New perspectives on capital, sticky prices, and the Taylor principle

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Abstract

Our main result is that dynamic new-Keynesian (DNK) models with firm-specific capital feature a substantial amount of endogenous price stickiness. We use this insight to assess the desirability of alternative interest rate rules, and make the case for combining active monetary policy with interest rate smoothing and/or some responsiveness of the nominal interest rate to real economic activity. The key mechanism behind our results is also useful from a positive point of view: the feature of firm-specific capital increases the empirical appealingness of DNK models, as documented by a growing body of literature.

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1. Introduction

According to the Taylor principle monetary policy should be active.\textsuperscript{1} Specifically, the nominal interest rate should be adjusted by more than one-for-one in response to changes...
in current inflation. Most of the existing literature supports the view that by following this simple recommendation a central bank can avoid being a source of unnecessary fluctuations in economic activity. The reason is that many dynamic new-Keynesian (DNK) models imply that the Taylor principle is a sufficient condition for determinacy, i.e. local uniqueness of rational expectations equilibrium (REE). Given its apparent robustness Clarida et al. [11], and a large subsequent literature, use the Taylor principle to judge the conduct of monetary policy in practice.

In the present paper we reassess the usefulness of the Taylor principle. Our model features Calvo pricing, combined with two standard restrictions on capital accumulation at the firm level: the additional capital resulting from investment becomes productive with a one period delay and there is a convex capital adjustment cost. In this sense capital is firm-specific in our model. Surprisingly, we find that active monetary policy is not a sufficient condition for determinacy. This is interesting because most of the existing literature supports the view that the Taylor principle is robust with respect to the modeling of capital accumulation. An exception is Dupor [13]. His result that a passive interest rate rule is required to guarantee determinacy appears, however, to be specific to the continuous time framework he employs. In discrete-time models Galí et al. [18], Lubik [20], and Carlstrom and Fuerst [8] find that endogenous capital per se does not challenge the Taylor principle, if conventional values are assigned to the relevant parameters.

How is it possible that we reach a different conclusion in the present paper? The answer is that the convenient and widely used assumption of a rental market for capital is not innocuous: it hides an indeterminacy problem. The reason is as follows. First, with sufficiently high price stickiness REE is indeterminate in a DNK model with endogenous capital. This has been pointed out by Carlstrom and Fuerst [8]. Second, the difference between a specification with firm-specific capital and an alternative formulation with a rental market boils down to a difference in implied price stickiness, as we show. For any given exogenous restriction on price adjustment there is less price stickiness, if a rental market for capital is assumed. The difference in implied price stickiness is therefore a useful metric: a Calvo parameter of about 0.9 is needed in the rental market model in order to recover the equilibrium dynamics resulting form a value of 0.75 in the model with firm-specific capital, if conventional values are assigned to the remaining parameters. Our theoretical result regarding the price stickiness metric has also triggered some empirical research. Altig et al. [1] and Eichenbaum and Fisher [15] argue that the feature of firm-specific capital increases the empirical appealingness of the Calvo model.

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2 See, e.g., [32,34].

3 Sveen and Weinke [29,30] explain the economic mechanism through which firm-specific capital affects inflation and output dynamics in the Calvo model. The latter has been obscured by a conceptual mistake in Woodford [35, Ch. 5], as we note. Since we wrote and circulated our papers there have been other contributions that stress the fruitfulness of assuming firm-specific capital in a model with staggered price setting. See, e.g., [1,9,12,15,21,36].

4 Carlstrom and Fuerst [8] find, however, that forward-looking interest rate rules do generally not guarantee determinacy in a DNK model with capital accumulation.

5 The intuition is analog to the one that explains the difference in implied inflation dynamics resulting from assuming either constant returns to scale or decreasing returns to scale in a DNK model, along the lines discussed in [26,17].
In summary, with a rental market for capital the resulting price stickiness will generally be too low to make the indeterminacy issue appear to be relevant from a practical point of view. This conclusion changes if capital is assumed to be firm-specific: if a central bank respects the Taylor principle and follows a rule according to which the nominal interest rate is set as a function of inflation only, then indeterminacy appears to be the regular case.

Next, we explain how monetary policy rules should be designed in order to guarantee macroeconomic stability. Based on our insights we make the case for combining active monetary policy with interest rate smoothing and/or some responsiveness of the nominal interest rate to real economic activity.\(^6\)

The remainder of the paper is organized as follows: Section 2 outlines the model structure with firm-specific capital and explains how it changes under the alternative assumption of a rental market for capital. Section 3 presents our results. Section 4 concludes.

2. The model

The economy is populated by households and firms. In what follows we consider a variant of the model with firm-specific capital outlined in Sveen and Weinke [30].\(^7\) In the present paper we assume that there is no aggregate uncertainty except for sunspots according to which economic agents agree on a particular equilibrium. A short description of the alternative specification with a rental market is left for the last paragraph.

2.1. Households

Households choose consumption, supply labor in a competitive market, and have access to complete financial markets. A representative household seeks to maximize expected discounted utility:

\[
E_t \sum_{k=0}^{\infty} \beta^k U (C_{t+k}, N_{t+k}) ,
\]

where \(U(\cdot)\) denotes the period utility function, \(\beta\) is a discount factor, \(N_t\) denotes hours worked in period \(t\), and \(C_t\) is a Dixit–Stiglitz consumption aggregate as of that time. Specifically,

\[
C_t \equiv \left( \int_0^1 C_t(i) \frac{e^{1}}{\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}} ,
\]

where \(\epsilon\) is the elasticity of substitution between different varieties of goods \(C_t(i)\).

\(^6\) Interestingly, rules of this kind appear to be empirically plausible for the US economy under Volcker and Greenspan. For a comprehensive overview, see [35, Ch. 1].

\(^7\) In [30] we solve the model using an iterative procedure. In the present paper we follow Woodford [36] and use the method of undetermined coefficients, which is computationally more efficient.
We assume the following period utility function:
\[
U (C_t, N_t) = \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\phi}}{1 + \phi}.
\]  
(3)

Parameter \(\sigma\) denotes the household’s relative risk aversion, or equivalently, the inverse of the intertemporal elasticity of substitution, and parameter \(\phi\) can be interpreted as the inverse of the Frisch labor supply elasticity.

The maximization is subject to the following sequence of budget constraints:
\[
\int_0^1 P_t (i) C_t (i) \, di + E_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t + T_t, \tag{4}
\]

where \(W_t\) is the time \(t\) nominal wage, \(Q_{t,t+1}\) is the stochastic discount factor for random nominal payments, \(D_{t+1}\) is the nominal payoff of the portfolio held at the end of period \(t\), and \(T_t\) denotes profits resulting from ownership of firms.

For each variety of goods the consumption demand function reads:
\[
C_d^t (i) = \left( \frac{P_t (i)}{P_t} \right)^{-\varepsilon} C_t, \tag{5}
\]

where \(P_t \equiv \left( \int_0^1 P_t (i) (1-\varepsilon) \, di \right)^{\frac{1}{1-\varepsilon}}\) denotes the price index. The latter has the property that the minimum expenditure required to purchase a bundle of goods resulting in \(C_t\) units of the composite good is given by \(P_t C_t\).

The remaining first-order conditions associated with the household’s problem are:
\[
C_t^\sigma N_t^\phi = W_t, \tag{6}
\]
\[
\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = Q_{t,t+1}. \tag{7}
\]

The first equation is the optimality condition for labor supply, and the second one is a standard intertemporal optimality condition. Finally, let us note that the time \(t\) gross nominal interest rate, \(R_t\), is related to the stochastic discount factor by the equilibrium condition \(R_t^{-1} = E_t \{ Q_{t,t+1} \}\).

2.2. Firms

There is a continuum of monopolistically competitive firms, indexed on the unit interval. Each firm \(i\) has access to a Cobb–Douglas technology:
\[
Y_t (i) = K_t (i)^{\alpha} N_t (i)^{1-\alpha}, \tag{8}
\]

where \(\alpha\) is the capital share in the production function, and \(K_t (i)\) and \(N_t (i)\) denote, respectively, firm \(i\)’s capital stock and labor input used in its period \(t\) production denoted \(Y_t (i)\).

We assume staggered price setting à la Calvo [7], i.e. each firm faces a constant and exogenous probability, \((1 - \theta)\), of getting to reoptimize its price in any given period. This
structure implies that firm $i$’s time $t$ nominal price, $P_t(i)$, is either the one that was posted the period before or the optimally chosen price $P^*_t(i)$.

Moreover, we follow Woodford [35, Ch. 5] in assuming two restrictions on capital adjustment. First, the additional capital resulting from an investment decision becomes productive with a one period delay. Second, firms face a convex capital adjustment cost. This is summarized in the following equation:

$$I_t(i) = I \left( \frac{K_{t+1}(i)}{K_t(i)} \right) K_t(i),$$

where $I_t(i)$ denotes the amount of the composite good purchased by firm $i$ at time $t$, and $K_t(i)$ denotes this firm’s capital stock as of that period. Moreover, function $I(\cdot)$ is assumed to satisfy the following: $I(1) = \delta$, $I'(1) = 1$, and $I''(1) = \epsilon$. Parameter $\delta$ denotes the depreciation rate. Eichenbaum and Fisher [15] interpret parameter $\epsilon$ as the elasticity of the investment to capital ratio with respect to Tobin’s q, evaluated in steady state. Parameter $\epsilon$ is assumed to be strictly larger than zero and it measures the convex capital adjustment cost in a log-linear approximation to the equilibrium dynamics.

Cost minimization by firms and households implies that demand for each individual good $i$ in period $t$ can be written as follows:

$$Y^d_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y^d_t,$$

where $Y^d_t(i)$ denotes aggregate demand at time $t$, which is given by

$$Y^d_t(i) \equiv C_t + I_t,$$

and $I_t \equiv \int_0^1 I_t(i) \, di$ denotes aggregate investment demand.

Let us now consider a price setter’s problem. Given its time $t$ capital stock, $K_t(i)$, a price setting firm $i$ chooses contingent plans for $\{P^*_t(i), K_{t+k+1}(i), N_{t+k}(i)\}_{k=0}^{\infty}$ in order to solve the following:

$$\max \sum_{k=0}^{\infty} E_t \left\{ Q_{t+k} \left[ Y^d_{t+k}(i) P_{t+k}(i) - W_{t+k} N_{t+k}(i) - P_{t+k} I_{t+k}(i) \right] \right\}$$

s.t.

$$Y^d_{t+k}(i) = \left( \frac{P_{t+k}(i)}{P_{t+k}} \right)^{-\epsilon} Y^d_{t+k},$$

$$Y^d_{t+k}(i) \leq N_{t+k}(i)^{1-\alpha} K_{t+k}(i)^{\alpha},$$

$$I_{t+k}(i) = I \left( \frac{K_{t+k+1}(i)}{K_{t+k}(i)} \right) K_{t+k}(i),$$

$$P_{t+k+1}(i) = \begin{cases} P^*_t(i) & \text{with prob. } (1 - \theta), \\ P_{t+k}(i) & \text{with prob. } \theta. \end{cases}$$

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8 Sveen and Weinke [29] consider a model with just the first restriction on a firm’s capital accumulation, namely the one period delay.

9 The relevant elasticity of substitution is assumed to be the same as in the consumption aggregate.
A firm $j$ that is restricted to change its price at time $t$ solves the same problem, except for the fact that it takes $P_t(j)$ as given.

The first-order condition for capital accumulation reads:

$$\frac{dI_t(i)}{dK_{t+1}(i)} P_t = E_t \left\{ Q_{t,t+1} \left[ MS_{t+1}(i) - \frac{dI_{t+1}(i)}{dK_{t+1}(i)} P_{t+1} \right] \right\}, \quad (12)$$

where $MS_{t+1}(i)$ denotes the nominal reduction in firm $i$’s labor cost associated with having one additional unit of capital in place in period $t + 1$. The only non-standard feature of the last equation is that the marginal return to capital is not measured by the nominal marginal revenue product of capital, but instead by $MS_{t+1}(i)$. The reason is that firms are demand constrained, as discussed in Woodford [35, Ch. 5].

The following relationship holds true:

$$MS_t(i) = W_t \frac{MPK_t(i)}{MPL_t(i)}, \quad (13)$$

where $MPK_t(i)$ and $MPL_t(i)$ denote, respectively, the marginal product of capital and labor of firm $i$ in period $t$.

The first-order condition for price setting is given by

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k}^d(i) \left[ P_t^*(i) - \mu MC_{t+k}(i) \right] \right\} = 0, \quad (14)$$

where $\mu = \frac{\varepsilon}{\varepsilon - 1}$ denotes the frictionless mark-up over marginal costs, and $MC_t(i)$ denotes the nominal marginal cost of firm $i$ in period $t$.\footnote{We follow a large literature on the Calvo model in using the notation $E_t$ in Eq. (14) to indicate an expectation that is conditional on the time $t$ state of the world, but integrating only over those future states in which firm $i$ has not reset its price since period $t$. Woodford [36] uses $\hat{E}_t(i)$ in order to denote this expectation.}

The latter is given by

$$MC_t(i) = \frac{W_t}{MPL_t(i)}. \quad (15)$$

Eq. (14) reflects the forward-looking nature of price setting: firms take into account not only current but also future expected marginal costs in those states of the world where the chosen price is still posted.

2.3. Market clearing

Clearing of the labor market requires that hours worked, $N_t$, are given by the following equation, which holds for all $t$:

$$N_t = \int_0^1 N_t(i) \, di. \quad (16)$$

Finally, market clearing for each variety $i$ requires at each point in time:

$$Y_t(i) = C_t^d(i) + I_t^d(i), \quad (17)$$

where $I_t^d(i)$ denotes time $t$ investment demand for good $i$.\footnote{We follow a large literature on the Calvo model in using the notation $E_t$ in Eq. (14) to indicate an expectation that is conditional on the time $t$ state of the world, but integrating only over those future states in which firm $i$ has not reset its price since period $t$. Woodford [36] uses $\hat{E}_t(i)$ in order to denote this expectation.}
2.4. Some linearized equilibrium conditions

We restrict attention to a linear approximation around a steady state with zero inflation. Throughout, a hat on a variable denotes the percent deviation of the original variable with respect to its steady-state value.

2.4.1. Households

From the household’s problem we obtain, respectively, an Euler equation and a labor supply equation. They read:

$$\hat{C}_t = E_t \hat{C}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho),$$  \hspace{1cm} (18)

$$\left(\frac{W_t}{P_t}\right) = \phi \hat{N}_t + \sigma \hat{C}_t,$$  \hspace{1cm} (19)

where parameter $\rho \equiv -\log \beta$ is the time discount rate, $i_t \equiv \log R_t$ denotes the time $t$ nominal interest rate, and $\pi_t \equiv \log \left(\frac{P_t}{P_{t-1}}\right)$ is time $t$ inflation.

2.4.2. Firms

Aggregating and log-linearizing the first-order condition for investment (12) and combining the resulting expression with the Euler equation (18), we obtain

$$\Delta \hat{K}_{t+1} = \beta E_t \Delta \hat{K}_{t+2} + \frac{1}{\epsilon_{K}} \left[ \left(1 - \beta(1 - \delta)\right) E_t \hat{m}_{St+1} - (i_t - E_t \pi_{t+1} - \rho) \right],$$  \hspace{1cm} (20)

where $\Delta$ is the first-difference operator, $K_t \equiv \int_0^1 K_t(i) \, di$ is the aggregate time $t$ capital stock, and $m_{St} \equiv \int_0^1 MSt(i) \, di$ denotes the average real marginal savings in labor cost.

We follow Woodford [36] and derive the inflation equation by employing the method of undetermined coefficients. He shows that it takes the following simple form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{m}_c,$$  \hspace{1cm} (21)

where $\kappa$ is a parameter which is computed numerically, and $m_c \equiv \int_0^1 MC_t(i) \, di$ is the average real marginal cost.\(^\text{11}\)

Aggregating and log-linearizing the production functions of individual firms (8) results in

$$\hat{Y}_t = \alpha \hat{K}_t + (1 - \alpha) \hat{N}_t,$$  \hspace{1cm} (22)

where $Y_t \equiv K_t^{\alpha} N_t^{1-\alpha}$ is aggregate production, up to the first order.

2.4.3. Market clearing

Aggregating and log-linearizing the goods market clearing condition for each variety (17), and invoking (8) and (9), we obtain

$$\hat{Y}_t = \zeta \hat{C}_t + \frac{1 - \zeta}{\delta} \left[ \hat{K}_{t+1} - (1 - \delta) \hat{K}_t \right],$$  \hspace{1cm} (23)

\(^{11}\) See the Appendix A for an outline of the Woodford [36] solution.
where $\zeta \equiv 1 - \frac{\delta x}{\mu (\rho + \delta)}$ denotes the steady-state consumption to output ratio, and $\frac{(1 - \zeta)}{\delta}$ is the steady-state capital to output ratio.

2.5. Rental market

Let us now assume that households accumulate the capital stock and rent it to firms. This is the only change with respect to the specification with firm-specific capital. The household maximizes the objective function given in (1) subject to the following sequences of constraints:

$$\begin{align*}
P_t (C_t + I_t) + E_t \left\{ Q_{t,t+1} D_{t,t+1} \right\} &\leq D_t + W_t N_t + R^k_t K_t + T_t, \\
I_t &= I \left( \frac{K_{t+1}}{K_t} \right) K_t.
\end{align*}$$

(24) \hspace{1cm} (25)

Again, function $I (\cdot)$ is assumed to have the characteristics outlined above, and $R^k_t$ denotes the time $t$ rental rate of capital. Hence, $R^k_t K_t$ is the income that accrues to the household in period $t$ for renting the capital stock $K_t$. Finally, $P_t I_t$ denotes nominal expenditure on investment.

The first-order conditions associated with the household’s choices over leisure and consumption are identical to the ones given, respectively, in (6) and (7). The first-order condition associated with the household’s investment decision reads:

$$\frac{dI_t}{dK_{t+1}} P_t = E_t \left\{ Q_{t,t+1} \left[ R^k_{t+1} - \frac{dI_{t+1}}{dK_{t+1}} P_{t+1} \right] \right\}. \hspace{1cm} (26)$$

Cost minimization implies that each firm produces at the same capital labor ratio. The marginal cost is therefore common to all firms, and this allows us to write the rental rate of capital as follows:

$$R^k_t = W_t \frac{MPK_t}{MPL_t}. \hspace{1cm} (27)$$

Log-linearizing equation (26) and invoking (18) we recover the same log-linearized law of motion of capital as the one given in Eq. (20). It should be noted that, up to a log-linear approximation to the equilibrium dynamics, the set of equilibrium conditions associated with the rental market specification is identical to the one implied by the firm-specific capital model, except for the inflation equation: with a rental market for capital a firm’s marginal cost is independent of its price setting decision. The resulting inflation equation therefore takes the following standard form

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \dot{m} c_t, \hspace{1cm} (28)$$

where $\lambda \equiv (1 - \beta \theta) (1 - \theta) \theta$. This means that, given a specification of monetary policy, the equilibrium processes for the nominal interest rate, consumption, real wage, capital, output, hours, and inflation are determined by Eqs. (18)–(20), (22), (23), and an inflation equation.

\textsuperscript{12} See, e.g., [18].
3. Results

Our goal is to explore what are desirable features of interest rate rules in the sense that they guarantee determinacy. To this end we use the theoretical framework developed so far and explain why some rules are more desirable than others.

3.1. Baseline calibration

The period length is one quarter. Consistent with empirical estimates of the intertemporal elasticity of substitution given in Basu and Kimball [3] we assume $\sigma = 2$. We set $\phi = 1$, implying a unit labor supply elasticity. We assign a standard value of 0.36 to the capital share in the production function, $\alpha$. Setting $\beta = 0.99$ implies an average annual real return of about 4 percent. We choose $\varepsilon = 11$ implying a frictionless markup of 10 percent, which is in line with the empirical estimate in Galí et al. [17]. Finally, we set $\varepsilon_\phi = 3$, as justified in Woodford [35, Ch. 5].

3.2. A simple interest rate rule

Our starting point is a simple rule according to which the nominal interest rate is set as a function of current inflation:

$$i_t = \rho + \tau_\pi \pi_t.$$  \hspace{1cm} (29)

We ask what combinations of values for the inflation response coefficient, $\tau_\pi$, and the price stickiness parameter, $\theta$, result in a determinate equilibrium. The result is shown in Fig. 1 for the model with firm-specific capital: a large range of parameter values that meet the Taylor principle are inconsistent with determinacy. \footnote{There is also a standard indeterminacy region in Fig. 1. The latter is associated with the case where the Taylor principle is not met. As one may expect, the dimension of the standard indeterminacy is one. In what follows we focus on the non-standard indeterminacy region.} An inflation response coefficient, $\tau_\pi$, strictly larger than one is necessary but not sufficient for determinacy.

Moreover, our results are reasonably robust with respect to the particular values assigned, respectively, to the labor supply elasticity and the intertemporal rate of substitution. This is shown in Fig. 2.

We develop the intuition behind this finding along five questions. Why does the Taylor principle guarantee macroeconomic stability in the absence of capital accumulation? Why

\footnote{To solve the dynamic stochastic system of equations we use Dynare (http://www.cepremap.cnrs.fr/dynare/). Thanks to Larry Christiano for providing us with Matlab code which we have used in the computation of $\kappa$.}
is this not necessarily so in the presence of capital accumulation? Why is price stickiness crucial? What is the role of the inflation response coefficient? Why is firm-specific capital crucial?

First, think of a simple economy without capital accumulation, and suppose households increase consumption without any change in the economy’s fundamentals justifying this. Could this consumption boom be consistent with equilibrium? The answer is no, if the central bank follows the Taylor principle: the associated increase in aggregate demand will
lead to an increase in the marginal cost, the latter results in an increase in inflation, and hence real interest rates will increase, if monetary policy is active. But this shows that the boom cannot be consistent with equilibrium since the consumption Euler equation would be violated, if the boom were to occur.

Second, consider an economy with capital accumulation and make the analog thought experiment of an investment boom. Could this boom be potentially consistent with equilibrium? The answer is yes, and the reason is simple. Investment has counteracting effects on the determination of the marginal cost. On the one hand investment demand increases marginal cost, on the other hand the resulting additional capital tends to decrease marginal cost by the time when it becomes productive. The associated inflation dynamics inherit the marginal cost pattern. In particular, there will be some period of deflation in the aftermath of an investment boom. To the extent that the central bank follows the Taylor principle, real interest rates will therefore drop in the deflationary period. The latter could potentially result in a drop in the long real interest rate relevant for investment. If the drop is sufficiently large, then it may rationalize the investment boom ex post.

Third, the latter will only happen if there is enough price stickiness in the model. With higher price stickiness the expected future reduction in marginal cost affects price setting more strongly when the investment boom kicks in. Hence, higher price stickiness dampens the initial increase in inflation and in the real interest rate. If the current real rate is sufficiently stable, then the long real rate drops. Indeed, under an interest rate rule that respects the Taylor principle, a price stickiness parameter, \( \theta \), of about 0.63 is needed to obtain indeterminacy, as shown in Fig. 1. This value corresponds to an average lifetime of a price of less than 3 quarters. Of course, the exact extent to which prices are sticky in actual economies remains controversial. However, a value of \( \theta \) as high as 0.75 is often considered to be empirically plausible.

Fourth, we find that from among the rules which meet the Taylor principle very aggressive rules and intermediate rules, as measured by the relative size of the respective inflation response coefficients, have crucially different properties: the former rules guarantee determinacy, whereas the latter may not if prices are sufficiently sticky. The reason for why very aggressive rules tend to stabilize macroeconomic outcomes is as follows. We observe that the central bank is more effective in reducing future deflation than in reducing current inflation: a more aggressive monetary policy decreases future deflation, which in itself tends to increase current inflation. Hence, if monetary policy is sufficiently active and future expected deflation is low, then the relevant long real interest rate must increase rather than decrease if an investment boom occurs. A maybe somewhat surprising result in Fig. 1 is that

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15 The indeterminacy region associated with the case where the Taylor principle is met does not lend itself for a simulation of the sunspot since the dimension of indeterminacy is two. For a discussion of the last point see [16] and the references herein. Therefore, our thought experiment illustrates only one from among a continuum of possible responses of the endogenous variables to a sunspot shock. In doing so it highlights, however, the key economic mechanism behind our results, namely the role of capital accumulation for the marginal cost dynamics.

16 The reason for the word ‘sufficiently’ is that the average marginal savings in labor costs will also tend to decrease in the considered economic situation. We will come back to this point.

there also exists a determinacy region associated with rules that respect the Taylor principle but prescribe a very gentle interest rate response to inflation. Our explanation is as follows. If the long real rate does not change by much then the drop in marginal savings associated with an investment boom will render REE determinate.

Fifth, we turn to the role of firm-specific capital. It is useful to start with some positive analysis before addressing the normative question which is the main focus of the present paper. With firm-specific capital a price setter is more reluctant to change its price in response to a change in the average real marginal cost. The reason is that a firm takes into account that its marginal cost is affected, to some extent, by the chosen price: due to the restrictions on capital adjustment a price increase is associated with a decrease in the firm’s marginal cost. This mechanism has been discussed by Sbordone [26] and Galí et al. [17] for models with decreasing returns to scale resulting from a fixed capital stock at the firm level. This effect is absent if a rental market for capital is assumed. In that case each firm produces at the same marginal cost, which is independent of the quantity an individual firm supplies. This means that for any given exogenous restriction on price adjustment there is more price stickiness in the firm-specific capital model with respect to the rental market specification.

We have already noted that this is the only difference between the two specifications. It is therefore natural to construct a simple metric: for each value of the Calvo parameter in the firm-specific capital model we compute the corresponding parameter value in the rental market model, which has the property that it makes the resulting equilibrium dynamics coincide in the two models. This is shown in the upper panel of Fig. 3. In particular, for our baseline calibration of the other parameters, assuming a rental market for capital instead of taking firm-specific investment into account is as important as a change in the average expected lifetime of a price from 4 to about 10 quarters. Of course, the adjustment of the price stickiness parameter that is needed in the rental market model in order to generate the same equilibrium dynamics as in the firm-specific capital model depends on the calibration. This is shown in the two lower panels of Fig. 3. First, if the elasticity of substitution between goods, $\varepsilon$, increases then a price setter is more reluctant to change its price in the firm-specific capital model. The reason is that a higher value of $\varepsilon$ implies that a firm’s price setting decision has a stronger impact on its marginal cost. Therefore, more price stickiness is needed in the rental market model in order to make the two models coincide. Second, an increase in the capital share in the production function, $\alpha$, has a similar effect: it increases the price setters’ reluctance to change their prices in the firm-specific capital model.

Recently, it has been argued (on intuitive grounds) that the assumption of a rental market for capital in a Calvo-style sticky price model might be problematic because the researcher who uses such a model for empirical analysis would tend to overestimate the degree of price stickiness. For instance, Smets and Wouters [28] amend their empirical analysis with a caveat of this kind. Their estimate of the expected lifetime of a price is two and a half years, which is far fetched. Our theoretical result shows that this somewhat puzzling finding might reflect the quantitative consequences of the rental market assumption. Our result sheds also light on a finding by Christiano et al. [10]. Their empirical estimate of the price stickiness parameter in a Calvo-style model with capital accumulation and a rental market is ‘driven

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18 For an early model which features differences in marginal costs among producers, see [33].
to unity’. They claim that this is an unappealing feature of sticky price models. However, we tend to interpret their finding as an artefact of the rental market assumption. 19 From our theoretical result regarding the metric it is also plain that the estimated price stickiness must be considerably smaller, if the aggregate data are analyzed through the lens of a DNK model with firm-specific capital.

For the indeterminacy issue analyzed in the present paper the difference in implied price stickiness is crucial: to the extent that a rental market for capital is assumed price setting is generally not forward-looking enough to imply indeterminacy, unless extreme assumptions regarding the frequency of price adjustment are made, as noted by Carlstrom and Fuerst [8].20

What is the relevance of our results? In related literature Edge and Rudd [14] and Røisland [24] make the case against too gentle interest rate rules, while Orphanides [22] points out

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19 It should be noticed, however, that both Smets and Wouters [28] and Christiano et al. [10] assume an investment adjustment cost combined with other features that are not present in the models we compare in the present paper.

20 However, Benhabib and Eusepi [5] show in the context of a rental market model that achieving local indeterminacy might not be sufficient to rule out global indeterminacy.
that too aggressive rules are undesirable. Combining their findings with ours we conclude that the Taylor principle is a poor guide for the design of monetary policy. What form should simple interest rate rules then take in order to prevent the central bank from becoming a source of macroeconomic instability?

3.3. More prominent interest rate rules

We analyze the desirability of some interest rate rules that have been proposed in the literature, either on normative grounds or as an empirically relevant description of the conduct of monetary policy in practice. As in the previous section our criterion to assess the performance of a particular interest rate rule is whether or not it guarantees determinacy.

3.3.1. Responding to economic activity

Let us first consider the indeterminacy regions associated with an interest rate rule that allows for an output response, in the spirit of Taylor [31]:

\[ i_t = \rho + \tau_\pi \pi_t + \tau_y \hat{Y}_t. \] (30)

A relatively small size of the output response coefficient is sufficient to reduce dramatically the importance of the indeterminacy issue, as shown in Fig. 4. The intuition is straightforward from the thought experiment of an investment boom. The latter is associated with an increase in current output. If the central bank reacts with its interest rate instrument directly to this, then the expectation of an investment boom will generally not be self-fulfilling. The last result amends a recent finding by Schmitt-Grohé and Uribe [27] with a caveat. They study the welfare properties of alternative interest rate rules across a rich variety of DNK models. Using a second-order approximation they argue that responding to output is costly in welfare terms. However, based on our analysis, reacting to some measure of real activity will generally prevent the central bank from becoming a source of unnecessary fluctuations in the economy. A caveat is that rules prescribing a reaction to output do not necessary guarantee a globally unique equilibrium, as noted by Benhabib and Eusepi [5].

3.3.2. Interest rate smoothing

Let us analyze next the performance of interest rate rules which take the following form:

\[ i_t = \rho_i i_{t-1} + \left(1 - \rho_i\right) \left(\rho + \tau_\pi \pi_t\right). \] (31)

With interest rate smoothing the definition of the Taylor principle becomes that monetary policy should be active in the long run. In a model without capital the so defined Taylor principle guarantees determinacy. This means that the particular value of the interest rate

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21 Edge and Rudd [14] and Røisland [24] observe that taxes are paid on nominal capital income, which calls for a strengthening of the Taylor principle, while Orphanides [22] argues that very aggressive interest rate rules have the undesirable property of amplifying mistakes in the conduct of monetary policy.

22 It should be noted that the analysis in Schmitt-Grohé and Uribe [27] does not imply that it would be costly in welfare terms to respond to some output gap measure. However, it is unclear a priori how natural output should be defined in a model with endogenous capital, as discussed in Woodford [35, Ch. 5].
smoothing coefficient, $\rho_I \in (0, 1)$, is irrelevant for indeterminacy, as long as the inflation response coefficient, $\tau_{\pi}$, is strictly larger than one. Schmitt-Grohé and Uribe [27] argue that this insight is robust with respect to the modeling of capital accumulation. We find, however, that the role of interest rate smoothing changes substantially if capital is firm-specific. This is shown in Fig. 5. For a value of $\tau_{\pi}$ strictly larger than one and a sufficiently high price stickiness, it is not true that determinacy would obtain for all $\rho_I \in (0, 1)$. We therefore find that inertial rules enhances macroeconomic stability. This contributes to an already extensive literature on the role of interest rate smoothing.23

The intuition behind this finding is in line with our previous interpretations of the model. Let us reconsider the thought experiment of an investment boom. To the extent that the central bank behaves in a backward-looking manner the initial increase in inflation associated with the boom will keep being relevant for the determination of future (real) rates. Hence, indeterminacy can be ruled out in this case. We therefore find that interest rate smoothing and responding to real activity are both desirable properties of interest rate rules, in the sense that they help guaranteeing determinacy. Clearly, a second-order approximation to the equilibrium dynamics is required in order to tell which one of the two features is preferable from a welfare point of view.24 This is an interesting line for future research.25

23 Benhabib et al. [6] conduct a global analysis and make the case for super-inertial rules, i.e. rules where $i_t$ on the left-hand side of Eq. (31) is replaced by $\Delta i_t$. Rules of this type have also been advocated based on local analysis. See, e.g., [25].

24 It should be emphasized that the results from such an analysis are not trivial given the findings in Schmitt-Grohé and Uribe [27]. The reason is that a rental market model and a specification with firm-specific capital do not just differ in the inflation equation if the order of approximation to the equilibrium dynamics is higher than one.

25 Purely practical considerations might also be important for the design of monetary policy rules. In this context, Orphanides and van Norden [23] emphasize that real-time measures of GDP are often subject to large revisions.
4. Conclusion

According to the Taylor principle a central bank should adjust the nominal interest rate by more than one-for-one in response to changes in current inflation. This recommendation is generally believed to be a useful guide for the design of monetary policy. We find, however, that by following the Taylor principle a central bank does not necessarily avoid becoming a source of macroeconomic instability. More importantly, to the extent that a central bank adjusts the nominal interest rate in response to inflation only, indeterminacy appears to be the regular case. This challenges much of the conventional wisdom regarding desirable features of interest rate rules.

The reason for why our results differ from those that have been obtained in the existing literature lies in the fact that we take firm-specific capital into account. We use our model to identify desirable properties of monetary policy, and make the case for combining the Taylor principle with interest rate smoothing and/or some responsiveness of the nominal interest rate to real economic activity.

The key mechanism behind our results is also useful from a positive point of view: the feature of firm-specific capital increases the empirical appealingness of DNK models, as documented by a growing body of literature.

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Appendix A. Inflation dynamics

Analyzing firm-specific capital accumulation in a Calvo-style DNK model has been originally proposed by Woodford [35, Ch. 5]. However, Sveen and Weinke [29] note a conceptual mistake in his text. Basically, a price setter’s expectation regarding its capital holdings in the relevant future states of the world is computed in an incorrect way. Sveen and Weinke [29,30] use an iterative procedure to solve DNK models with firm-specific capital. In the present paper we use the methodological contribution in Woodford [36]. The latter is computationally more efficient than our original solution. In what follows we briefly outline how to solve our model using Woodford’s strategy, which is based on the method of undetermined coefficients.  

We start by considering the log-linearized marginal savings and marginal cost for an individual firm $i$. Invoking (8) and (10) we obtain

$$\hat{mst}(i) = \hat{mst} - \frac{\varepsilon}{1 - \alpha} \hat{p}_t(i) - \frac{1}{1 - \alpha} \hat{k}_t(i),$$  \hspace{1cm} (A.1)

$$\hat{mct}(i) = \hat{mct} - \frac{\varepsilon}{1 - \alpha} \hat{p}_t(i) - \frac{1}{1 - \alpha} \hat{k}_t(i),$$  \hspace{1cm} (A.2)

where $p_t(i) \equiv \frac{P_t(i)}{P_t}$ and $k_t(i) \equiv \frac{K_t(i)}{K_t}$ denote, respectively, firm $i$’s relative price and relative to average capital stock.

Log-linearizing the first-order conditions for investment (12) and price setting (14) and combining them with (A.1) and (A.2), it is straightforward to derive the following two equations:

$$\hat{p}_{t+1}(i) = \hat{k}_t(i) + \beta E_t \hat{k}_{t+2}(i) - \frac{(1 - \beta (1 - \delta)) \varepsilon}{\varepsilon \varphi (1 - \alpha)} E_t \hat{p}_{t+1}(i),$$  \hspace{1cm} (A.3)

$$\hat{p}^*_t(i) = \sum_{k=1}^{\infty} (\beta \theta)^k E_t p_{t+k} + \frac{(1 - \beta \theta) (1 - \alpha)}{1 - \alpha + \varepsilon \alpha} \sum_{k=0}^{\infty} (\beta \theta)^k E_t \hat{mct}_{t+k}$$

$$- \frac{(1 - \beta \theta) \alpha}{1 - \alpha + \varepsilon \alpha} \sum_{k=0}^{\infty} (\beta \theta)^k E_t \hat{k}_{t+k}(i),$$  \hspace{1cm} (A.4)

where $\varphi \equiv \varepsilon \varphi (1 - \alpha) (1 + \beta) + (1 - \beta (1 - \delta)) / \varepsilon \varphi (1 - \alpha)\varepsilon \varphi (1 - \alpha)$ and $p^*_t(i) \equiv \frac{P^*_t(i)}{P_t}$.

Next, we invoke the price index and obtain the following relationship:

$$\pi_t = \frac{1 - \theta}{\theta} \hat{p}^*_t,$$  \hspace{1cm} (A.5)

where $p^*_t \equiv \int_0^1 p^*_t(i) \, di$.

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26 For an alternative description of Woodford’s [36] method, see [9].
Finally, we follow Woodford [36] and posit rules for price setting and investment:

\[ \hat{p}^*_t (i) = \hat{p}^*_t - \tau_1 \hat{k}_t (i), \]  
\[ \hat{k}_{t+1} (i) = \tau_2 \hat{k}_t (i) + \tau_3 \hat{p}_t (i), \]  
(A.6)  
\[ (A.7) \]

where \( \tau_1, \tau_2, \) and \( \tau_3 \) are unknown parameters.\(^27\)

Combining (A.3) with (A.5)–(A.7) it is possible to write \( \hat{k}_{t+1} (i) \) as a function of \( \hat{p}_t (i) \) and \( \hat{k}_t (i) \), as in the investment rule (A.7). This imposes two restrictions on parameters \( \tau_2 \) and \( \tau_3 \). However, these two parameters do also depend on \( \tau_1 \). We turn to this next.

Combining (A.4) with (A.5)–(A.7) we can express \( \hat{p}^*_t (i) \) as a function of \( \hat{k}_t (i) \) and aggregate variables. Averaging this expression and using the fact that price setters are randomly selected, we obtain

\[ \hat{p}_t = \sum_{k=1}^{\infty} (\beta \theta)^k E_t \pi_{t+k} + \frac{(1 - \beta \theta)}{\omega} \sum_{k=0}^{\infty} (\beta \theta)^k E_t \tilde{m}_{ct+k}, \]  
(A.8)  

where \( \omega \equiv 1 + \frac{\theta \epsilon}{1 - 2 \theta - \frac{\epsilon}{1 - 2 \theta - \beta \theta \tau_2}}. \)

In order to pin down \( \tau_1 \) we combine (A.8) with the expression for the newly set relative price of an individual firm (A.4). After rearranging we obtain an equation that takes the same functional form as the pricing rule (A.6). This yields the required restriction on \( \tau_1 \).

Forwarding (A.8) by one period, multiplying the result by \( \beta \theta \), subtracting the outcome from (A.8), and employing (A.5) we derive the inflation equation (21), where \( \kappa \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \frac{1}{\omega}. \)

References


\(^{27}\) Woodford [36] notes that Eqs. (A.5)–(A.7) imply three stability conditions for \( \tau_1, \tau_2, \) and \( \tau_3. \)