Abstract—In the uplink, while macrocell mobile stations (mMSs) are synchronized to the macrocell base station (mBS), their signals will arrive asynchronously at the femtocell base station (fBS). For OFDMA, if the timing misalignment among the mMSs is greater than the cyclic prefix duration, intersymbol interference (ISI) and intercarrier interference (ICI) will occur in the received signal at the femtocell. In this paper, we show that the average performance of closed access OFDMA femtocell under timing misalignment depends on its distance to the mBS and on the number of mMSs. Simulation results show performance degradations in ergodic capacity and symbol error rate due to ISI and ICI induced by timing misalignment of the mMSs.

I. INTRODUCTION

Femtocell has received considerable attention recently from both academia and industry [1]–[3]. An operator benefits from femtocell in extending coverage to indoor environment without implementing a new base station. By exploiting Internet backhaul, femtocell offloads the indoor cellular traffic and hence saves precious cellular bandwidth for other outdoor users. Indoor users benefit from the femtocell from improved quality-of-service (QoS) due to their shorter distances between their mobile stations and the femtocell.

Due to limited coordination between the macrocell and the femtocell, ‘cross-tier’ interference becomes an important issue and has been discussed in several works recently. Uplink and downlink cross-tier interference were discussed in [2] for an HSPA+ network. Femtocell carrier selection and power control were proposed for downlink transmission while adaptive attenuation at femtocell and limited mobile transmit power were proposed for uplink transmission. Macrocell and femtocell outage probabilities were derived in [4]. The paper also proposed load balancing for an open access CDMA network showing significant capacity improvement. OFDMA network with co-channel assignment between macrocell and femtocell was studied in [5]. Sensing the spectrum opportunity and adjusting power and subcarrier assignment techniques were proposed to handle the cross-tier interference. Comparison between open and closed access femtocells was made recently in [6] for uplink TDMA/OFDMA/CDMA transmission. It was concluded that the preferred configuration of open/close access OFDMA/TDMA femtocell depends on the cellular user density while open access CDMA femtocell is favorable.

All the above works consider the ideal case in which clock synchronization issue is neglected. In practice, timing misalignment between multiple received signals occurs due to different propagation delays. It is known that, in a multiuser OFDMA system, when the timing misalignment between uplink users is greater than the cyclic prefix (CP) period, intersymbol interference (ISI) and intercarrier interference (ICI) will degrade the received subcarrier SINR [7], [8]. The reason is that the FFT window of the desired user may overlap with part of the previous symbol of some other user signals arriving later (See Fig. 1). Cyclic structure is not preserved for those late arrival signals and orthogonality between subcarriers is lost.

An interesting problem was identified recently in the uplink OFDMA network when macrocell mobile stations (mMSs) are synchronized to the macrocell base station (mBS) while their signals arrive at the femtocell base station (fBS) asynchronously [9]. Although the problem was identified, the main focus in [9] was about detection of spectrum opportunity in a cellular network setting where multiple mMSs without power control interfere with each other due to timing misalignment. Also in a cellular setting, [10] proposed an algorithm to obtain the synchronization point that minimizes ICI under timing misalignment. In [11], the statistics of arrival times of macrocell-synchronous femtocell-asynchronous was derived. Another related work was found in [12] where performance of multiuser ad hoc network was evaluated under timing misalignment.

In this paper, closed access OFDMA femtocell performance is studied under timing misalignment of uplink mMSs. Closed access prevents the mMSs to be handed over to a nearby femtocell. By orthogonal subcarrier assignment, the performance of femtocell is normally unaffected by the transmissions of the mMSs. However, we show that the femtocell performance under timing misalignment depends on the number of mMSs transmissions. We derive the condition for the location of a mMS to cause ICI and the femtocell critical distance below

![Timing misalignment generates ISI and ICI](image-url)
which the ICI is not experienced at the fBS. Furthermore, the performance of femtocell is evaluated in terms of the ergodic capacity and symbol error rate. Both show the detrimental effect of timing misalignment to the performance of the femtocell.

II. SYSTEM MODEL

We study an uplink cellular network consisting of one mBS, $N_m$ mMSs and one fBS serving a single femtocell mobile station (fMS). The macrocell radius is $R$. Without loss of generality, the fBS is located on the $x$-axis at the distance, $R_f$, away from the mBS centered at the origin. The location of the $j$th mMS is represented by $r_j$, the distance from the mBS, and $\theta_j$, the angle from the $x$-axis. The geometry of the network is shown in Fig. 2. For uniform distribution of the mMS locations, the pdf of $r_j$ is $f(r) = \frac{2r}{R^2}$ where $r \in [0, R]$ and the pdf of $\theta_j$ is $f(\theta) = \frac{1}{2\pi}$. For multiple access, we consider an OFDMA system, where the total number of subcarriers (FFT size), $N$, is divided into two sets: one set for the macrocell use, and the other set for the femtocell use. The subcarriers for the macrocell use, denoted as $\Gamma$, are split for each mMS use.

We consider a power control network where the transmit power of each mMS is adapted so that the receive power at the mBS is maintained at $P_m$. Similar to [6], we consider only path-loss attenuation for the channel. No multipath components are considered. Thus, the channel power gain depends only on the distance between a transmitter and a receiver. With power control, the transmit power of the mMS is $P_m r_j^{-\alpha}$, where $\alpha$ is the path-loss exponent. The received power at the fBS is then $P_m (d_j/r_j)^{-\alpha}$, where $d_j$ is the distance between the $j$th mMS and the fBS.

In terms of synchronization, we assume that the transmissions from the mMSs are synchronized at the mBS. The mMS further away from the mBS transmits earlier than the mMS closer to the mBS. Assume that the transmission of the mMS at the cell edge occurs at time zero. The transmission time of the mMS at distance $r_j$ is delayed by $\frac{R-r_j}{c}$, where $c$ is the speed of light. Due to timing misalignment, the fBS should synchronize to the earliest arrival of the signals from mMSs, otherwise severe ISI and ICI will occur due to earlier signal arrivals [9], [12]. Thus, the synchronization point is assumed to be at the start of the OFDMA frame of the first arrival. This point is close to the optimal point that achieves the minimum ICI [10] while it will simplify the analysis. In Fig. 2, the mMSs corresponding to the earliest arrival are on the $x$-axis on the right of the fBS (not shown). The transmission from the mMS will arrive at the fBS with the discrete-time delay:

$$ D_j = \left\lceil \frac{d_j + R/r_j - R/r_j}{c T_{samp}} \right\rceil = \left\lceil \frac{d_j - r_j + R_f}{c T_{samp}} \right\rceil, \quad (1) $$

where $T_{samp}$ is the sampling duration, $\lceil x \rceil$ is an integer greater than or equal to $x$. The condition for the mMS to generate ISI and ICI is $D_j > N_{cp}$ where $N_{cp}$ is the CP length. Since the performance of the femtocell is a function of subcarrier SINR, we will refer to the interference collectively as ICI.

III. EFFECTS OF TIMING MISALIGNMENT ON FEMTOCELL

In the following, we present the results on the fBS distance from the mMS such that no ICI occurs and the location where an mMS will generate ICI. Then, the expressions for femtocell ergodic capacity and symbol error rate under timing misalignment will be given.

A. Femtocell Distance with no ICI

If the distance between the fBS and mBS is small enough, we can guarantee that all signal arrivals from mMSs will reach the fBS within the CP period. This insight is stated precisely in the next Theorem.

**Theorem 1:** Femtocell will experience no ICI when $R_f \leq R^*$ where

$$ R^*_f = \frac{\beta T_{sym}}{2} = \frac{\xi}{2}, \quad (2) $$

is the **femtocell critical distance**, $\beta = \frac{N_{cp}}{N}$ is the ratio between CP length and the number of OFDM subcarriers. $T_{sym}$ is the useful OFDM symbol duration. $\xi = \beta T_{sym}$.

**Proof:** See Appendix. \(\square\)

We can interpret $T_{sym}$ as a symbol-length equivalent distance and $\xi$ as a CP-length equivalent distance. The femtocell distance with no ICI is the distance below the straight line with slope $\frac{\xi}{T_{sym}}$ as shown in Fig. 3. Therefore, to increase $R^*_f$, one can either increase $T_{sym}$ or $\beta$. Interestingly, the femtocell critical distance $R^*_f$ does not depend on the macrocell radius $R$ in the network where mMSs are synchronous at the mBS.

![Fig. 2. Macrocell and femtocell system model](image)
B. mMS Location Causing ICI

In the next Theorem, we relate the mMS location with \( D_j > N_{cp} \) that will cause ICI at the fBS.

**Theorem 2:** For a mMS to generate ICI, its location in the polar coordinate, \((r, \theta)\), is given by

\[
 r < \frac{\xi}{R_f (1 - \cos \theta) - \xi} ; \\ 0 \leq \theta < \theta_c \cup 2\pi - \theta_c < \theta < 2\pi \\
 r > 0 ; \\ \theta_c \leq \theta \leq 2\pi - \theta_c,
\]

where \( \theta_c = \cos^{-1} \left( 1 - \frac{c_p}{R_f} \right) \) and \( \xi = N_{cp} c_T \text{samp} \).

**Proof:** See Appendix.

C. Femtocell Ergodic Capacity

When the delay of the \( j \)-th mMS signal is larger than the CP length, without considering path-loss, the ICI power at the femtocell subcarrier \( k \) can be written as [9]

\[
 I_2^f(k) = \frac{2}{N_T^2} \sum_{p \in \mathcal{J}_j, p \neq k} \frac{1 - \cos \left( \frac{2\pi(p-k)(D_j - N_{cp})}{N} \right)}{1 - \cos \left( \frac{2\pi(p-k)}{N} \right)}.
\]  

(4)

where \( \mathcal{J}_j \) is the set of subcarriers assigned to the mMS \( j \). The actual amount of ICI is \( I_2^f(k) \) weighted by the received power, \( P_{\text{m}} \left( d_j / r_f \right)^{-\alpha} I_2^f(k) \).

Assume that the receive signal power at the fBS from the fBS is \( P_f \) and the noise power spectral density is \( N_0 \). The received SINR at the subcarrier \( k \) at the fBS is written as

\[
 \text{SINR}(k) = \frac{P_f}{\sum_{j \in \mathcal{J}} P_{\text{m}} \left( d_j / r_f \right)^{-\alpha} I_2^f(k) + N_0},
\]

(5)

where \( \mathcal{J} \) is a set of mMSs generating ICI. Suppose \( P_{\text{m}} = P_f = P \) and define ICI-free SNR as \( \gamma = \frac{P_f}{N_0} \). The SINR\((k)\) can be rewritten as

\[
 \text{SINR}(k) = \frac{\gamma}{\gamma \sum_{j \in \mathcal{J}} \left( d_j / r_f \right)^{-\alpha} I_2^f(k) + 1}.
\]

(6)

As \( \gamma \to \infty \), the SINR\((k)\) will reach an upper bound \( \sum_{j \in \mathcal{J}} \left( d_j / r_f \right)^{-\alpha} I_2^f(k) \), which sets a limit for an achievable capacity at high \( \gamma \).

Therefore, the femtocell ergodic capacity is given by

\[
 C = \frac{1}{N} \mathbb{E} \left[ \sum_{k \in \Gamma_{fc}} \log_2 (1 + \text{SINR}(k)) \right],
\]

(7)

where \( \Gamma_{fc} \) is the set of subcarriers given to femtocell use and the expectation is taken over different sets of mMS locations.

D. Femtocell Symbol Error Rate

For a given average SINR at subcarrier \( k \), the conditional probability of a symbol error for \( M \)-QAM modulation is [13]

\[
 Pr_s(k) = 1 - \left( 1 - \frac{2(\sqrt{M} - 1)}{\sqrt{M}} Q \left( \sqrt{3\text{SINR}(k)} \right) \right). 
\]

(8)

By averaging over all subcarriers in \( \Gamma_{fc} \) and over different sets of mMS locations, the femtocell SER is given by

\[
 \text{SER} = \frac{1}{N_{fc}} \mathbb{E} \left[ \sum_{k \in \Gamma_{fc}} Pr_s(k) \right],
\]

(9)

where \( N_{fc} \) is the cardinality of \( \Gamma_{fc} \).

IV. SIMULATION RESULTS AND DISCUSSION

The common simulation parameters are shown in Table I. Other parameters are adjusted according to the plot. The mBS is at the origin, \((0m, 0m)\). The mMSs are located randomly over the cell with uniform distribution. We assume the system is fully loaded, i.e., all subcarriers are fully occupied either by mMSs or fMS. Multiple mMSs share the subcarriers from the set \( \Gamma \) equally in blocks. For example, when \( N_m = 1 \), the mMS occupies the 0th-223rd subcarriers. When \( N_m = 2 \), the first mMS occupies the 0th-111st subcarriers while the second mMS occupies the 112nd-223rd subcarriers. The 224th-255th subcarriers are reserved for femtocell.

Fig. 4 illustrates the boundaries where ICI occurs as a function of fBS locations. The boundary is plotted from Theorem 2. The marker \( \times \) represents the fBS location for each boundary. The mMSs staying on the left side of the boundary will cause ICI at the fBS. As the fBS is further away from the mBS, the area where mMSs generating ICI enlarges, i.e., larger amount of ICI is expected at the fBS due to larger fraction of mMSs locating in that area. As an example, the shaded area is where mMSs can cause ICI when the fBS is located at \((750m, 0m)\) (obtained by simulation).

Fig. 5 shows the contour plot of the average ICI in dB (assuming \( P = 1 \)) when the fBS is at \((750m, 0m)\). Each contour has a sharp transition due to discrete values of delays. Note that the amount of ICI already includes the effect of attenuation from path loss. It appears that the mMSs locating close to the \( x \)-axis on the leftmost region creates highest amount of ICI under the chosen simulation parameters.

Fig. 6 shows the femtocell ergodic capacity under different numbers of mMSs when the fBS is located at \((750m, 0m)\). Solid lines represent a general case where mMSs can exist anywhere in the cell while dashed lines represent the case where all mMSs are in the region that generate ICI. Compared to the no-ICI case, \( N_m = 0 \), ICI slightly degrades the femtocell capacity at the high SINR region. The influence of different numbers of mMSs is small due to the assumption of fully load cell. If the dashed lines are extended at a larger \( \gamma \),
Fig. 4. The area of ICI users is on the left of the solid curved lines as a function of distance between fBS and mBS, $R_f$. The shade shows the ICI users from simulation when fBS is at (750m,0m).

Fig. 5. The ICI contour when the fBS is at (750m,0m). The numbers on the contours represent the ICI power in dB. Cool colors represent less ICI.

Fig. 6. Femtocell ergodic capacity under different numbers of mMS. fBS is located at (750m,0m). Dashed lines are the capacity conditioned on all mMS generating ICI.

Fig. 7. Femtocell ergodic capacity with different femtocell locations defined by $R_f$. The number of mMS is $N_m = 4$.

they will reach a bound where increasing $\gamma$ does not increase the capacity due to dominant ICI.

Fig. 7 shows the femtocell ergodic capacity when the fBS is located at different distances from the mBS. The number of mMSs is fixed at four. The figure shows that the ergodic capacity decreases as $R_f$ increases from 900m at high SINR. Note that when $R_f = 500, 700m$, their average capacities correspond to the case of no ICI since the $R_f$ in these cases are smaller than the critical distance $R_f^* = 703.13m$.

Fig. 8 shows the effect of ICI to the femtocell SER when 16-QAM is applied at the fMS. Note that while ICI is independent of the modulation at the mMS for symmetric constellations [9], the SER is dependent on the modulation at the fMS. We can see that timing misalignment of mMSs degrades the SER performance. Interestingly, the worst case degradation occurs when only one mMS is present. This is because the SER is very sensitive to the variance of the SINR in contrast to the ergodic capacity. From the simulation, we found that while the mean of SINR in the case of $N_m = 1$ is slightly greater than that in the case of $N_m = 4$ at the same $\gamma$, the variance of SINR in the case of $N_m = 1$ is much greater. For $N_m = 1$, the amount of ICI can be either zero, when the mMS does not cause ICI, or very large, when the mMS generates ICI from all occupied subcarriers. For $N_m > 1$, due to orthogonal spectrum sharing, the amount of ICI gradually changes according to different numbers of mMSs causing the ICI.

V. CONCLUSION

Closed access OFDMA femtocell is studied under timing misalignment of macrocell mobile stations which potentially causes ISI and ICI. The paper offers two analytical derivations: the critical distance for femtocell experiencing no ICI and the location of macrocell mobile station generating ICI. It is
found that the femtocell critical distance does not depend on the macrocell radius but rather on the CP length equivalent distance. The contour plot shows the amount of ICI and the effect of distance between femtocell and macrocell base station. In addition, the performance of femtocell is evaluated in terms of ergodic capacity and M-QAM symbol error rate showing the detrimental effect of timing misalignment.

APPENDIX

A. Proof of Theorem 1

If the worst case arrival is within the CP-length, no ICI is experienced at the femtocell. From (1) and omitting the subscript \( j \), the maximum \( d = \sqrt{r^2 + R_f^2 - 2rR_f \cos \theta} \) can be reached when \( \theta = \pi \). Then, the condition of the worst case arrival within the CP-length becomes

\[
\frac{\sqrt{r^2 + R_f^2 - 2rR_f \cos \theta}}{cT_{\text{samp}}} \leq N_{\text{cp}}.
\]

Hence, the worst case delay occurs from the users along the \( x \)-axis on the left side of the mBS independent of the distance to the mBS. Since \( N_{\text{cp}} \) is an integer, the \( [\cdot] \) can be removed. Then, \( R_t \leq \frac{N_{\text{cp}} cT_{\text{samp}}}{2} \). Replace \( T_{\text{samp}} \) with \( \frac{T_{\text{sym}}}{N} \) to arrive at Theorem 1. Similar result from different viewpoint was obtained in [11] by different derivation.

B. Proof of Theorem 2

ICI is generated when \( D_j > N_{\text{cp}} \). This is possible only if the femtocell distance is greater than the critical distance, \( R_f > \frac{\xi}{2} \). From (1) and omitting the subscript \( j \) for convenience, we have

\[
\frac{\sqrt{r^2 + R_f^2 - 2rR_f \cos \theta}}{cT_{\text{samp}}} > N_{\text{cp}}.
\]

Then, remove \( [\cdot] \) and write \( d \) in terms of \( r \) and \( \theta \), we have

\[
\sqrt{r^2 + R_f^2 - 2rR_f \cos \theta} > N_{\text{cp}} cT_{\text{samp}} + r - R_t.
\]

Now, suppose \( N_{\text{cp}} cT_{\text{samp}} > R_t \), squaring both sides of the above and rearranging terms yield

\[
2rR_t(1 - \cos \theta) > 2(r - R_t) N_{\text{cp}} cT_m + (N_{\text{cp}} cT_{\text{samp}})^2 \quad \text{and} \quad r(2rR_t(1 - \cos \theta) - 2\xi) > \xi^2 - 2R_t \xi.
\]

Depending on \( \theta \), the term multiplied with \( r \) can be either positive or negative. For \( \cos \theta > 1 - \frac{\xi}{R_t} \), i.e.,

\[
0 \leq \theta < \theta_c \cup 2\pi - \theta_c < \theta < 2\pi,
\]

\[
\theta_c = \cos^{-1} \left( 1 - \frac{\xi}{R_t} \right).
\]

We readily obtain

\[
r < \frac{\xi (\frac{\xi}{2} - R_t)}{R_t (1 - \cos \theta) - \xi}.
\]

For \( \theta_c \leq \theta \leq 2\pi - \theta_c \), the expression becomes

\[
r > \frac{\xi (\frac{\xi}{2} - R_t)}{R_t (1 - \cos \theta) - \xi}.
\]

Since the nominator is negative and the denominator is positive by assumptions, the term on the right is negative. Then, any \( r > 0 \) will be valid for \( \theta_c \leq \theta \leq 2\pi - \theta_c \). Similar results can be obtained for \( R_t > N_{\text{cp}} cT_{\text{samp}} \).

REFERENCES