Abstract—We focus on an uplink network composed of femtocell access points (FAPs), macrocell users (MUs) and a macrocell base station (MBS) employing orthogonal frequency division multiple access (OFDMA) signaling. Synchronization of MUs at the MBS results in asynchronous signal arrivals at the FAPs which may induce intercarrier interference (ICI). Given a fixed distance between an FAP and the MBS, we derive a closed-form expression for the probability of an MU causing ICI to the FAP. Then, we extend this to obtain the probability of an open and closed access FAP experiencing ICI in the presence of a fixed number of MUs, and the average number of open and closed access FAPs experiencing ICI.

Index Terms—Femtocell, OFDMA, uplink, Poisson process.

I. INTRODUCTION

Behavior of cross-tier interference is closely affected by the type of access control employed by FAPs [1], [2]. Closed access FAPs can only provide service to subscribed home users (HUs) and thus unsubscribed MUs nearby can cause strong interference to the FAPs in the uplink. In open access FAPs, strong interfering MUs can simply handover to nearby FAPs, thus reducing their transmission power and amount of cross-tier interference. In [1], open access FAPs were shown to increase the overall throughput of the network. However, fully open access approach could impose stringent requirements on the backhaul capacity since the backhaul network could potentially be serving a large number of MUs [2]. Instead of looking at network throughput or backhaul requirement, we aim to study how access control affects the timing misalignment problem in the uplink OFDMA network.

In an OFDMA network, timing misalignment of the uplink users causing ICI is a known problem [3]. When the arrival times of the uplink users span over the cyclic prefix (CP) duration, there will be an intersymbol interference (ISI) which generates intercarrier interference (ICI) at the receiver due to loss of orthogonality. Unlike the conventional OFDMA network, although the received signals from the MUs are perfectly synchronous at the MBS, these signals inherently appear to be asynchronous at the FAPs. This problem was first briefly mentioned in [4]. Some work along this line can be found in [5]–[7]. The distribution of arrival times at an FAP was derived in [5]. In [6], a new frequency assignment was proposed for an FAP that reuses the spectrum of remote MUs and avoids ICI due to timing misalignment. The effect of timing misalignment on the uplink ergodic capacity and symbol error probability of a closed access FAP was recently investigated in [7].

In this letter, we first derive the probability of an MU causing ICI to the FAP at a fixed distance. Then, we compute the probability of an open and closed access FAP experiencing ICI in the presence of a fixed number of MUs, and the average number of open and closed access FAPs experiencing ICI. Our results reveal that open access FAPs are beneficial in reducing the effect of ICI due to timing misalignment in the uplink. Also, there exists an FAP intensity in the open access case which results in the highest number of FAPs experiencing ICI.

II. SYSTEM MODEL

We consider a single MBS located at the origin with a circular coverage area and radius \( R \). FAPs are randomly deployed within the coverage area of the MBS according to a homogeneous Poisson point process (PPP) with intensity \( \lambda \). For open access model, each FAP has a circular coverage area and radius \( r_o \).

Without loss of generality, we first consider the FAP located at \((R_0,0)\) in a polar coordinate. A fixed number of MUs, \( N_m \), are randomly located over the macrocell coverage area. Denote \((r_j, \theta_j)\) as the location of the \( j \)th MU. For uniform distribution, the probability density functions of \( r_j \) and \( \theta_j \) are \( f(r) = \frac{2}{\pi R}, r \in [0, R] \), and \( f(\theta) = \frac{1}{2\pi}, \theta \in [0, 2\pi] \) respectively. In closed access FAPs, all FAPs can overhear the transmissions of any MUs which may cause ICI depending on their locations. In open access FAPs, when an MU is within the coverage of an FAP, handover occurs. After the handover, the MU becomes an HU and is assumed to transmit at sufficiently low power such that there is no interference to other FAPs.

With OFDMA, the MUs are sharing the same OFDM symbol with each other and with the HUs. Denote \( N \) and \( N_{CP} \) as the FFT size and CP length, respectively. The MUs are assumed to be perfectly synchronized at the MBS. To avoid severe ICI, the synchronization point of the FAP should be set at the start of the OFDMA frame of the earliest possible signal arrival. This point is close to the optimal point that minimizes ICI while it simplifies the problem significantly [8]. For simplicity, we assume a single path channel between a transmitter and a receiver [5], [7].

Denote as \( D_j \), the discrete time delay of the \( j \)th MU signal arrival at the FAP. ICI will occur at the FAP of interest if \( D_j > N_{cp} \). From [7], ICI can only occur at the FAP with \( R_i \) greater than \( \frac{\xi}{2} = N_{cp}T_{samp}/2 \), where \( T_{samp} \) is the sampling interval and \( c \) is the speed of light. The location of an MU contributing ICI was derived in [7] and we provide the following Lemma on which the results of this letter extend.
I in [7]. Since \( R_t > \frac{\xi}{2} \), since \( R_t < R \), then \( R_t - \frac{\xi}{2} < R \). Therefore, \( \left( \frac{1}{R_t} \left( R_t - \frac{\xi}{2} \right) + 1 \right) < 1 \) and this leads to \( \theta_d < \theta_c \).

**A. Probability of an MU Causing ICI to a Given FAP**

Denote \( p_m \) as the probability of an MU causing ICI to a given FAP. Since the MU is uniformly distributed, this probability is the ratio between the area of ICI region and the MBS coverage area. Since the ICI region is symmetric along \( \theta = \pi \), the area of the ICI region is determined by

\[
2 \left( \int_0^{\theta_d} \int_0^{\frac{\xi}{R_t}} r \, dr \, d\theta + \int_0^{\pi} \int_0^R r \, dr \, d\theta \right) = \int_0^{\theta_d} \left( \frac{\xi}{R_t} \left( \frac{\xi}{2} - R_t \right) \right) \, d\theta + R^2 (\pi - \theta_d) = \left( \frac{\xi}{R_t} \left( \frac{\xi}{2} - R_t \right) \right) \int_0^{\theta_d} \frac{1}{\cos(\theta) + \frac{\xi}{R_t} - 1} \, d\theta + R^2 (\pi - \theta_d).
\]

Using the following integral [9],

\[
g(x) \triangleq \frac{1}{(\cos(x) + a)^2} \int \frac{dx}{\sqrt{1 - a^2 \cos^2 x}} = \frac{\sin(x)}{a + \sqrt{a^2 - 1}},
\]

with \( a = \frac{\xi}{R_t} - 1 \), we obtain a closed-form expression

\[
p_m = \frac{1}{\pi R^2} \left( \frac{\xi}{R_t} \left( \frac{\xi}{2} - R_t \right) \right)^2 g(\theta_d) + R^2 (\pi - \theta_d)
\]

**B. Probability of an FAP Experiencing ICI**

Denote \( p_t \) as the probability of an FAP experiencing ICI from any of the \( N_m \) MUs. For closed access FAPs, this is equivalent to the probability that at least one of the MUs is in the ICI region. Therefore, we have

\[
p_t = 1 - (1 - p_m)^{N_m}.
\]

For open access FAPs, for an MU to cause ICI, it must be in the ICI region and it is not within any of the FAP coverage, i.e., no FAPs is within the distance \( r_o \) from the MU. From the property of the PPP, the probability of no FAPs are within the distance \( r_o \) from a given point is \( e^{-\lambda \pi r_o^2} \). Therefore, it follows that

\[
p_t = 1 - (1 - p_m \cdot e^{-\lambda \pi r_o^2})^{N_m}.
\]

Note that we have ignored the edge effect in the above derivation. The edge effect appears when an MU is near the macrocell boundary and the possible area around it that can contain a FAP is not circular.
C. Average Number of FAPs Experiencing ICI

Now we consider ICI that may occur at any FAPs in the MBS coverage area. For a fixed $N_m$, the MUs may not cause ICI to a set of FAPs but may cause ICI to another set of FAPs depending whether any of the MUs is in the ICI region of any specific FAP. From the FAP perspective, the probability of a given FAP having ICI depends mainly on its distance to the MBS. Therefore, the average number of FAPs experiencing ICI, $N_{ICI}$, can be found simply by integrating the number of FAPs weighted by $p_f$ over the area where they could experience ICI. Hence, for both closed and open access cases,

$$N_{ICI} = \int_{\xi/2}^{2\pi} \int_{0}^{R} \lambda p_f r dr d\theta = 2\pi \lambda \int_{\xi/2}^{R} pr dr,$$

which is evaluated by numerical integration.

IV. RESULTS AND DISCUSSIONS

The simulation parameters are as follows: $f_s = 1/T_{samp} = 3.84\text{MHz}$, $N = 256$, $N_{cp} = 18$, $R = 1500\text{m}$. For open access case, the femtocell radius $r_f$ is 100 m. Simulation results are shown in solid lines with markers while analytical results are shown in dashed and dash-dot lines for closed and open access cases, respectively. We consider three cases with different number of MUs, $N_m = 1, 4, 10$.

Fig. 3 illustrates $p_f$ of an FAP at distance $R_f$ from the MBS. An FAP further away from the MBS has a higher $p_f$ due to its larger ICI region. As $N_m$ increases, $p_f$ rises significantly as there is a higher chance that any single MU will fall in the ICI region. In the open access case, the intensity of the FAPs, $\lambda = 50\text{ FAPs/}\pi R^2$, i.e., the average number of FAPs is 50 over the MBS coverage area. It is clear that open access FAPs alleviate the probability of ICI at an FAP. The amount of reduction in $p_f$ depends on both $R_f$ and $N_m$. In addition, the simulation and analytical results are in good agreement in all cases shown.

Fig. 4 plots the average number of FAPs having ICI in the MBS coverage area. As $N_m$ increases, $N_{ICI}$ increases significantly. For closed access case, $N_{ICI}$ increases linearly with $\lambda$. The higher number of the FAPs, the more FAPs experience ICI. For open access case, it is interesting that $N_{ICI}$ keeps increasing until $\lambda$ reaches a critical value after which $N_{ICI}$ starts to decrease. At this critical point, handover of MUs causing ICI to any FAPs becomes effective, i.e., increasing the number of FAPs has a dominant effect in decreasing the number of MUs causing ICI than increasing the number of FAPs experiencing ICI. This critical point has a larger value for a larger $N_m$. Simulation and analytical results match well with each other although there appears slight discrepancies for the open access case when $N_m = 4, 10$ at high $\lambda$. These discrepancies come from the edge effect where, for a higher $N_m$ and $\lambda$, there is a higher chance that an MU and an FAP will be near the macrocell boundary.

REFERENCES