epsilon} = 1, \ \sigma^2_h = 1 \), and show \( f(Y \mid p_S, p_I) \) in Figure 2. Not surprisingly, this distribution is skewed positively, just as the conditional PDF.

Using numerical integration and nonlinear optimization to obtain the CDF and it’s inverse, the backoff may be obtained. These are given in Figure 3 as a function of the predicted error, power, outage, and error fixed. At low signal powers, the signal PDF is similar to an exponential, skewed right, while at higher signal powers it is symmetric like the Laplacian. This shift in the shape of the signal PDF results in need for additional backoff in Figure 4.

Another interpretation of the pdf in (20) is that of the density of the ratio of the powers of two Rician faded signals (13) or the ratio of two noncentral chi-square variates each with 2 degrees of freedom \( (\sigma^2_{\epsilon}/2) \cdot \frac{\chi^2(2, 2\hat{p}/\sigma^2_{\epsilon})}{2} \cdot \frac{2\sqrt{yp_{\epsilon}p_I}}{(y\sigma^2_{\epsilon} + \sigma^2_{\epsilon})} \). The ratio two such variables is a doubly noncentral F distribution. The F distribution is may be expressed by two summations making numerical evaluation difficult.

**Conclusions**

To mitigate the feedback delay in downlink packet access systems, the fading signal and interference were modeled as auto-regressive stochastic processes and predicted. We derived an expression for the desired posterior SIR, an identity to the doubly noncentral F with two degrees of freedom. The conditional distribution is a function of the mean square errors and the predicted values themselves. At the mobile, the backoff margin for any packet outage may be obtained using the inverse CDF. The backoff was found to be a strong function of the prediction errors and desired outage, and a weakly related to the predicted SIR.

**Acknowledgements**

We would like to thank Jerry Foschini for pointing out reference [13], and thank Dmitry Chizhik, Dragan Samardzija, Venkat Sivarama, and Didem Tureli for helpful discussions.

**References**

Figure 1. SIR fading when there is one interferer, w/pwr equal to the signal.
Fd=67 Hz, Average SIR=0dB.

Figure 2. Plot of $f(y \mid p_s = 2, p_I = 1)$, with SIR in linear units.
Figure 3. Backoff factors for $P_s = 2$, $P_i = 1$.

Figure 4. Backoff factor $\theta$ vs. SIR, $P_i = 1, \Omega = .1$.