Learning by Problem-Posing as Sentence-Integration and Experimental Use

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Abstract. In this paper, “problem-posing as sentence-integration” is proposed as a new design of learning using computer-based method. We also introduce an interactive learning environment for the problem-posing and report experimental use of the environment in elementary schools. We have already developed a computer-based learning environment for problem-posing as concept combination. However, we could understand that it was difficult for students of the lower grades. Problem-posing by sentence-integration is a framework to realize learning by problem-posing in the lower grades. Sentence-integration is a simple method than problem-posing by concept combination, but it is expected to keep the learning effect to refine problem schema. The learning environment was used by 132 second grade students of two elementary schools to examine whether the lower grade students can pose problems in the environment or not. The effect of refinement of problem schema by the problem-posing was also analyzed by pre-test and post-test with excessive information problems. The students and teachers who were observers of the experiment accepted the learning environment as a useful learning tool to realize learning by problem-posing through the questionnaires and additional comments. Although the results suggest the effect of refinement of problem schema by the problem-posing, it was only compared against a no instruction control condition. Therefore, as a future work, it is necessary to compare the learning against other instructions to examine its characteristics.

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Introduction

Learning by problem-posing is well recognized as an important way to learn mathematics [1-4]. However, despite the importance of problem posing, it is not popular as a learning method in reality. This is due to various factors. First, learners can make various kinds of problems, but some may be wrong and also some of the learners might repeatedly make similar problems, or make problems that are too simple to be useful for learning. In such cases, adequate feedback for each problem is required. However, because learners can make a large range of problems, it is difficult to prepare adequate feedback for every problem that learners might make. In problem posing, assessment of each posed problem and assistance based on the assessment is necessary. Because the above task puts a heavy burden on teachers, it is very difficult for them to use problem posing as a learning method. From this point of view, if a diagnosis function of problems posed by learners is realized, learning by problem posing will come to be used more.

Based on this consideration, we are investigating computer-based learning environment for interactive problem-posing that is composed of (1) problem-posing interface, (2) problem diagnosis function, and (3) help function for correcting or
completing the posed problems. We have already developed several learning environments for interactive problem-posing in arithmetical word problems [5, 6] and confirmed that exercise of problem-posing with the learning environment is effective to improve both problem-solving and problem-categorization abilities [7]. These results suggest that computer-supported interactive problem-posing is a promising approach to realize the learning by problem-posing.

One of the most important issues of these researches, however, is that the students who could pose problems with this learning environment are the fourth grade or above in elementary schools. Since learning of arithmetical word problems starts from lower grade, to realize learning by problem-posing in the lower grade is one of the most important issues. To realize such learning environment, then, it is necessary (1) to simplify the task of problem-posing, and (2) to keep the worth of problem-posing as a learning method.

Problem-posing in the previous environments is carried out as a combination of concepts. In the problem-posing, therefore, the process of making simple sentences by combining concepts and the process of making a problem by combining simple sentences are intermingled. The former process is corresponded to "transformation process" and the latter to "integration process" in a well-known process model of problem-solving of arithmetical word problems [9]. A simple sentence is interpreted linguistically in the transformation process, and the linguistic interpretations are integrated mathematically in the integration process. From the viewpoint of learning of mathematics, the integration process is far more important than the transformation process. Based on this consideration, we have proposed "problem-posing as sentence integration."

In the problem-posing as sentence integration, several simple sentences are provided to a learner. The learner, then, selects necessary sentences and arranges them in an appropriate order. Although the process to integrate simple sentences remains as the characteristics of problem-posing, the process to make simple sentences is simplified to the process to understand them. Therefore, the "problem-posing as sentence integration" is a promising approach to satisfy both (1) simplification of problem-posing task for lower grade students in the elementary schools, and (2) keeping the worth of problem-posing as a learning method. In other words, this is a problem-posing focused on sentence-integration process.

In this paper, in Section 1, learning by problem-posing is discussed in detail. In Section 2, a learning environment for problem-posing as sentence-integration is described. An experimental use of the learning environment in two elementary schools is also reported. Although the results suggest the effect of refinement of problem scheme by the problem-posing, it was only compared against a no instruction control condition. Therefore, as a future work, it is necessary to compare the learning against other instructions to examine its natures.

1. Learning by Problem-Posing

In our research of "Learning by Problem-Posing", the "problem" is defined as follows.

\[ \text{Problem} = \text{"Given-Information" + "Required-Information"} \]

"Problem-Solving", is an activity to derive required-information from the given-information and then the problem solving should be able to complete deductively.

"Solution Method" is, then, the method that is used in the problem solving. In the case of
an arithmetical word problem composed of the following three sentences {I) *Tom had five pencils.* (II) *Ken received three pencils from Tom.* (III) *How many pencils does Tom have?*}, Sentence-I and –II are the given-information and Sentence-III is the required information. The solution method, then, is “5-3”. These definitions cover most of the problems used in problem exercises in arithmetic, mathematics, physics and so on. Based on this definition, problem-posing can be completed by adequately combining the following three elements: (1) given-information, (2) required-information, and (3) solution method. These elements, then, are used as constraints that a student has to satisfy in problem-posing. Problem-posing that dealt with in this paper is “solution-based problem-posing”. In solution-based problem-posing, a learner is required to make a problem and the problem should be composed of given-information and required-information that can be solved by a specified solution method. To realize this exercise, it is important to provide the way to let a learner to make the composition of given-information and required-information.

In the learning of arithmetical word problem, it is assumed that a learner has already had the ability to understand the meaning of each sentence in a problem and it is expected that the learner acquires the ability to understand the problem mathematically. In learning from problem-posing, therefore, it isn't indispensable to pose a problem by using natural language written by a learner. Based on these considerations, we have proposed three types of problem-posing methods as follows: (A) sentence template method [5], (B) problem template method [6, 7], and (C) sentence card method [8]. In this subsection, because of page limitation, the outline of only the sentence card method has been explained.

In the sentence card method, a learner is provided with several cards with a sentence in each card. The learner, then, is required to select some of them and to sort it out in a proper order. Although the process to make a sentence by combining concepts in other methods is substituted for the process to understand the sentences on the cards, the process to integrate simple sentences is same with the other problem posing methods. Therefore, the template card method is an approach to realize (1) to keep the worth of problem-posing as a learning method and (2) to simplify the task of problem-posing.

A typical process model of problem-solving of arithmetical word problems [9] is shown in Figure 1. The processes of *transformation* and *integration* are characteristics process of word problem. In the *transformation* process, natural language is interpreted linguistically. A linguistic interpretation of a natural language sentence is corresponds to “*sentence*” in Figure 1. In the *integration* process, they are integrated mathematically. The structure made by the *integration* process is often called “problem structure” and it is describes the understanding of the problem. Problem schema is mainly used in this process. From the viewpoint of learning of mathematics, the *integration* process is far more important than
the transformation process in the model. In the problem-posing by the sentence template method and the problem template method, transformation and integration are intermingled. Because the problem-posing with sentence card method is the problem-posing focused on the integration process, we call this type as "problem-posing as sentence integration". In the next section, a learning environment for interactive problem-posing as sentence integration, named MONSAKUN, is described.

2. Learning Environment for Problem-Posing: MONSAKUN

2.1 Interface for Problem-Posing

The interface of problem-posing in MONSAKUN is shown in Figure 2. The area in left side, imaged blackboard, is "problem-composition area". At the top of the area, a calculation expression is presented. A learner is required to pose a problem that is able to solve by the calculation expression. Here, the expression is the solution-method. Although the expression itself is easy, the learner has to consider the combination of not only a number but also a subject, object and predicate in each sentence. The three blanks in the area are the ones to put sentence cards. Sentence cards are presented at right side of the interface. A learner can move the card by drag&drop method freely in the interface. When a learner pushes "diagnosis button" under the problem-composition area, the system diagnoses the combination of sentences. The results of the diagnosis and message to help the learner's problem-posing is presented by another window.

In the case of Figure 2, the calculation expression is "5+3". To pose a problem that can be solved by this calculation, a learner has to put the cards of "Tom has five pencils", "Ken gave three pencils to Tom", and "How many pencils does Tom have?" into the blanks in this order. In this case, a learner sometimes selects "Ken received three pencils from Tom" instead of the second sentence. This is a typical error, and it is assumed that the cause is the association from "addition" as an operation to "receive" as a predicate.

![Figure 2. Problem-Posing Interface of MONSAKUN.](image-url)
2.2 Diagnosis of a Posed Problem

A problem posed by a learner is diagnosed by the following three viewpoints: (1) problem type, (2) relations of concepts, and (3) numerical relation. The problem type is diagnosed by the combination of sentence type. For example, in the change problem, the first sentence has to describe a number of something exists, and the second sentence has to describe a change of the number. By this diagnosis, the type of problem the learner tried to pose can be detected. If the type can not be detected, that is, the combination of sentences is wrong, the system gives advises that what kind of sentence should be put in each blank.

When the problem type is detected, the relations of the concepts in the sentences can be diagnosed. Depending on each problem type, the conditions that the relations have to satisfy are different. For example, in the combination problem, the numbers should belong to the concepts that are able to add mutually, like the number of apples and the number of oranges. In the change problems, the object of the change in the second sentence should be appeared in the first and the third sentence. By using such conditions, the relations of concepts are diagnosed. When the problem satisfies the conditions, a calculation expression can be made from the problem. If the problem doesn't satisfy a condition of the problem type, the system judge the problem is wrong. Then, the system gives advises that what kind of condition should be specified.

In the diagnosis of numerical relation, a calculation expression is derived from the problem and then compared with the one that is the target of the problem posing. When the two expressions are the same, the posed problem is correct. If the derived expression is different only in the number, the system indicates the difference. When the expression is different in the operation, the system indicates that the posed problem is solved by the difference operation. When the calculation expression derives a minus number, the system indicates that the calculation is impossible because minus number has not been taught in elementary school. Since this case is happened only in subtraction, the system indicates that "it is impossible to subtract this number from that number".

3. Experimental Use of MONSAKUN

After trial uses of the learning environment by teachers in elementary schools and getting prospects that it might be usable and useful in learning of arithmetic for the second grade students, the environment was practically used in arithmetic classes. The purposes of the practice were to examine that (1) the students can pose problems by using the system, and (2) effect of the learning by problem-posing. Whether the students could pose problems in the environment was analyzed by using the logs of problem posing and the results of questionnaires. The learning effect was analyzed by the change of an experimental group and no instruction control group between pre-test and post-test for the student’s scores of problem solving test of “extraneous problems” including an extraneous information that is not necessary to solve the word problems. A student who has sophisticated problem schema can perform well to solve them. This test, therefore, is useful to examine the learning effect for sophisticating problem schema by problem-posing as sentence integration. Comparison against other instruction condition is one of the most important future works.
3.1 Situation of the Experimental Use

The participants of our experiment are 132 students in the second grade of two elementary schools. They are divided into two groups: an experimental group and a control group. Because there are six classes originally, we assigned three classes to each group. The experimental group composed of 66 students was given a pre-test and a post-test before and after of the environment use. The control group composed of 66 students was given the pre-test and the post-test without the environment use.

In the experimental group, the pre-test was given at the beginning of the first class time of the first day. There was no task related to this use in the second day. In the third day, a series of two class times (One class = 45 minutes) were used to the use. Then, the post-test was given to them at the beginning of the first class of the fourth day. In control group, the pre-test was given at the beginning of the first class of the first day. There was no class related to problem-posing in the second and third day. Then, the post-test was given to them at the beginning of the first class of the fourth day. During this experiment, there was no holiday. Therefore, the difference between the experimental group and the control group is that whether they used the learning environment or not (that is, learning by the environment vs. no instruction). The control group also used the system for two class times after the post-test, in order to even up the learning opportunities following to the request of schools.

The experimental use was carried out in a computer room at each school, where every student was provided with one computer. In the experimental use, two staff supervised them and only answered them if there is any problem in the operations of the learning environment. To adjust the explanation time of the operations, time of answering the questionnaires, time of transfer from a normal classroom to the computer room and so on, the students used the systems for 50 minutes. The questionnaire was also given to the control group. In the analysis of the logs of system use and questionnaire, we don’t distinguish the control group and the experiment group.

3.2 Results of System Use

In this subsection, we analyze whether the students can pose problems by using the learning environment based on the logs of the system use and the results of the questionnaire. Table 1 shows the results of the use. The number of “diagnosis requests” is the number that a student pushes “diagnosis button”. This number can be regarded as the number of posed problems. When a problem that is required to diagnose is the same as the previously diagnosed problem, the request is not counted.

<table>
<thead>
<tr>
<th>Total Time</th>
<th>Students</th>
<th>Posed Problems</th>
<th>Correct Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 minutes</td>
<td>132</td>
<td>71 (in average)</td>
<td>50 (in average)</td>
</tr>
</tbody>
</table>

A student could pose 71 problems in average. In the posed problems, 50 problems were correct and 21 problems were incorrect in average. The student who uses the learning environment for the first time poses one correct problem in a minute. Therefore, the student can pose problems by using the environment. The number of incorrect problems suggests that the necessity of diagnosis function.
Table 2 shows the results of questionnaire. Most students enjoyed the problem-posing and interested to use MONSAKUN again. They also thought that the activity to pose problems was useful for their learning. The helps received from the system were also judged useful. Besides the teachers who were present at the class commented that most of the students kept their motivation to pose problems and their activities were valuable as arithmetic learning. These results suggest that MONSAKUN can be easily used by the second grade students and could be accepted as a useful learning tool by the both teachers and students.

Table 2. The Results of Questionnaires.

<table>
<thead>
<tr>
<th>Questions</th>
<th>Answers</th>
<th>Yes</th>
<th>No</th>
<th>No idea</th>
</tr>
</thead>
<tbody>
<tr>
<td>Did you enjoy the problem-posing?</td>
<td></td>
<td>131</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Do you like to use the system again?</td>
<td></td>
<td>129</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Do you think the problem-posing is important in learning?</td>
<td></td>
<td>120</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Were the helps from the system useful?</td>
<td></td>
<td>121</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Have you improved in posing problems?</td>
<td></td>
<td>123</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

3.3 Learning Effect

We used extraneous problems in both pre-test and post-test to access subject’s problem solving performance. The extraneous problem includes extraneous information that is not necessary to solve the word problem. Figure 3 is an example of the extraneous problem. It is more difficult for children to solve the extraneous problem than to solve the standard word problem [10]. To solve the extraneous problem, children have to judge the relevance of sentences and find the sentence including the extraneous information. In this process, problem schema plays a crucial role. Therefore, the extraneous problem is useful to access children’s problem schema.

Both pre-test and post-test are composed of 12 extraneous word problems. We made two test versions and these two tests were used with counterbalance. For scoring the each problem, a correct numerical expression was given as 1 and an incorrect numerical expression was given 0 for each problems. The magnitude of score for both tests is 0 - 12.

Table 3 shows the mean scores of the pre-test and post-test as a function of condition and group. The difference of the mean scores between experiment group and control group was high (0.88), and the score of pre-test and post-test were correlated (0.83). A one way analysis of covariance, 2 (condition: experiment vs. control, covariance: pre-test) was carried out for the score of post-test. The trimmed means of the post-test in each condition, as the covariance is pre-test are 7.72 in the case of control and 8.55 in the case of experiment. The results revealed that the main effect of condition was significant ($F_{(1,129)}=84.71, p<0.5$) for the scores of post-test. This result indicated that the problem solving performance of the experimental group was higher than the control group in the post-test and the children in the experimental group have obvious gains from pre-test to
post-test in the learning with MONSAKUN. In conclusion, MONSAKUN improves the children’s problem solving performance in the experimental group.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment Group (n=66)</td>
<td>6.56 (SD=4.43)</td>
<td>8.21 (SD=4.01)</td>
</tr>
<tr>
<td>Control Group (n=66)</td>
<td>7.44 (SD=4.28)</td>
<td>8.06 (SD=4.03)</td>
</tr>
</tbody>
</table>

4. Conclusion Remarks

We have already developed several computer-based interactive environments for learning by problem-posing. However, it is often difficult to pose word problems in the environment for lower grade students. In this paper, we described a learning environment for learning by problem-posing as sentence-integration, in order to keep the worth of problem-posing as a learning method and also to simplify the task of problem-posing. In the problem-posing, several simple sentences are provided to the students. The students, then, select necessary sentences and arrange them in an appropriate order. Although the process to integrate simple sentences remains as the characteristics of problem-posing, the process to make simple sentences is simplified to understand them. Through experimental use of the learning environment in arithmetic classes, we confirmed that teachers and students accepted it as a useful tool for learning. Besides, analysis of scores of the pre-test and post-test as a function of condition and group suggests that the learning environment improve the children’s problem solving performance in the experimental group. However, the result was derived by the comparison against a no instruction control condition. Therefore, as a future work, it is necessary to compare the learning against other instructions to examine its characteristics.

References