Adaptive Network Coding in Two–Way Relaying MIMO Systems

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Abstract—We investigate physical–layer network coding for two–way relaying systems which employ multiple–input multiple–output (MIMO) techniques. For single–antenna cases, it has been previously verified that adaptive network coding, whose mapping rule is dependent on channel state information (CSI), can offer good performance for two–stage relaying protocols. In this paper, we evaluate gains achieved by adaptive network coding over the conventional network coding in MIMO channels. We consider several different scenarios: any CSI is not available, only local CSI is available, and global CSI is available at transmitters. Optimal precoding is derived for each scenario to minimize pairwise error probability. We confirm that the adaptive network coding still outperforms the fixed network coding even for MIMO communications.

I. INTRODUCTION

In the past few years, multi–way relaying which exploits network coding [1] at the physical layer has received a significant amount of attention [2–7]. We consider a bidirectional relaying system, in which there are three nodes a, b and r. The terminal a has traffic to send data towards the terminal b and vice versa, while the relaying node r assists this two–way communication. There are various forwarding schemes in this scenario: 4–, 3–, and 2–stage protocols with amplify–and–forward (AF), decode–and–forward (DF), joint decode–and–forward (JDF), compress–and–forward (CF), estimate–and–forward (EF), demodulate–and–forward (DMF) and denoise–and–forward (DNF) schemes. In this paper, we focus on the 2–stage DNF, which was originally introduced in [3], and extend our previous works [10–13] to the case of multiple–input multiple–output (MIMO) systems.

The information–theoretic potential of DNF is analyzed in [6, 7]. The use of structured codes to enable efficient denoising has been considered in [8, 9]. Focusing on the communication–theoretic aspects of the finite–length packets and practical modulation schemes, the authors have optimized signalling constellations for DNF in [10, 11], and extended it for convolutionally–coded systems in [12] and for adaptive modulations in [13]. We have found that physical–layer network coding used at the relay r should be adaptively changed according to the channel state information (CSI), and that unconventional 5–ary network coding can significantly improve throughput. In [10–13], we have mainly considered single–antenna communications or multiple–antenna diversity receptions. It is rather natural to extend our previous works to multiple–antenna systems exploiting MIMO techniques. In this paper, we evaluate a performance advantage of adaptive network coding in two–way relaying MIMO systems where all the nodes a, b and r are equipped with multiple antennas.

For MIMO systems, precoding techniques, such as eigen–beamforming based on singular–value decomposition (SVD) of the channel matrix, play an important role to realize spectrally efficient communications. Such a transmitter precoding requires CSI in principle. Since all the nodes a, b and r in two–way relaying systems should operate in a distributed manner, it is often hard for all the nodes to acquire full CSIs. In this paper, we discuss adaptive network coding with precoding techniques for some different scenarios, more specifically, i) any CSI is not available at transmitters, ii) only local CSI is available, and iii) global CSI is available. For each scenario, we optimize network coding and precoding to minimize the pairwise error probability. Through performance evaluations, we demonstrate that adaptive network coding still improves throughput even for MIMO systems. For many antenna cases, we confirm that precoding techniques have more impact on throughput performance than optimized network coding if CSIs are available at transmitters.

Notations: We write matrices including vectors by bold italic characters. The notations C and Zn are the complex field and the non–negative integer ring of a modulo n. The Frobenius norm, the trace, the Hermitian transpose and the inverse of a matrix X are expressed by \( \|X\|, \text{tr}[X], X^\dagger \) and \( X^{-1} \), respectively. The vector operation \( \vec{[X]} \) generates a single column vector by stacking all columns of a matrix X in a left–to–right fashion. Given matrices X, Y and Z of a proper size, we can write \( \vec{(XYZ)} = (Z^\dagger \otimes X)\vec{[Y]} \), where \( \otimes \) denotes the Kronecker product. The matrix \( I_n \) denotes the n–dimensional identity matrix. The expectation function and the ceiling function are denoted by \( E[\cdot] \) and \( \lceil x \rceil \), respectively.

II. TWO–WAY RELAYING MIMO SYSTEMS

We focus on two–stage forwarding protocols for two–way relaying communications between the terminals a and b via the relay r, as illustrated in Fig. 1. The nodes a, b and r are equipped with \( N_a, N_b \) and \( N_r \) antennas, respectively. At the first step termed multiple–access (MA) stage, two terminal nodes a and b simultaneously transmit data towards the relay r, and the relay r broadcasts the data to both the terminals a and b at the second step termed broadcast (BC) stage. Due
to the half-duplex constraint, the terminals cannot receive any data at the MA stage.

A. Multiple–Access (MA) Stage

We let $s_a \in \mathbb{Z}_{Q_a}$ and $s_b \in \mathbb{Z}_{Q_b}$ be the source data to be exchanged during one symbol duration, where $Q_a$ and $Q_b$ denote the alphabet size of the digital data. The source data $s_a$ (and $s_b$) are split into $N_a$ (resp. $N_b$) parallel data, and individually modulated by $Q_a$-ary ($Q_b$-ary) phase shift keying (PSK) modulation signals for each transmitting antenna. For simplicity, we suppose that the cardinality of the modulation per antenna is identical to $Q'_a = \lceil Q_a^{1/N_a} \rceil$ ($Q'_b = \lceil Q_b^{1/N_b} \rceil$). Let $x_a = \mathbf{M}_a(s_a) \in \mathbb{C}^{N_a \times 1}$ and $x_b = \mathbf{M}_b(s_b) \in \mathbb{C}^{N_b \times 1}$ be the modulated signals at the nodes $a$ and $b$, respectively, where $\mathbf{M}_a(\cdot)$ and $\mathbf{M}_b(\cdot)$ are the spatial modulation functions.

Each node $k \in \{a, b\}$ uses a linear precoding matrix $P_k \in \mathbb{C}^{N_k \times N_a}$ before transmitting the signals in order to improve performance. At the MA stage, the relay $r$ then receives

$$y_r = H_{ra} P_a x_a + H_{rb} P_b x_b + z_r,$$

where $y_r \in \mathbb{C}^{N_r \times 1}$, $H_{ra} \in \mathbb{C}^{N_r \times N_a}$, $H_{rb} \in \mathbb{C}^{N_r \times N_b}$, and $z_r \in \mathbb{C}^{N_r \times 1}$ denote the received signal at the relay $r$, the channel matrix from the terminal $a$ to the relay $r$, the channel matrix from the terminal $b$ to the relay $r$, and the additive Gaussian noise at the relay $r$, respectively.

B. Physical–Layer Network Coding

The relay $r$ jointly estimates the data $s_a$ and $s_b$ through the use of the maximum–likelihood (ML) detection. The ML estimate is given as

$$\hat{s}_a, \hat{s}_b = \arg\min_{s'_a \in \mathbb{Z}_{Q_a}, s'_b \in \mathbb{Z}_{Q_b}} \| y_r - H_{ra} P_a \mathbf{M}_a(s'_a) - H_{rb} P_b \mathbf{M}_b(s'_b) \|^2 .$$

With the ML estimates $\hat{s}_a$ and $\hat{s}_b$, the relay $r$ generates the network–coded data $s_r \in \mathbb{Z}_{Q_r}$ as follows:

$$s_r = C_r(\hat{s}_a, \hat{s}_b),$$

with $C_r(\cdot)$ being the physical–layer network coding function. As discussed in [10, 11], the following condition is necessary for successful forwarding:

$$C_r(\hat{s}_a, \hat{s}_b) \neq C_r(\hat{s}_a, \hat{s}'_b), \quad \text{for } \hat{s}_a \in \mathbb{Z}_{Q_a}, \hat{s}_b \neq \hat{s}'_b \in \mathbb{Z}_{Q_b},$$

$$C_r(\hat{s}_a, \hat{s}_b) \neq C_r(\hat{s}'_a, \hat{s}_b), \quad \text{for } \hat{s}_a \neq \hat{s}'_a \in \mathbb{Z}_{Q_a}, \hat{s}_b \in \mathbb{Z}_{Q_b},$$

which we refer to as exclusive law of network coding. This condition implies that the cardinality of the network–coded data, $Q_r$, must fulfill

$$Q_r \geq \max(Q_a, Q_b).$$

The conventional network coding generally uses a fixed function based on the modulo addition as follows: $C_r(\hat{s}_a, \hat{s}_b) = \hat{s}_a + \hat{s}_b \mod \max(Q_a, Q_b)$, which can achieve the maximum compression gains of the cardinality.

In our previous works [10, 11], it has been verified that the network coding function $C_r(\cdot)$ should be adaptively determined according to CSIs ($H_{ra}$ and $H_{rb}$) for reliable forwarding. For the case of $Q_a = Q_b = 4$ (and single antenna, $N_a = N_b = 1$), we have discovered an interesting fact that the unconventional network coding, whose cardinality is $Q_r = 5$, can outperform the modulo–based 4-ary network codes for some specific channel conditions. The contribution of this paper includes an extension of the result to the MIMO systems, and an evaluation of adaptive network coding (whose coding rule is optimized by our proposed closest–neighbor clustering method [10, 11]).

C. Broadcast (BC) Stage

The network–coded data $s_r \in \mathbb{Z}_{Q_r}$ are split into $N_r$ parallel data, and individually modulated by $Q'_r$-ary PSKs, where $Q'_r$ is chosen to be $Q'_r = \lceil Q_r^{1/N_r} \rceil$. Letting $\mathbf{M}_r(\cdot)$ be the spatial modulation function at the relay $r$, the modulated signals are expressed as $x_r = \mathbf{M}_r(s_r) \in \mathbb{C}^{N_r \times 1}$. With a precoding matrix $P_r \in \mathbb{C}^{N_r \times N_r}$, the relay $r$ broadcasts the data towards both the terminals $a$ and $b$, each of which in turn receives

$$y_a = H_{ar} P_r x_r + z_a, \quad y_b = H_{br} P_r x_r + z_b,$$

during the BC stage. Here, $y_a \in \mathbb{C}^{N_r \times 1}$, $y_b \in \mathbb{C}^{N_r \times 1}$, $H_{ar} \in \mathbb{C}^{N_r \times N_a}$, $H_{br} \in \mathbb{C}^{N_r \times N_b}$, $z_a \in \mathbb{C}^{N_r \times 1}$, and $z_b \in \mathbb{C}^{N_r \times 1}$ are the received signal at the terminal $a$, the received signal at the terminal $b$, the channel matrix from the relay $r$ to the terminal $a$, the channel matrix from the relay $r$ to the terminal $b$, the noise at the terminal $a$ and the noise at the terminal $b$, respectively. Here, we do not make any assumption of channel reciprocity as it can be applied to frequency–division duplex as well as time–division duplex in practice.

At the terminal $a$, the ML detection is performed to estimate $s_b$ by using the own information $s_a$ as follows:

$$\hat{s}_b = \arg\min_{s'_b \in \mathbb{Z}_{Q_b}} || y_a - H_{ar} P_r \mathbf{M}_r(C_r(s_a, s'_b)) ||^2 .$$

In an analogous way, the terminal node $b$ obtains the ML estimate of $s_a$ by using the self data $s_b$. 

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D. Channel Assumptions

Here, we describe some assumptions of the signal models. Any one of elements in the additive noise vector $z_k$ at node $k \in \{a, b, r\}$ follows i.i.d. complex Gaussian distribution such that

$$
\mathbb{E}[z_k z_k^*] = \sigma^2 I_{N_k},
$$

(7)

where $\sigma^2$ denotes the noise variance. Each element in the modulated signals $x_k$ has unity energy on average, and its covariance becomes an identity matrix as follows:

$$
\mathbb{E}[x_k x_k^*] = I_{N_k}.
$$

(8)

The transmission power is controlled by precoder matrix $P_k$. Letting $E_k$ be the transmission energy per symbol at the node $k$, we have

$$
\mathbb{E}\left[\|P_k x_k\|^2\right] = \text{tr}\left[P_k P_k^*\right] = E_k.
$$

(9)

The channel matrix $H_{jk}$ ($j k \in \{ra, rb, ar, br\}$) is the Nakagami–Rice fading MIMO channels with Rician factor of $K_R$. More specifically, $\text{vec}(H_{jk})$ follows $N_j N_k$-dimensional complex Gaussian distribution with mean of $\mu_{jk}$ and covariance of $1/(1+K_R)I_{N_j N_k}$. Each entry in the mean vector $\mu_{jk}$ (corresponding to the stationary or the line-of-sight component) has a magnitude of $\sqrt{K_R}/(1+K_R)$ and its phase is uniformly distributed over $[0, 2\pi]$.

E. CSI Availability

We consider several different scenarios regarding the available CSI when transmitting:

- No–CSIT: All the transmitters do not have any CSI prior to transmissions.
- Local–CSIT: At the MA stage, the terminal node a knows CSI $H_{ra}$, and the terminal node b knows CSI $H_{rb}$. At the BC stage, the relay node $r$ has both CSIs $H_{ar}$ and $H_{br}$ before transmissions.
- Global–CSIT: At the MA stage, both the CSIs $H_{ra}$ and $H_{rb}$ are available at both terminals a and b. At the BC stage, the relay node $r$ has both CSIs $H_{ar}$ and $H_{br}$ before transmissions.
- Full–CSIT: All the CSIs ($H_{ra}$, $H_{rb}$, $H_{ar}$, and $H_{br}$) are available at all the terminals a, b, and r when transmitting.

Although no–CSIT is the most feasible scenario in practice, we cannot enjoy any beamforming gains because the most appropriate precoding matrix becomes the identity matrix. When CSIs are available at transmitters, we can use some good precoders such as eigen–beamforming and minimum mean–square error (MMSE) pre–filtering. Note that full–CSIT and global–CSIT are much harder to be realized in practice than local–CSIT. However, some applications including fixed wireless access (FWA) can adopt those scenarios with high reliability because of the slow channel fluctuation.

Likewise, we may be interested in the following scenarios for receiving:

- No–CSIR: All the receivers do not have any CSI when receiving the signals.
- Local–CSIR: At the MA stage, the relay $r$ knows $H_{ra}$ and $H_{rb}$ for receiving. At the BC stage, the terminal node a has only the local CSI $H_{ar}$, and $H_{br}$ is only available for the terminal node b.
- Global–CSIR: At the MA stage, the relay $r$ knows $H_{ra}$ and $H_{rb}$ for receiving. At the BC stage, both the CSIs $H_{ar}$ and $H_{br}$ are available at both the terminals a and b.
- Full–CSIR: All the CSIs ($H_{ra}$, $H_{rb}$, $H_{ar}$, and $H_{br}$) are available at all the terminals a, b, and r when receiving.

For the case of no–CSIR, we require some sophisticated non–coherent communication techniques like differential space–time constellations [14], unitary space–time modulations [15], and non–coherent Grassmann codes [16]. Such a scenario is beyond the scope of this paper because the adaptive network coding basically requires CSI at the relay $r$. Therefore, we leave it as a future work. The local–CSIR is the most general scenario for practical communications which employ coherent detections. The full–CSI requires more overhead to realize. It should be noted that some combinations, such as full–CSIT and local–CSIR, make no sense.

III. ADAPTIVE NETWORK CODING IN MIMO SYSTEMS

In this section, we extend our previous works on adaptive network coding [10–13] to MIMO systems. The key idea behind the adaptive network coding lies in the fact that the network coding function $C_{r}(\cdot)$ which is dependent on the CSIs ($H_{ra}$ and $H_{rb}$) can minimize the pairwise error probability at the MA stage.

A. Pairwise Error Probability

At the relay $r$, the pairwise error probability between the correct data $(s_a, s_b)$ and the wrong data $(s_a', s_b')$ is given by

$$
\Pr((s_a, s_b) \rightarrow (s_a', s_b')) = \frac{1}{2} \text{erfc}\left(\frac{d^2_{(s_a,s_b)}-(s_a',s_b')}{4\sigma^2}\right),
$$

(10)

where $d^2_{(s_a,s_b)}-(s_a',s_b')$ is the squared distance defined as

$$
d^2_{(s_a,s_b)}-(s_a',s_b') \triangleq \left\|H_{ra} P_r \delta_a(s_a, s_a') + H_{rb} P_b \delta_b(s_b, s_b')\right\|^2,
$$

(11)

with $\delta_k(s_k, s_k')(k \in \{a, b\})$ is the difference vector between the spatial constellations

$$
\delta_k(s_k, s_k') \triangleq \mathcal{M}_k(s_k) - \mathcal{M}_k(s_k').
$$

(12)

The most dominant pair to determine the error rate performance is the one which has the minimum squared distance over all the possible error pairs $(s_a, s_b)$ and $(s_a', s_b')$ such that $C_{r}(s_a, s_b) \neq C_{r}(s_a', s_b')$. 

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B. Zero–Distance Occurrence

As shown in (11), the squared distance is highly dependent on the channel parameters \(H_{ra}\) and \(H_{rb}\) as well as the precoders \(P_a\) and \(P_b\). It should be noted that some pairs are not uniquely distinguishable for specific channel conditions because the squared distance can become zero even though the channels are not faded. This is explained as follows: Letting \(H_t = [H_{ra}P_a, H_{rb}P_b] \in \mathbb{C}^{N_t \times (N_a + N_b)}\) be the effective compound channel matrix, the matrix \(H_t^H H_t\) can have zero in its eigenvalue even though \(H_t \neq 0\) for some channel realizations. If there exists a compound difference vector \(\delta(s_a,s_b) - (s'_a,s'_b)\), which is originally proposed by the authors for single–antenna systems, the squared distance for such pairs becomes zero because we have \(d^2(s_a,s_b) - (s'_a,s'_b) = \|H_t \delta(s_a,s_b) - (s'_a,s'_b)\|^2 = 0\). This effect is interpreted as a multiple–access interference (MAI).

C. Closest–Neighbor Clustering

It is of great importance for reliable relaying that we appropriately design the network coding function \(C_r\) to avoid the zero distance occurrence. Well–designed network coding should generate the same network–coded data as \(C_r(s_a,s_b) = C_r(s'_a,s'_b)\) for the dominant pairs \((s_a,s_b)\) and \((s'_a,s'_b)\) which have shorter distance. It can be done by clustering such dominant pairs through the closest–neighbor clustering method which is originally proposed by the authors for single–antenna systems in [10, 11]. In the clustering method, we successively search for the dominant pairs and cluster them as long as those pairs fulfill the exclusive law. Considering all the possible channel realizations and precoding, we can obtain the best network codebook by the clustering method as discussed in [10, 11].

IV. PRECODING MATRIX

A. Identity Precoding for No–CSIT

As we mentioned before, the most proper precoding for the case of no–CSIT is

\[ P_k = \sqrt{\frac{E_k}{N_k}} I_{N_k}, \]

for each node \(k \in \{a, b, r\}\). Note that this identity precoding has no beamforming gains.

B. Optimal Precoding for Global–CSIT

Here, we derive optimal precoding for the global–CSIT scenario. The optimal precoding which minimizes the upper bound of the pairwise error probability at the MA stage is given as follows:

\[
\max_{P_a P_b} t,
\text{s.t. } d^2(s_a,s_b) - (s'_a,s'_b) \geq t, \quad \forall C_r(s_a,s_b) \neq C_r(s'_a,s'_b),
\text{tr}[P_k P_k^H] = E_k, \quad \forall k \in \{a, b\},
\]

where \(s_a, s'_a \in \mathbb{Z}_{Q_a}\) and \(s_b, s'_b \in \mathbb{Z}_{Q_b}\). The Lagrangian is written as

\[ \mathcal{L} = t + \sum_{m} \nu_m (d_m^2 - t) - \sum_{k \in \{a,b\}} \mu_k (\|P_k\|^2 - E_k), \]

where \(\nu_m\) and \(\mu_k\) are Lagrange multipliers. Here, we use a value \(m \in \mathbb{Z}_{Q_a} \times \mathbb{Z}_{Q_b} \times \mathbb{Z}_{Q_a} \times \mathbb{Z}_{Q_b}\) to conveniently represent \((s_a, s_b) - (s'_a, s'_b)\). Taking the derivative with respect to \(P_a^H P_a\) and \(P_b^H P_b\) and zeroing can yield the Karush–Kuhn–Tucker (KKT) conditions necessary for the solution as follows:

\[ H_{ra}^H H_{ra} P_a Q_{aa} + H_{ra}^H H_{rb} P_b Q_{ba} = \mu_a P_a, \quad (15) \]
\[ H_{ra}^H H_{ra} P_a Q_{ab} + H_{rb}^H H_{rb} P_b Q_{bb} = \mu_b P_b, \quad (16) \]

where

\[ Q_{ij} \triangleq \sum_m \nu_m \delta_{i,j'} \delta_{j,j'}^T \in \mathbb{C}^{N_t \times N_t}, \]

for node indicators \(i, j \in \{a, b\}\). Using the vector operation, we can rewrite the KKT conditions as

\[ \begin{bmatrix} \Phi_{aa} & \Phi_{ab} \\ \Phi_{ba} & \Phi_{bb} \end{bmatrix} - \begin{bmatrix} \mu_a I_{N_a^2} & 0 \\ 0 & \mu_b I_{N_b^2} \end{bmatrix} \begin{bmatrix} \text{vec}(P_a) \\ \text{vec}(P_b) \end{bmatrix} = 0, \]

(18)

where \(\Phi_{ij} \triangleq Q_{ij}^T H_{ij}^\dagger H_{ij} \in \mathbb{C}^{N_t \times N_t}\). The derivative in terms of \(t\) yields \(\sum_m \nu_m = 1\).

After some manipulations, it turns out that the optimal precoder is obtained by a linear mixture of two eigenvectors associated with the maximum and second maximum eigenvalues of a matrix \(\Phi_{ij}\) as follows:

\[ \begin{bmatrix} \text{vec}(P_a) \\ \text{vec}(P_b) \end{bmatrix} = \mathbf{V}[2] \max \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \begin{bmatrix} \Phi_{aa} & \Phi_{ab} \\ \Phi_{ba} & \Phi_{bb} \end{bmatrix} \]

(19)

where \(\mathbf{V}[n]_{\max}(\cdot)\) generates an orthonormal matrix which contains the maximum \(n\) eigenvectors. The Lagrange multipliers \(\nu_m\) and the mixture factors \(\alpha_1\) and \(\alpha_2\) are chosen to satisfy the power constraint \((\alpha_1^2 + \alpha_2^2 = E_a + E_b)\) such that \(\|P_k\|^2 = E_k\) and \(\sum_m \nu_m = 1\).

C. Eigen–Beam Precoding for Local–CSIT

Here, we describe the eigen–beam precoding scheme. The channel matrix is decomposed by SVD as follows:

\[ H_{rk} = U_{rk} \Lambda_{rk} D_{rk}^H, \]

(20)

for \(k \in \{a, b\}\). Here, \(U_{rk} \in \mathbb{C}^{N_r \times N_t}\) is unitary, \(\Lambda_{rk} \in \mathbb{C}^{N_r \times N_k}\) is diagonal, and \(D_{rk} \in \mathbb{C}^{N_k \times N_k}\) is unitary.

The eigen–beam precoding employs the following precoder matrix:

\[ P_k = D_k V_k, \]

(21)

where \(V_k \in \mathbb{C}^{N_k \times N_k}\) is a diagonal matrix which is corresponding to the power assignments for each eigen–beam. Note that local–CSIT is required for implementing it.

We consider four different power allocations as follows:
1) **Water–Filling Allocation:** The conventional water–filling technique can maximize the conditional mutual information, \( I(y_k; x_a | x_b) \) and \( I(y_k; x_b | x_a) \), constrained on the power \( \text{tr}[P_k P_k^H] = E_k \). The power allocation is written as

\[
v^2_{k,n} = \left( \frac{\mu_k - \sigma^2}{\lambda_{k,n}} \right)_+, \tag{22}
\]

where \( v_{k,n} \) and \( \lambda_{k,n} \) are the \( n \)-th diagonal entries of matrices \( V_k \) and \( \Lambda_k \). The parameter \( \mu_k \) denotes the water level which is chosen to fulfill the power constraint.

2) **Equal Allocation:** In this paper, any capacity–approaching channel codes nor adaptive modulations are not considered. It implies that the water–filling power allocation may not be appropriate for improving throughput performance. We may use the following equal power allocation

\[
V_k = \sqrt{\frac{E_k}{N_k}} I_{N_k}. \tag{23}
\]

It suggests that the precoding matrix \( P_k = D_{rk} V_k \) becomes a simple unitary matrix, which we often call unitary precoding.

3) **Inverse Allocation:** For the inverse allocation, we put the power for each eigen–beams as follows:

\[
v^2_{k,n} = \frac{H_k}{\lambda_{k,n}^2}. \tag{24}
\]

The parameter \( \mu_k \) is chosen to be satisfied with the power limitation. The inverse allocation yields a unitary matrix for the effective channel gains, more specifically, \( H_{rk} P_k = U_{rk} \) for \( N_r = N_k \).

4) **Maximal–Ratio Allocation:** Using the concept of maximal–ratio combining for diversity, we may use the following power allocation:

\[
v^2_{k,n} = \mu_k \lambda_{k,n}^2. \tag{25}
\]

**D. Generalized MMSE Precoding for Local–CSIT**

The terminal node \( k \in \{a,b\} \) may use the generalized MMSE precoding whose precoder matrix is written as

\[
P_k = \frac{1}{\eta_k} \left( H_{rk}^H H_{rk} + \beta_k I_{N_k} \right)^{-1} H_{rk}^H, \tag{26}
\]

where \( \beta_k \) is a parameter controlling the mean–square error. When we set \( \beta_k = \sigma^2 \), it becomes the conventional MMSE precoder. Setting \( \beta_k = 0 \) yields the zero–forcing (ZF) precoder. The parameter \( \eta_k \) is the power normalization factor such that

\[
E_k \eta_k^2 = \text{tr} \left[ H_{rk}^H H_{rk} \left( H_{rk}^H H_{rk} + \beta_k I_{N_k} \right)^{-2} \right]. \tag{27}
\]

This precoding only requires local–CSIT for implementation.

**V. PERFORMANCE EVALUATION**

Now, we evaluate the end–to–end throughput performance to compare between the adaptive network coding optimized by the closest–neighbor clustering and the conventional network coding based on the modulo addition. We assume that the channel \( H_{jk} \) for \( j,k \in \{ra, rb, ar, br\} \) follows i.i.d. Nakagami–Rice distribution with a Rician factor of 10 dB. The length of transmission blocks is 128 symbols long. We use QPSK signals for modulation schemes per transmitting antenna at the terminal nodes a and b. The transmission power is identical to be \( E_k = 1 \) for all the nodes. The average SNR is defined as \( (\mathbb{E}[\|H_{ra}\|^2]/N_r N_a + \mathbb{E}[\|H_{rb}\|^2]/N_r N_b)/2\sigma^2 \). The end–to–end throughput is evaluated by packet success probability.

The adaptive network coding can improve the distance profile at MA stage by clustering replica points which has short Euclidean distance. In Fig. 2, we show the cumulative
QPSK MIMO systems are ties of the adaptive network coding for network coding as addressed in Fig. 2. The average cardinality of adaptive network coding can be decreased by precoding. It indicates that precoding technique has more impact on the performance improvement than adaptive network coding in MIMO systems.

VI. CONCLUSION

In this paper, we investigated physical–layer network coding for two–way relaying systems which employ MIMO techniques. We evaluated gains achieved by adaptive network coding over the conventional network coding. We considered several different scenarios: any CSI is not available at transmitters, local CSI is available, and global CSI is available at all transmitters. Precoder design was discussed for each scenario to minimize error probability. We confirmed that the adaptive network coding still outperforms the fixed network coding in two–way relaying MIMO systems.

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Fig. 4. End–to–end throughput performance of local–CSIT scenario as a function of average SNR for QPSK 3 × 3 MIMO systems in Nakagami–Rice fading channels with 10 dB Rician factor.