Versioning when customers can buy both versions:
An application to intertemporal movie distribution*

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Abstract
We re-consider the decision of a seller who can introduce two versions of a product to a population with differing valuations of quality. In contrast with the previous literature, we allow for the possibility that consumers buy both versions. This simple extension introduces novel results. It now becomes optimal to introduce both versions if products are not too substitutes, even when production costs are zero (pure information goods). The model is particularly appropriate for analyzing the movie industry, where consumers can both watch a movie in a theatre and a home video. The simultaneous introduction of both versions is also contrasted with their sequential release.

Keywords: Product segmentation; versioning; sequential introduction; movie industry.

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1 Introduction

This paper is motivated by versioning and sequencing in theatrical movie distribution. Films are typically first shown in a theatre, then followed by its video release. This is in accordance with the principle of the “second-best alternative”, according to which the content producer should first distribute the movie in the channel that generates the highest revenues over the least amount of time. Then, the movie should cascade in order of revenue contribution, down to markets with lower returns per unit of time (Waterman, 1985 and 2005; Owen and Wildman, 1992; Vogel, 2001). Historically, this has resulted in theatrical exhibition, followed by pay-TV programming, home video, network television, and finally local television syndication (Eliashberg et al., 2006). Staggered release schedule gives each distribution channel a “window” in which to profit from the movie.

This marketing strategy can also be seen as one in which a firm offers different versions of the same good, where each version has a different level of quality, as this enables the firm to create factors that segment consumers, thereby inducing them to self-select their preferred product variant. Versioning is therefore a case of second-degree price discrimination (Mussa and Rosen, 1978; see also Sundararajan, 2004a). Examples of versioning abound in many industries, other than the movie industry. Product line pricing for different sizes of coffee drinks, PCs, or for cars, are possible applications of the idea of versioning. Many more examples can be found for information goods - in fact movies are a relevant category of them, alongside with software, music, etc. (Shapiro and Varian, 1998).

The literature has sharpened our understanding of how versioning should be conducted, and in particular whether it should be introduced at all. When a firm commercializing a high-quality good introduces a lower-quality variant, it is aware that the new version expands the market, but also that it cannibalizes the existing version, as some consumers who would have bought the high-quality good can now switch to the low-quality one. In seminal works, Mussa and Rosen (1978) and Moorthy (1984) find that versioning is optimal, while Stokey (1979) provides conditions under which (second-degree) price discrimination is not optimal. A key difference in these works stems from the marginal cost function. Salant (1989) reconciles these
earlier studies and shows that price discrimination is optimal if the marginal cost function of improving quality is sufficiently convex. For example, in the case of information goods, where the costs of providing quality are of a fixed nature and the variable production costs are constant or even zero, the model specifications of Mussa and Rosen (1978) for the demand side suggest that versioning in not profitable and only the high-quality good should be supplied.\footnote{This result can change in the presence of piracy. Wu and Chen (2008) show that versioning can be an effective and profitable instrument to fight piracy for digital information goods when the piracy costs are within a certain range. Novel insights on versioning can also emerge in psychologically richer models of consumer behavior: when consumers care about the relative standings of products (choice-set-dependent preferences), context management considerations will affect the versioning strategy (Orhun, 2009).}

Some recent related works have generalized this analysis to different degrees.\footnote{For a comprehensive review of the literature on product development, see Krishnan and Ulrich (2001).} Bhargava and Choudhary (2008) show that versioning is profitable when the optimal market share of the lower quality version, offered alone, is greater than the optimal market share of the high quality version, offered alone. Anderson and Dana (2009) find that an “increasing percentage differences condition” is needed for versioning to be optimal, that is, the percentage change in total joint surplus (joint between the consumers and the firm) associated with a product upgrade is increasing in consumers’ willingness to pay.

Why, then, another paper on versioning? All the works mentioned above assume that consumers buy at most one version of a good, e.g., one cup of coffee, one car model. However, this assumption does not really suit the movie industry. While some consumers may watch a movie only at the theatre, or only at home, we should allow the possibility that some consumers, especially those with a high willingness to pay, may want to watch both versions.\footnote{A feature of videos not considered in this paper is that they can be viewed several times (Varian, 2000).}

Our contribution to the literature is therefore to analyze a simple problem of versioning, where a seller can sell two different variants to a continuum of consumers with different willingness to pay. Both variants are produced at the same (constant) marginal cost and are (partial) substitutes. Our innovation is that we allow for consumers to buy both variants. This simple change makes versioning more profitable than otherwise found by the existing literature. In particular, we show that in the case of a monopoly, if the single unit purchase
assumption is imposed, versioning is never optimal. Instead, when consumers are allowed to buy both versions, versioning becomes optimal, with some consumers buying the high-quality good, some buying the low-quality good, and some consumers buying both. In this case, although introducing the low-quality good cannibalizes the high-quality version, the overall effect is positive for the seller because some consumers buy both versions. For this result to hold, the two versions must be not too substitute for each other as otherwise one version cannibalizes the other and our model boils down to standard results.

Our result that versioning can be an optimal strategy when consumers buy both versions, also introduces the possibility of sequencing, i.e., the two versions may be sold at different points in time. The basic model we propose shows that, when the seller and the consumer have the same discount factor, sequencing never arises when versioning is chosen. Both versions are always sold at the earliest possible date, because delaying the release of one version would only discount profits into the future.

The second part of this paper applies our basic model on versioning to analyze the movie industry, which is characterized by the vertical separation between movie exhibitors and distributors. As copyright holders of movies, distributors can commercialize movies through video stores, while they have to reach agreements with theatres to exhibit the theatrical version. Under this market configuration, we consider a bargaining model where distributors and exhibitors negotiate over the contractual terms that specify a rental price to the distributor and the moment in which the video version is released. We show that when the distributor and the exhibitor are not perfectly coordinated (e.g., a linear rental price is used) they do not maximize their joint profits and both versioning and sequencing may appear, even when the two versions are perfect substitutes. The reason is that the vertical separation of the industry prevents the distributor from fully internalizing the profits by selling just one version.

The rest of the paper is organized as follows. Section 2 reviews the literature and presents relevant stylized facts about the movie industry. Section 3 sets up the model and its main assumptions. Section 4 analyzes the optimal versioning and sequencing strategies of a monopolist. Section 5 re-assesses the main results when the two versions are sold in the vertically separated movie industry. Section 6 concludes and offers directions for future research.
2 Literature review and the movie industry

2.1 Literature review

The option of a joint purchase of two versions of a product that we consider in this paper has barely been addressed in the literature.\textsuperscript{4} The extant literature has nevertheless tackled other fundamental questions which are immediately relevant to the movie industry, such as the decision of simultaneously or sequentially introducing the different versions. Moorthy and Png (1992) use the framework of Mussa and Rosen (1978) to analyze the optimal introduction of a product for a monopoly seller. They demonstrate that with a simultaneous introduction of two products, the lower quality product would cannibalize demand for the higher quality. The authors show that an alternative strategy of the seller is to delay the introduction of the low quality product, although this implies the postponement of profits. The sequential introduction of the products wherein the monopoly first serves the consumers with high preferences and afterwards the consumers with lower preferences might be profitable for the firm when consumers are relatively more impatient than the seller (i.e., when consumers have a higher time discount rate). Notice that Moorthy and Png (1992) employ a model with increasing marginal costs of producing each unit of a certain quality, which possibly does not fit particularly well information goods. If marginal costs were constant (or zero in the limit) versioning, and even more sequential versioning, would not be optimal in their framework.

Riggins (2004) extends the model of Moorthy and Png (1992) to consider the case where the seller markets its products in two channels simultaneously, the online (Internet) channel and the offline (bricks-and-mortar) channel.\textsuperscript{5} He assumes that there is a digital device and that a different fraction of the low- and the high-type consumers migrate to the online channel. The seller can potentially offer simultaneously high and low-quality versions of the good in both channels, resulting in, at most, four quality versions. The author shows that, even when

\textsuperscript{4}Exceptions are Gabszewicz and Wauthy (2003) who study a duopoly model of competition, and Martínez-Sánchez (2008) whose focus is on the monopolist optimal choice about the degree of substitution or complementarity of the versions.

\textsuperscript{5}Wildman (2008) also discusses the optimal strategy of content producers for combining distribution through the Internet and traditional channels.
cannibalization is low, if the digital device is important, low types consumers are served only in the offline market, since there will not be enough low-type consumers in the online market to make it profitable for the seller to offer a low-quality good online.

Padmanabhan et al. (1997) analyze the monopolist’s marketing strategy when it is endowed exogenously with some demand externality and consumers are uncertain about it. The monopolist earns a higher profit with a one-shot new product introduction strategy when consumers are informed about demand externality. Conversely, sequential introduction of the products is optimal when consumers are not fully informed. In this case, the firm offers a credible signal of high network externalities by first introducing a product with less than full quality and afterwards an upgrade. For example, a part of a software product can be given away for free to signal the attractiveness of the commercially sold version. Under-provision of introductory quality serves as a signal of high externality, and upgrades serve as the mechanism for implementation of the signaling strategy. In this context, when the firm’s product enjoys high potential demand, it follows a sequential introduction. But when network externalities are low, the firm offers a product with efficient quality in the first period and does not offer any upgrades in the second period.6

The basic model that we propose does not rely on increasing marginal costs, network externalities, signaling, piracy, or differences between the discount factors of the firm and its consumers in order to explain versioning and sequential introduction, although each one of these factors would contribute to refine segmentation. In our simple model with joint purchase, the optimal versioning strategy is to introduce either one or two products simultaneously, depending on the degree of substitution between the two versions. If versioning occurs, a group of consumers buys the high-quality version, another the low-quality version and some consumers buy both. This model offers a richer set of circumstances under which versioning will occur than the standard literature on information goods. In addition, we find

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6The literature reviewed above typically considers models with only two periods, and analyzes whether to introduce variants in the first or in the second period. These two periods are pre-determined. Prasad et al. (2004) are an exception in that they examine the issue of when to introduce a new variant. We also endogenize optimal sequencing in Section 5.
that, when the discount factor of the seller and the consumer are the same, the sequential introduction of the two versions is never optimal. This latter result contrasts with the present practice in the movie industry of exhibiting the movies in theatres in advance of their DVD release. In order to explain this stylized fact, we adapt our basic model to take into account the vertical structure of the movie industry, where distributors can directly sell the video version to consumers but need exhibitors to commercialize the theatrical version.

2.2 The movie industry

It is customary to divide the movie industry into three vertically-related sectors: production, distribution, and exhibition. In the U.S., production and distribution are largely done by the same studios, which we simply call “distributors” in our model. The Hollywood major studios (Universal Pictures, Paramount Pictures, MGM, Fox Film Corporation, Columbia Pictures, Disney, and Warner Brothers) control the distribution of theatrical and video versions. They account for 80 to 90% of the total receipts from the distribution of movies to theatres and other media in the United States (Waterman, 2005, p. 15). Independent studios perform the same basic functions, although they can subcontract distribution to foreign markets. Exhibitors, instead, run theaters and place movies on their screens to attract audiences.

The vertical structure of the movie industry has changed over the last century. Before the late 1940’s, all the major studios owned chains of movie theatres. Orbach (2004) and Waterman (2005) discuss how in the 1920’s and 1930’s, some representatives of the majors and theatre owners constituted cartels to control local markets. These associations assigned the run stage, run length intervals, minimum admission price, and geographic and temporal clearances to each theatre in urban areas. In 1948, the U.S. Supreme Court in United States v. Paramount considered that these cartels were violating the Sherman Act and required majors to divest themselves of their theatre chains. The Courts established a number of regulations to allow the entry of independent exhibitors and to prevent majors from setting admission prices and exclusivity contracts with theatre chains.7

7Interestingly, these restrictions only apply to the signatories of the Decree, but Sony and other new entrants are free to own theatres.
The vertical separation between distributors and theatre exhibitors has persisted to the present days. When theatres are interested in a movie they can lease it in different ways (De Vany, 2004, p. 12). In the most frequently used contract, distributors and theatres agree to share a percentage of the theatre box-office receipts.\(^8\)

The interests of distributors and exhibitors are not perfectly aligned, and the contractual forms between them are incomplete and give room to tensions. For instance, Vogel (2001) and McKenzie (2008) argue that, while distributors get an important percentage of the revenues generated by theatrical versions, exhibitors have incentives to keep their tickets prices low in order to raise their popcorn and other concession prices. This situation creates a constant struggle between distributors and exhibitors over admission prices, with distributors wanting higher admission prices than theatres. In a similar spirit, Gil (2009) argues that the vertical separation of distributors and exhibitors causes misaligned incentives when choosing the optimal movie run length, and tests this hypothesis using Spanish data.

What is important for the purpose of our paper is that, after the *Paramount* anti-trust decision, distributors have less than perfect control of exhibitors. Exhibitors enjoy some market power because they may be the only theater in town or because the movies they show are licensed exclusively to them in their geographic area. As a result, two mark-ups appear in the industry, the first imposed by a distributor because of its exclusive rights over the movie, and the second by the exhibitor because of its monopolistic condition. Distributors cannot avoid this with resale price maintenance or with vertical integration, given the strict anti-trust provisions. It is not coincidental that the seminal paper of Spengler (1950) on the double marginalization distortion was inspired by the 1948 *Paramount* case.

Another important channel for distributors to exhibit movies is the home video. In the last decade, the home video market has experienced an extraordinary growth with the introduction of DVDs, and nowadays video rentals and DVD sales are the largest source of domestic revenue.

\(^8\)Theatres pay the distributor a fee per week and keep a “house nut” (approximately the exhibitor’s weekly cost of operating the theatre). In addition, contracts include a sliding scale for sharing box office receipts that exceed the house nut. A typical contract for a four-week run might offer distributors a minimum box-office percentage of 70-90% in the first two weeks and thereafter distributor’s shares may decrease to 60%.
for studios. Mortimer (2007) and Ho et al. (2008) analyze different pricing mechanisms that distributors use in their contracts with video stores. They show that Blockbusters Video adopted revenue-sharing agreements with several studios in 1998, and quickly other retailers adopted the same mechanism. These arrangements are not regulated and can be quite sophisticated. In our model, we abstract from these contractual arrangements between studios and video stores, and we consider that videos are directly commercialized to consumers by the distributor.  

A final but key feature of the movie industry is the sequence in which distributors release the movie in each media. The time lag between the theatrical release of a movie and its video/DVD rental release is called “video window”. Distributors may benefit from a quicker video release because potential consumers are still influenced by the publicity from the theatrical release and because this strategy moves ahead their video revenues. However, if the video window is too short consumers may decide not to go to the theatre and wait for the video version. Waterman (2005, p. 124) argues that, in the U.S., video windows are very important and, in the past, the industry may have cooperated to control the release sequence of movies. The relevance of the video window cannot be underestimated and is a testground of the frictions between exhibitors and distributors generated by the vertical separation on the supply side. The possibility of the simultaneous release of the two versions has been discussed for some years and producers and distributors announced a series of experiments with simultaneous theater, video, and pay-per-view television release.  

The movie industry, as a whole, has been gradually shortening the video window, which is now approximately 4 months, compared to 6-7 months a decade ago. This trend has worried in particular the theatre owners.  

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9 Dana and Spier (2001) show how, in the presence of stochastic demand, revenue sharing agreements, combined with a low input price, can align the incentives of the vertical chain of the video rental industry.


11 See the comments of John Fithian, President & CEO of the National Association of Theatre Owners. “I believe that the two biggest threats to the movie business are shrinking theatrical release windows and movie theft (or “piracy”).” He also adds “Interestingly, short windows are increasingly associated with movies that do not perform well.” http://www.natoonline.org
In European countries, the window coordination problem has been addressed by industry-wide agreements or has even been set by law. Waterman et al. (2010) show that in the mid-1990’s the window in most European countries was set at either 6 or 8 months but it was 12 months in France and Portugal, which have statutory windows. In recent years, the extent of the window has abruptly decreased, coinciding with the growth of the video market.

Several authors have empirically analyzed the factors affecting the video window. Frank (1999) and Lehmann and Weinberg (2000) show that large windows reduce the cannibalization of the first version, but theatrical marketing and word-of-mouth effects from cinema are not used to increase video sales. Prasad et al. (2004), Luan and Sudhir (2007), and Waterman et al. (2007) consider that movie viewers form expectations about the extent of the video window, and that this leads distributors and exhibitors to coordinate around longer windows. In particular, Waterman et al. (2010) show that distributors that belong to the Motion Picture Association of America have had significantly longer windows on average than non-members of this association. Henning-Thuran et al. (2007) analyze the optimal determination of windows in a market with three or more channels to exhibit movies. They also consider how order changes will affect studios revenues and account for regional differences. None of these works, however, take into account the vertical separation of distributors and exhibitors.

3 The model

We consider a single firm that offers two versions of a product, a high-quality version denoted as \(H\) and a low-quality version denoted as \(L\). Our departure from the extant literature is that we allow consumers to buy both versions, if they wish to do so. Thus consumers can buy \(H\) alone, or \(L\) alone, or both versions (we refer to this case as \(B\), a mnemonic for ‘both’), or they can also decide to buy nothing, indicated as 0. In our application to the movie industry \(H\) represents watching a movie in a theatre and \(L\) renting a home video.

Let \(u_i\) denote the quality of product \(i = H, L, B\). When both \(H\) and \(L\) are bought, the
resulting quality of both versions consumed jointly is

\[ u_B = u_H + u_L(1 - s), \]

where \( s \) represents the level of substitutability between \( H \) and \( L \). When \( s = 0 \) goods are independent, when \( 0 < s < 1 \) the two products are partial substitutes, and when \( s = 1 \) the two products are partial substitutes if \( u_H > u_L \) and perfect substitutes if \( u_H = u_L \). Notice that \( s = 1 \) corresponds to the standard case in the literature, where consumer are limited to a single-unit purchase. In fact, when \( s = 1 \) if a consumer has already bought \( H \), buying \( L \) confers no additional utility on top, and therefore it never buys the two versions.

The more interesting case is when \( 0 < s < 1 \). Consider, for example, the possibility of watching a movie in a theater (the high-quality version) and/or renting a home video (the low-quality version). The case where \( 0 < s < 1 \) represents the situation where consumers are willing to watch at home a movie that they have already watched in a theater, though the additional benefit they enjoy is not as high as if they were watching the movie at home for the very first time.\(^{13}\) When \( s < 0 \), the two products are complements. This situation reflects, for example, the case where consumers obtain more utility from a concert if they have previously listened to the same music in a CD.

Preferences of consumers are heterogeneous. Each consumer is represented by her type \( \theta \), which is uniformly distributed over the segment \([0, 1]\). Following Mussa and Rosen (1978), the surplus of a consumer that buys a product of quality \( u_i \) at price \( p_i \) is given by \( \theta u_i - p_i \).

The two products are sold separately. Thus, when a consumer of type \( \theta \) buys both versions, her net surplus is \( \theta u_B - p_H - p_L \). This is the more interesting and realistic case for the movie industry, especially when products are sold sequentially and separately by exhibitors and distributors (we analyze this case in Section 5). In other markets, the seller can consider the possibility of bundling the two versions, but in the movie industry this would

\(^{13}\) Luan and Sudhir (2007) estimate a flexible structural model of versioning that allows products to be substitutes (and, possibly, even complements). They find that, on average, a consumer’s utility from a DVD would be reduced after having viewed the movie in a theater. The degree of substitutability changes with quality and genre, with highly-rated movies and animation movies showing less substitutability between the theatre and DVD versions.
be difficult to enforce.\textsuperscript{14}

We can now illustrate how the market is split between the two versions. Define $\theta_{ij}$ as the consumer that is indifferent between buying good $i$ and $j$, where $i = \{H, L, B, 0\}$ and $j \neq i$, at a price $p_i = \{p_H, p_L, p_H + p_L, 0\}$ respectively. We thus obtain that $\theta_{LH} = (p_H - p_L)/(u_H - u_L)$ is the consumer that is indifferent between buying $L$ and $H$ separately, $\theta_{HB} = p_L/(u_H - u_H) = p_L/(u_L(1 - s))$ is the consumer that is indifferent between buying the high quality version and both versions, and $\theta_i = p_i/u_i$ is the consumer that is indifferent between buying product $i$ alone, and not buying anything. Different market shares for the two products can arise according to the relative magnitude of the various indifferent consumers $\theta_{ij}$.

The quality of the two versions is given, and we denote by $k = u_H/u_L > 1$ the quality ratio. The firm has a constant marginal cost of supplying each variety of the product, denoted as $c \geq 0$. The assumption that costs are equal across products is useful for our objective of analyzing price discrimination since this implies that differences in prices are due to differences in willingness-to-pay. The case $c = 0$, as in the rest of the literature, corresponds to pure information goods that can be offered in different versions.

4 Simultaneous and sequential introduction of the two versions

We start with a single-period analysis. The monopolist sells both versions simultaneously in one period. The second part of this section analyzes the possibility of releasing the two versions sequentially.

4.1 Simultaneous product introduction

In order to show the main mechanism at work, we simplify the analysis here by considering first the simple case where $0 \leq s \leq 1$ and $c = 0$. That is, we consider the case of pure information goods that can be (partial) substitutes. The quality of versions $H$ and $L$ is

\textsuperscript{14}In the proof of Proposition 1 below we briefly consider the case where the firm bundles the two versions.
exogenously given and the firm can not modify it. The firm sets the prices to maximize its profits, anticipating the way consumers decide to purchase one particular version, both, or none. The following proposition describes the optimal strategy.

**Proposition 1.** Imagine $c = 0$. When $0 < s \leq 2/3$, the firm offers the pattern $L/H/B$ at prices $p_L = \frac{(1-s)u_L}{(2-s)}$ and $p_H = \frac{u_H}{2} - \frac{s(u_L+u_H)}{2(2-s)}$. When $2/3 < s \leq 1$, it offers the pattern $H$ at a price $p_H = u_H/2$.

**Proof.** See the Appendix.

As already anticipated, for $s = 1$, our model corresponds to the standard case of information goods analyzed, e.g., by Bhargava and Choudhary (2001). In line with the received literature, the cannibalization effect of introducing the lower quality variant always prevails over the market expansion effect, and the monopolist is better off by supplying only the high-quality version. Indeed, we find that this result persists when the degree of substitutability is sufficiently high ($s > 2/3$). When the level of substitutability is low, however, the firm finds
it profitable to offer both variants, and consumers self select the variant(s) that maximize individual utility. The pattern that emerges is $L/H/B$ (see Figure 1). Consumers with a very low $\theta$ buy nothing, those with a low $\theta$ buy only $L$, those with an intermediate $\theta$ buy $H$, and those with a high $\theta$ buy both versions, $B$.

This market segmentation is novel to the literature and is sustainable as long as the degree of substitutability is not too high. When $s$ is high the model boils down to a standard model with zero production costs and uniform distribution of types, making it optimal to sell only $H$. The segmentation also disappears if products are completely independent ($s = 0$): In this case it is optimal to sell both versions at a price $(u_L + u_H)/2$ to every buyer.

The fact that, at $s = 0$, there is the pattern $B$ depends on having assumed zero marginal costs. To see why, imagine products are independent ($s = 0$), but the marginal cost is now $c > 0$ for both versions. Since products are independent, they are both charged the monopoly price $(c + u_i)/2, i = H, L$. The consumer indifferent between buying product $i$ and not buying at all is $\theta_{i0} = p_i/u_i = 1/2 + c/(2u_i)$. Since $u_H > u_L$ it follows that $\theta_{H0} < \theta_{L0}$ (these indifferent types coincide only for $c = 0$). Thus, at $s = 0$, there must be now the following pattern: $H/B$. Consumers with a very low type buy nothing, those with $\theta_{H0} < \theta < \theta_{L0} \equiv \theta_{HB}$ buy only $H$, and those with $\theta \geq \theta_{L0} \equiv \theta_{HB}$ buy both versions.

The following proposition makes this reasoning more precise as it generalizes our previous result to the general case where $c \geq 0$ and $s \leq 1$.

**Proposition 2.** Imagine $c \geq 0$. The optimal segmentation strategy and the profit maximizing prices take different values according to the degree of substitution $s$ between versions:

- When $s^1 < s \leq 1$, the firm supplies only the $H$ version at a price $p_H = (c + u_H)/2$;
- When $s^2 < s \leq s^1$, the firm supplies the pattern $L/H/B$ at prices $p_H = \frac{c + u_H}{2} - \frac{su_L}{2(2-s)}$ and $p_L = \frac{c}{2} + \frac{u_L(1-s)}{(2-s)}$;
- When $s^2 < s \leq s^3$, the firm supplies the pattern $H/B$ at prices $p_H = \frac{u_H}{2} - \frac{u_H[s(1-s) + su_L]}{2[u_L + u_H(1-s)]}$ and $p_L = p_H u_L/u_H$;
- When $s^3 < s \leq s^3$, the firm supplies the pattern $H/B$ at prices $p_H = \frac{c + u_H}{2}$ and $p_L = \frac{c + u_L}{2}$.
\[
c + u_L(1-s); \\
\]

- When \( s \leq s^3 \), the firm supplies the pattern \( B \) at prices \( p_H + p_L = (c + u_B)/2 \),

where the cut-off points are

\[
s^1 = \frac{u_L\{c^2(3 - 4u_H) + 6cu_H - 5u_Lu_H + [(u_Lu_H - 2cu_H + c^2)(u_Lu_H + 6cu_H + c^2)]^{1/2}\}}{2c^2(u_L - u_H) + u_Lu_H(4c - 6u_L)}; \\
s^2 = \frac{2c}{c + u_L}; \quad s^3 = 1 - \frac{u_H}{u_L} < 0; \quad \hat{s} = c\left(\frac{1}{u_L} - \frac{1}{u_H}\right),
\]

with \( s^1 > s^2 > s^3 \) when \( c \) is not too large.

**Proof.** See the Appendix.

The previous proposition shows that, for \( s^1 < s \leq 1 \), there is a region \( A \) where only the version \( H \) is provided. For \( s^2 < s \leq s^1 \) there is a region \( B \) where the pattern \( L/H/B \) is offered. The mechanisms at work in these two regions are the same ones as those described in Proposition 1. For \( s^3 < s \leq s^2 \) there is a region \( C \) where the pattern \( H/B \) is offered. Finally, for \( s \leq s^3 \) there is a region \( D \) where only \( B \) is supplied. (See Figure 2.)

![Figure 2: Market segmentation when \( c > 0 \) and \(-1 < s < 1\)](image-url)
The existence and the size of these regions depend on the value of $c$ and on the utility generated by each version. When $c = 0$, $s^2 = 0$ and region $C$ disappears. As a result, for $0 \leq s < s^1 = 2/3$ there is the pattern $L/H/B$, as already found in Proposition 1. Proposition 2 concentrates on the case when $c$ is not too high and the difference between $u_L$ and $u_H$ is not too important. When $c$ is high, or if $u_L$ is much lower than $u_H$, it would be that $s^2 > s^1$. Region $B$ would thus disappear and for $s^1 \leq s < 1$ only service $H$ would be provided.

Finally, note that when $c > 0$ the level of substitutability for which versioning is profitable depends on the quality ratio $k = u_H/u_L$. This situation has interesting implications if the firm is able to modify the quality of the services offered. For example, if $c > 0$ and $s > s^1$ (thus starting from region $A$) the firm might find it profitable to invest in $u_L$, thereby shifting $s^1$ to the right, and adopting the versioning strategy in region $B$.

### 4.2 Sequential product introduction

We now consider the possibility of introducing two versions sequentially. Imagine that $H$ is offered in the first period and $L$ is available for purchase, possibly in the second period. With this extension we are assuming two dimensions of product substitutability: one is exogenous and is represented by $s$ and the other is endogenous and is created by releasing $L$ in the second period. The monopolist commits to its subsequent pricing strategy and viewers form correct expectations about the time in which $L$ is offered. We assume for now that the firm and the consumers are both forward looking and have the same discount factor $\delta \in (0, 1)$. Hence, $\theta_{L,H} = (p_H - \delta p_L)/(u_H - \delta u_L)$ is the consumer that is indifferent between buying separately either the high quality version in the first period or the low quality version in the second period. The expressions for the other indifferent types are unchanged.

To facilitate the presentation of the results we assume again that $0 \leq s \leq 1$ and $c = 0$. The following proposition describes the optimal strategy of the firm.

**Proposition 3.** Imagine that $c = 0$. Also assume that the firm commits in advance to its pricing strategy and that it has the same discount factor as consumers. Then, sequential versioning never arises and products are offered as in Proposition 1.
Proof. See the Appendix.

The firm never introduces the products sequentially because the loss generated by the postponement of profits of \( L \) cannot be compensated by an improvement in extracting information rents. More precisely, when screening among customers, the discount factor \( \delta \) affects only the decision of \( \theta_{L,H} \). However, by deferring the low-quality version to the second period, this negatively affects the firm’s profits not only from those who indeed buy the low quality version alone in the second version, but also from those who decide to buy both versions in the two periods. This effect always prevails and sequencing never occurs.

In Moorthy and Png (1992) sequential introduction can arise due to the difference of discount factors between the producer (\( \delta^p \)) and the consumers (\( \delta^c \)), which we assumed to be the same. Recall that they employ a model with single-unit purchase (\( s = 1 \)) and a convex cost function. One should then wonder if sequentiality can occur also in our framework for pure information goods, just by changing the discount factors. We show next that the answer is no. In the standard case with \( s = 1 \), sequential introduction never arises, even if consumers are more impatient than the firm. Thus, in Moorthy and Png (1992), once again it is the convex cost function that plays a major role. Instead, sequential introduction can emerge for pure information goods as long as products are imperfect substitutes and \( \delta^c < \delta^p \).

The following proposition shows the optimal versioning and sequencing strategy of the firm, using the simplifying assumption that \( u_H = u_L = u \), i.e., the same good can be offered twice, though the two units have some degree of substitution.

**Proposition 4.** Imagine that \( c = 0 \), \( u_H = u_L = u \), and \( \delta^c < \delta^p \). The firm’s optimal sequencing strategy is:

- When \( s^1 < s \leq 1 \), the firm only supplies \( H \) in the first period;
- When \( s^2 < s \leq s^1 \), the firm offers the pattern \( L/H/B \) sequentially;
- When \( s^3 < s \leq s^2 \), the firm offers the pattern \( L/B \) sequentially;
- When \( 0 < s \leq s^3 \), the firm offers the pattern \( L/B \) simultaneously;
with \( s^1 > s^2 > s^3 \) when \( \delta^c / \delta^p \) is not too large.

**Proof.** See the Appendix.\(^\text{15}\)

The proposition shows that when \( \delta^c < \delta^p \), for \( s^1 < s \leq 1 \), there is a region \( A \) where only one good is provided. Therefore, and contrary to Moorthy and Png (1992), it is not enough that consumers are more impatient than the supplier in order for sequencing to arise, in case of pure information goods. In our case, when the cost of production is zero, even if consumers are much more impatient, versioning is never optimal, let alone sequencing. Instead, the ability to possibly buy both goods, introduces the possibility of sequencing for consumers that are relatively impatient. For \( s^2 < s \leq s^1 \) there is a region \( B \) where the firm offers the pattern \( L/H/B \): a low segment of consumers only buys the good in the second period as this is sold at a cheaper price than the same good in the first period, an intermediate segment of consumers only buys \( H \) in the first period, and the high segment of consumers buy both goods sequentially. For \( s^3 < s \leq s^2 \) there is a region \( C \) where the firm offers the pattern \( L/B \): the two goods are released again sequentially, but no consumer purchases only one product in the first period. Finally, for \( 0 \leq s \leq s^3 \) there is a region \( D \) where the firm offers the pattern \( L/B \) simultaneously: The two goods in this case are almost independent, and thus they are both released instantly as there is no gain in waiting until the second period. Some consumers buy only one good, and those with a high willingness-to-pay buy both.

The existence and the size of the regions depends of the value of the ratio \( \delta^c / \delta^p \). Proposition 4 concentrates on the case where the ratio is not too high. When the ratio is equal to 1, then \( s^1 = s^2 = s^3 = 2/3 \). This is the same result as Proposition 3, where regions \( B \) and \( C \) do not emerge, and sequencing never occurs. When \( \delta^c < \delta^p \) region \( B \) always exists.\(^\text{16}\)

The main interest of these results is that, when versions are not too independent or too substitutes, our model predicts sequentiality for information goods, to the extent that discount factors differ. However, we believe it is difficult to validate this rationale empirically in the movie industry, as it is hard to calculate the degree of patience of producers and

\(^{15}\) In the proof, we also provide the expressions of the prices in each region, and of the threshold values of \( s \).

\(^{16}\) In the appendix we also show that, when the ratio is close to 1, region \( C \) disappears. Importantly, also in this case note that there is always a range of values of \( s \) that makes sequencing possible.
viewers. Of course, there are models in the literature that predict sequential introduction. In Padmanabhan et al. (1997) for instance, sequential introduction is offered to create a signal to uninformed customers that the product has some demand externalities. Afterward, the seller offers an upgraded version. Levinthal and Purohit (1989) consider the case where the current version of the product loses its value due to obsolescence. The firm thus faces a trade-off between the cost of waiting for new products sales and the cost of cannibalizing these sales. While these are interesting reasons for justifying sequential offers, in our view, they do not appear to be first-order explanations for the movie industry. Instead, we propose next a new and simple reason which relies on its vertical structure.

5 Versioning and sequencing in the vertically-separated movie industry

This section applies our basic model of versioning to the movie industry. We show that the vertical separation between the movie distributor and the theatre exhibitor changes the incentives for versioning and, crucially, introduces the possibility of sequencing. To this end, imagine there is a distributor (which could be also the producer) that holds all the rights over the movie. This firm also decides whether to release the movie in theaters and/or through DVD stores. Following the notation of Section 3, we call $H$ the movie exhibited in theaters (high quality version) and $L$ the movie viewed with a DVD (low quality version). Reflecting the situation of the U.S. movie industry after the Paramount decision, we assume that theaters and distributors are vertically separated, and the distributor gets a rental price from theaters. We therefore assume that $H$ is exhibited in theatres and, for simplicity, that the distributor directly sells $L$ to consumers.\footnote{Equivalently, the distributor is able to write efficient complete contracts only with DVD video stores. Mortimer (2007) and Ho et al. (2008) discuss at some length how some majors have established sophisticated agreements with video stores.} In order to make the model more realistic we assume that consumers rent the DVD from the distributor and only watch it once.

We are therefore studying a situation where versioning might be affected by the vertical
structure of the industry. Theatres have some market power that can be used when they negotiate the contract of each movie with distributors. The distributor cannot fully appropriate the revenues associated with the theatrical version, for which an independent movie theatre is needed. The lack of complete internalization of the effect of $H$ introduces important changes, both to the optimal versioning strategy and to sequencing, compared to the basic full monopoly case.

5.1 Negotiations between the distributor and the exhibitor

We assume that the distributor and the exhibitor bargain over both the rental price for the movie and the release time for the video version, while theatre tickets and DVD prices are set independently by the exhibitor and distributor, respectively. These specifications produce the basic double mark up distortion and incentive misalignment produced by vertical separation. This bargaining feature contrasts with alternative models where one of the parties (typically, the upstream distributor) makes a take-it-or-leave it offer to the other party. Our assumption is particularly realistic when discussing the length of the video window. On the one hand, the distributor holds the rights over a movie and has full control over the DVD release. On the other hand, the exhibitor typically can decide when to stop showing the movie, which affects the length of the window. Both distributor and exhibitor play an important part when deciding on the video window, thereby making bargaining a natural approach.

We model bargaining using the generalized Nash axiomatic approach, whereas the two players maximize the weighted product of the payoffs perceived by the players in excess of their disagreement payoffs. We denote by $\alpha$ the degree of bargaining power of the distributor and by $1 - \alpha$ the degree of bargaining power of the exhibitor. Some factors that affect bargaining power are the presence of other theatres in the region, the affiliation to an association of exhibitors, or the degree of patience of players during negotiations. The distributor has a non-zero outside option in case of disagreement, as it can always supply the DVD version by itself. On the contrary, the exhibitor has a zero outside option if negotiations fail.

The game played has the following timing. First, at time $t_{-1}$ the distributor and the
exhibitor jointly decide whether or not to show the movie. If \( H \) is not shown in theatres, the distributor releases \( L \) at time \( t_0 \), and the game ends. If instead the two parties agree to show \( H \), they negotiate over the contractual terms that specify a linear rental price \( a \) to the distributor and the release time \( t_1 \geq t_0 \) for the video version. After these terms are set at time \( t_{-1} \), at time \( t_0 \) the exhibitor and the distributor independently set the retail prices \( p_H \) and \( p_L \) respectively. The theatre then releases \( H \) at time \( t_0 \) and the distributor offers \( L \) at a possibly later period, \( t_1 \). With a slight abuse of notation, we denote as \( d \) the compound discount factor for \( t_1 \), so that choosing \( d \) implicitly also determines \( t_1 \). If \( t_0 = t_1 \), then \( d = 1 \) and \( H \) and \( L \) are supplied simultaneously. If \( d < 1 \), this means that \( L \) is introduced some time after \( H \). That is, the DVD is released some months after the display of the movie in theaters. The period of time that separates \( t_0 \) and \( t_1 \) can be measured by its discount factor \( d \). The smaller is \( d \) the longer is the lapse of time taken to introduce \( L \).\(^{18}\)

### 5.2 Versioning and sequencing results

Consumer preferences are assumed to be the same as in Section 4. Thus the surplus of a consumer that buys \( H \) only is \( \theta u_H - p_H \), and if she buys \( L \) only obtains a surplus \( d(\theta u_L - p_L) \). If a consumer buys both versions, her surplus is \( \theta u_B - p_H - dp_L \), where \( u_B = u_H + du_L(1 - s) \) and \( s \) is the level of substitutability between \( H \) and \( L \).

Next, we present the optimal strategies for the firms. In order to keep the model as simple as possible, in what follows we assume that the distributor’s marginal production costs are zero, \( c = 0 \), and we present the results for the extreme cases where \( s = 0 \) (independent products) and \( s = 1 \) (standard case of single-unit purchase).

**Proposition 5.** Imagine that the distributor and the exhibitor negotiate a linear rental price, \( a \), and and the time in which \( L \) must be released, \( d \). When \( s = 0 \), the profit maximizing prices and the optimal segmentation strategy are the following:

- The pattern offered is \( L/B \) at prices \( p_L = \frac{u_L}{d} \), \( p_H = \frac{(2 + c)u_H}{d} \) and \( a = \frac{u_B}{1 - s} \).

\(^{18}\)Using the notation from Section 4.2, if \( \delta \) is the per-period discount factor, then \( d = \delta^{t_1 - t_0} \). In Section 4.2, we allowed only for a one-period sequencing, i.e., we simply set \( t_1 = t_0 + 1 \).
Both versions are offered simultaneously, \( d = 1 \).

Proof. See the Appendix.

When \( s = 0 \), versions are independent and there is no cannibalization. The distributor thus commercializes the movie through both theaters and DVD stores. Because of the double mark up imposed by movie theaters, the price \( p_H \) is particularly high, which explains why the versioning pattern \( L/B \) emerges now, instead of the pattern \( B \) that we found in Proposition 1 for \( s = 0 \), or the pattern \( H/B \) that we found in Proposition 2 for \( s = 0 \) and \( c > 0 \).\(^{19}\) When the products are independent \((s = 0)\), the two firms are interested in introducing the two products as soon as possible. Clearly, the introduction of \( L \) at \( t_0 \) does not cannibalize \( H \), thus \( d = 1 \) and sequencing is never profitable. Finally, the rental price is higher the higher the quality of \( H \) and the higher the bargaining power of the distributor.

When \( s = 1 \) \((B \text{ and } H \text{ are perfect substitutes})\) several strategies for introducing the theatre and the DVD versions are possible. One or both versions might be released, and the two versions might be introduced simultaneously or sequentially.\(^{20}\) The two firms have different preferences over the introduction of version \( L \), once \( H \) is released at time \( t_0 \). The theater exhibitor obtains more profits by delaying the introduction of \( L \), as this reduces the cannibalization of \( L \) over \( H \). However, the distributor may want just the opposite because the increase of rental revenues obtained by delaying the introduction of the video may not compensate the revenue loss in this market. In this situation, only if the bargaining power of the theater is high enough sequencing may arise. The following proposition shows how the bargaining power of firms determines the results on versioning and sequencing.

Proposition 6. Imagine that the distributor and the exhibitor negotiate over the linear rental price, \( a \), and the time in which \( L \) must be released, \( d \). When \( s = 1 \), the firms’ segmentation strategy is the following:

\(^{19}\)Recall that in Proposition 2 the positive marginal cost applied to both versions, while now the access charge is imposed to \( H \) only.

\(^{20}\)The "video" window we consider concerns \( L \) being sold after \( H \). In principle, one could also analyze an alternative timing where \( L \) is released first, and then \( H \). However, it is easy to show that in our model a "movie" a window never arises under this alternative timing, as \( H \) would always be introduced with \( L \).
• If $\alpha > \alpha_1$, the pattern offered is $L/H$, both versions are released simultaneously ($d = 1$), and the rental price is $a_1 = \frac{2+16k^2-(4k-1)(\alpha+2\alpha k-(16k(k-1)(1-\alpha)+(\alpha+2\alpha k)^2)^{1/2})}{4+32k} u_L$.

• If $\alpha_2 < \alpha \leq \alpha_1$, the pattern offered is $L/H$, both versions are released sequentially ($0 < d^* < 1$) and the rental price is $a^* = (2k^2 + 8d^*k - d^*^2) \frac{u_H}{18k}$.

• If $\alpha \leq \alpha_2$, only $H$ is shown ($d = 0$) with a rental price $a_2 = [(2 + \alpha)k - k^{1/2}|k(2 - \alpha)|^2 - 8(1 - \alpha)^{1/2}] \frac{u_H}{4}$, where: $k = \frac{u_H}{u_L} > 1$; $\frac{2}{5} < \alpha_1 = \frac{16k^2+19k+1}{72k^2+9k} < \frac{4}{5}$; $0 < \alpha_2 = \frac{16k-81}{72k-81} < \frac{2}{5}$ for $k > \frac{81}{16}$, otherwise $\alpha_2 = 0$.

**Proof.** See the Appendix.

When $s = 1$, if the bargaining power of the distributor is high enough, the firms segment the marked and offer the pattern $L/H$. This case reflects that the rental price obtained by the distributor from the theater compensates the cannibalization from $H$. The vertical separation does not allow the distributor to fully internalize the profits by selling just $H$, but it uses the rental price charged to the theatre to extract part of its revenues. The higher is the bargaining power of the distributor, the higher the rental price is set (Figure 3).

If the distributor’s bargaining power is very high, then the two versions are offered simultaneously at $t_0$, $d = 1$. If the bargaining power of the distributor is not so high, then the rental price charged to the exhibitor is smaller and the firms agree on introducing the two versions sequentially. This creates a pattern $L/H$ and $d < 1$. Finally, if the bargaining power of the distributor is small and if the quality ratio $k = u_H/u_L$ is high enough, then the distributor never releases $L$. The small revenues raised by $L$ make it profitable to delay its introduction as much as possible to avoid the cannibalization of the high quality version (still, the distributor is better than in its outside option).
The sequencing decision of firms also depends on their bargaining power (Figure 4). All in all, the release of the DVD might be delayed for intermediate or low values of \( \alpha \): either when the quality of the DVD is not too low compared to the theatrical version, or when the quality differential is very significant, but \( \alpha \) is in the intermediate range.

The previous Proposition identifies the tension between firms over sequencing. In our setting, in order to obtain sequencing, we need quite crucially both the vertical structure and incomplete contracts. From Proposition 1 we know that when \( s \) is high (e.g., \( s = 1 \)), if versions were controlled by a single monopolist (or if the distributor and the exhibitor could write perfectly efficient contracts to replicate the monopolist’s preferred solution) only the high-quality version would be introduced, and thus there would be no room for versioning at all. Instead, Proposition 6 shows that with a vertical structure, even if \( s \) is high, the distributor accepts to introduce the two versions and to delay the introduction of the low-quality version because he cannot fully internalize the profits by selling just one version. These findings are novel in the literature and appear in a setting without externalities and with identical
discount factors between the firms and customers.

Figure 4: Negotiated video window

For simplicity we have derived our results only in the extreme cases of perfectly separate or perfectly substitutable versions. More in general, it is possible to identify a threshold value of \( s \) such that, for values of \( s \) below this level, both exhibitor and distributor prefer the simultaneous introduction of the two versions. Instead, for values above this threshold, while the exhibitor prefers sequencing, the distributor prefers the simultaneous introduction of the two versions and thus versioning may occur.

Notice that we considered that the parties negotiate a linear rental price for each ticket sold by the exhibitor. Linear rental prices are arguably a very simple type of wholesale contract, though this assumption reflects the imperfect rent extraction that the distributor faces with respect to theatres. We also considered alternative contracts, in particular we studied the case where the parties bargain over a revenue share of each ticket receipts.\(^{21}\) This

\(^{21}\)We analyzed an alternative bargaining game where the firms bargain over the revenue share of ticket receipts.
contractual form is potentially more efficient than the linear access charge analyzed above, though we argued there that the latter seems to capture in a better way the incomplete nature of contracts over several aspects. When bargain is over revenue shares, and $s = 0$, of course Proposition 5 is no longer valid. Goods are independent, and there is efficient contracting over the exhibition of movies: The optimal versioning strategy is once again as in Proposition 1, and sequencing does not occur. Instead, when versions are substitutes, the qualitative results of Proposition 6 still go through, even under theatre revenue sharing. Intuitively, when $s = 1$ we found that, when the bargaining power of the distributor is high, it gets a high revenue share and the video is released quickly after the theatrical version. An increase in the bargaining power of the exhibitor, implies instead that the exhibitor gets a combination of a higher revenue share together with a delayed introduction of the video.

6 Conclusions

The main result of this paper is that versioning can happen even for information goods with zero marginal costs and simple uniform distribution of types, when consumers are able to buy the two versions of the product. The key parameter for this finding is the degree of substitutability between versions.

In the case of the movie industry, this result raises an empirical question: To what extent are theatrical and non-theatrical consumer segments overlapping. If DVDs deter people from going to the theater, then versioning is less likely. However, consumers can enjoy consuming the same information goods or cultural products several times and using different versions, because the utility they derive is not lost with repetition, or because consumption of different versions allow them to appreciate different aspects of the same product. If this is the case, theatrical movies and DVDs can be partial substitutes, or even complements, and versioning should occur more often than otherwise found in the literature.

Receipts that accrues to the distributor, $r$, on top of the window, $d$. Formally, in this case the profits of the exhibitor are $\pi^e = (1 - r)p_i q_i u$ and those of the distributor $\pi^d = r p_i q_i u + d q_i L$, where $q_i$ are the market shares of the two versions under various segmentation strategies. Results are available from the authors.
Versioning is also necessary, but not sufficient, for sequencing to occur. The second contribution of our paper is that, in a vertically-separated movie industry, the distributor accepts to supply the two versions if the quality differential between them is not too high, even when the two versions are perfectly substitutes. In addition, if the distributor’s bargaining power is sufficiently low, firms introduce the products sequentially. In this case, intertemporal segmentation can arise due to inefficient vertical contracting.

The possibility of delaying the introduction of a version still raises many questions both at a theoretical and at an empirical level. Waterman et al. (2010) find how the “video window” has been quite stable (around 6 months) between 1988 and 1997, but it has been falling steadily since then, to around 4 months at present. Our model predicts this trend, either when markets are not subject to cannibalization, or when distributors have stronger bargaining power than exhibitors when deciding on the length of the video window. Luan and Sudhir (2007) calculate that the optimal window should be even shorter, around 2.5 months. Their approach, though, considers the profits of an integrated distributor/exhibitor, while we suggest that the vertical separation might be one reason for longer video windows.

Several relevant aspects for the movie industry have not been considered in this paper. More analysis is needed for understanding how spillovers between versions (e.g., marketing campaigns) and word-of-mouth communications affect the commercialization strategy of movie distributors, in particular with respect to the sequencing decision. Piracy is another important problem that could also be analyzed using our simple framework. Pirate copies are themselves a different variant (Sundararajan, 2004b). They should be a very close substitute for DVDs, but a very poor substitute for the theatrical experience.

We have also taken the quality attributes, and in particular the quality ratio $k = u_H/u_L$, as exogenous. It would be interesting to understand the incentives of the firm to alter this ratio, which has a key effect on the versioning and sequencing decisions. It has been suggested, for instance, that lower-rated movies are released faster on DVD than a highly-rated movie. However, it is not clear if, for a given movie, the quality ratio between the theatre and the DVD versions should differ with ratings. We have also considered the level of substitutability between versions as independent from the eventual movie window. In practice, viewers
have imperfect recall, and the longer the time period elapsing between viewing the different versions, the less substitutable versions could become. These are additional aspects that call for further research.

Finally, in this paper we analyzed the case when all the versions are produced by a monopolist, although they can be distributed by different firms. Introducing competition among content producers is also an essential next step we envisage in our future research.
7 Appendix

Proof of Proposition 1. The firm’s problem, when it only offers $H$, is to set the price $p_H$ that maximizes $\pi_H = p_H(1 - \theta_{H0})$. The price that solves this problem is $p_H = u_H/2$ and the firm obtains $\pi_H = u_H/4$.

Next consider the case where the firm offers $L$ to the low segment of consumers, $H$ to the intermediate segment, and $L$ and $H$ to the high segment. It then sets $p_H$ and $p_L$ to maximize:

$$\pi_{LHB} = (p_H + p_L)(1 - \theta_{HB}) + p_H(\theta_{HB} - \theta_{LH}) + p_L(\theta_{LH} - \theta_{L0}).$$  \hfill (1)

Solving this problem, we obtain the following prices:

$$p_L = \frac{(1-s)u_L}{2-s}, \quad p_H = \frac{2u_H - s(u_L + u_H)}{2(2-s)},$$

and the firm’s profits are:

$$\pi_{LHB} = \frac{u_H}{4} + \frac{(2-3s)u_L}{4(2-s)}. \hfill (2)$$

It is simple to verify that $\pi_H < \pi_{LHB}$ for $s < 2/3$. In this range, it also immediate to confirm that at the equilibrium prices, $\theta_{HB} = \frac{1}{2-s} > \theta_{LH} = \frac{1}{2} > \theta_{L0} = \frac{1-s}{2-s}$.

Imagine now that the firm is able to bundle together the two products. In this case, the firm’s problem is to set the price $p_B$ that maximizes $\pi_B = p_B(1 - \theta_{B0})$. Solving this problem we obtain that the optimal price is $p_B = (u_H + u_L(1-s))/2$ and the firm’s profits are $\pi_B = (u_H + u_L(1-s))/4$. Finally, observe that $\pi_B > \pi_{LHB}$ and $\pi_B > \pi_H$ for any value of $s$. Thus, if feasible, the firm would bundle the two versions together. Q.E.D.

Proof of Proposition 2. Following the same steps as in Proposition 1, the firm’s expression for the profit when it only offers $H$ and $c > 0$ is $\pi_H = (p_H - c)(1 - \theta_{H0})$. Maximizing this with respect to the price we obtain $p_H = (c + u_H)/2$ and the corresponding profit $\pi_H = (u_H - c)^2/(4u_H)$. Consumers between $\theta_{H0} = 1/2 + c/(2u_H)$ and 1 buy only $H$, and the others buy nothing. We call this region $A$.

When the firm offers the pattern $L/H/B$, the prices and the associated profits are:

$$p_H = \frac{c + u_H}{2} - \frac{su_L}{2(2-s)}; \quad p_L = \frac{c}{2} + \frac{u_L(1-s)}{(2-s)}.$$

28
\[
\pi_{LHB} = \frac{u_H}{4} + \frac{(2-3s)u_L}{4(2-s)} + \frac{c[c(2-s) - 4(1-s)u_L]}{4(1-s)u_L}.
\] (4)

It can be computed that the value of \(s\) that equals \(\pi_H\) and \(\pi_{LHB}\) is

\[
s^1 = \frac{u_L\{c^2(3-4u_H) + 6cu_H - 5u_Lu_H + [(u_Lu_H - 2cu_H + c^2)(u_Lu_H + 6cu_H + c^2)]^{1/2}\}}{2c^2(u_L - u_H) + u_Lu_H(4c - 6u_L)}.
\] (5)

Thus, as long as \(\theta_{HB} = \frac{1}{2} - \frac{c}{2u_L(1-s)} > \theta_{LH} = \frac{1}{2} > \theta_{L0} = \frac{1}{2} - \frac{c}{2u_L}\), then (3) is a candidate solution. We call this region \(B\). This solution is valid until \(s\) is not too low.

When \(s = s^2 = \frac{2c}{c + u_L}\), then at the prices given by (3) it is the case that \(\theta_{L0} = \theta_{H0} = \theta_{LH}\). This implies that for \(s < s^2\), if the prices are as in (3), the first marginal buyers will choose \(H\) instead of \(L\), since now \(\theta_{H0} < \theta_{L0}\), and no one customer buys \(L\) alone. When this occurs, the firm will choose \(p_H\) and \(p_L\) to maximize:

\[
\pi_{HB} = (p_H + p_L - 2c)(1 - \theta_{BH}) + (p_H - c)(\theta_{HB} - \theta_{H0}).
\] (6)

The prices and the associated profit would be:

\[
p_H = \frac{c + u_H}{2}; \quad p_L = \frac{c + u_L(1-s)}{2},
\]

\[
\pi_{HB} = \frac{u_H + u_L(1-s)}{4} - c + \frac{c^2[(1-s)u_L + u_H]}{4(1-s)u_Hu_L}.
\] (7)

In this region, that we call \(C\), the pattern is \(H/B\) and the indifferent types are \(\theta_{HB} = \frac{1}{2} + \frac{c}{2u_L(1-s)}, \theta_{H0} = \frac{1}{2} + \frac{c}{2u_H}\). This solution holds as long as \(\theta_{HB} \geq \theta_{H0}\), which results in \(s \geq s^3 = 1 - \frac{u_H}{u_L} < 0\). Notice, however, that with these prices some consumers may want to buy \(L\) instead of \(H\). Thus we also have to check that \(\theta_{L0}u_L - p_L \leq 0\) at the prices given by (7). This is satisfied when \(s \leq \hat{s} = c(1/u_L - 1/u_H) < s^2\), and (7) is the solution for \(s^3 \leq s \leq \hat{s}\).

When \(\hat{s} \leq s \leq s^2\), we are still in region \(C\) as the segmentation pattern is \(H/B\), but the prices take a different expression. In particular, the firm sets \(p_L = p_H(u_L/u_H)\) in order to make sure that \(\theta_{L0} = \theta_{LH}\). The prices that satisfy this condition and maximize (6) are

\[
p_H = \frac{u_H}{2} - \frac{u_H[c(s-2) + su_L]}{2[u_L + u_H(1-s)]}, \quad p_L = p_H(u_L/u_H).
\] (8)
In this part of region \( C \) the indifferent types are \( \theta_{\text{HB}} = \frac{1}{2} + \frac{c(2-s)+s(1-s)u_H}{2(1-s)[u_L+u_H(1-s)]}, \) \( \theta_{\text{H0}} = \frac{1}{2} + \frac{c(2-s)-su_L}{u_L+u_H(1-s)}. \)

Finally, when \( s \) is very negative (strong complementarity), all consumers who buy prefer to buy \( B \). For \( s < s^3 \) the firm maximizes the following profit \( \pi_B = (p_H + \delta p_L - 2c)(1 - \theta_{\text{B0}}). \) The optimal price is \( p_H + p_L = c + u_B/2 \) and the firm’s associate profit is \( \pi_B = (u_B - 2c)^2/(4u_B) \), where \( u_B = u_H + u_L(1-s) \). The indifferent type is \( \theta_{\text{B0}} = \frac{1}{2} + \frac{c}{u_B}. \) This is region \( D \).

What remains to be shown is that the four regions are non-empty. We have already shown that \( s^2 > \tilde{s} > s^3 \). We must therefore only discuss when \( 1 > s^1 > s^2 \). The expression for \( s^1 \) given by (5) is a bit cumbersome, but it can be shown to decrease in \( c \) in the relevant range. Thus it takes a maximum when \( c = 0 \), in which case it simplifies to \( s^1 = 2/3 < 1 \). Secondly, still at \( c = 0 \), \( s^2 = \frac{2e}{c+u_L} \) simplifies to \( s^2 = 0 < s^1 \). By continuity, the four regions always exist for sufficiently low levels of \( c \). As \( c \) increases, region \( B \) shrinks, until it disappears when \( s^1 = s^2. ^{22} \) Also notice that, in order for the problem to make economic sense, \( c < u_L \), thus \( s^2 \) is always bounded below 1, and regions \( A, C, D \) are always non-empty. \( \text{Q.E.D.} \)

**Proof of Proposition 3.** The expression for the firm’s profit when it offers the products sequentially is
\[
\pi_{\text{SEQ}} = (p_H + \delta p_L)(1 - \theta_{\text{HB}}) + p_H(\theta_{\text{HB}} - \theta_{\text{LB}}) + \delta p_L(\theta_{\text{LB}} - \theta_{\text{L0}}). \tag{9}
\]

Solving this, we obtain that the optimal prices are
\[
p_H = \frac{u_H}{2} - \frac{\delta su_L}{2(2-s)}; \quad p_L = \frac{(1-s)u_L}{2-s};
\]
and the firm’s profits are
\[
\pi_{\text{SEQ}} = \frac{u_H}{4} - \frac{\delta(3s-2)u_L}{4(2-s)}.
\]

From equation (2) recall that the firm’s profit with simultaneous introduction of the versions when \( 0 \leq s \leq 2/3 \) is \( \pi_{\text{LHB}} = \frac{u_H}{4} + \frac{2(3-s)u_L}{4(2-s)}. \) It is thus \( \pi_{\text{SEQ}} < \pi_{\text{LHB}} \) for all \( \delta < 1 \). Also, recall that the profit of the firm when it only offers \( H \) in the first period is \( \pi_H = u_H/4 \). Thus \( \pi_H > \pi_{\text{SEQ}} \) for \( s > 2/3 \). \( \text{Q.E.D.} \)

\(^{22}\) As a numerical example, when \( u_H = 1 \) and \( u_L = 0.6 \), region \( B \) exists as long as \( c < 0.18 \).
Proof of Proposition 4. We use the same notation as in the previous propositions, despite a slight abuse, as the two versions have now an identical quality. When the firm only offers only one product \((H)\), it maximizes \(\pi_H = p_H(1 - \theta_{H0})\). The firm charges \(p_H = u/2\) and obtains \(\pi_H = u/4\). Consumers between \(\theta_{H0} = 1/2\) and 1 buy \(H\). We call this region \(A\).

Consider now that the firm releases the two versions sequentially: it offers one product \((H)\) in the first period to the intermediate and the high segment of consumers, and a second product \((L)\) in the second period which is appealing to the low and to the high segment of consumers. It then sets \(p_H\) and \(p_L\) to maximize:

\[
\pi_{LHB}^{SEQ} = (p_H + \delta^p p_L)(1 - \theta_{HB}) + p_H(\theta_{HB} - \theta_{LH}) + \delta^p p_L(\theta_{LH} - \theta_{L0}).
\]

The prices that solve this problem, and the corresponding profits, are:

\[
p_L = \frac{(1 - \delta^c)(\delta^c + 3\delta^p)(1 - s)}{\delta^p[4 - 2s - \delta^c(3 - s)] - (\delta^{p2} + \delta^c)^2(1 - s)} u/2, \\
\theta_{HB} = \frac{(1 - \delta^c)(\delta^c + 3\delta^p)}{2\delta^p[4 - 2s - \delta^c(3 - s)] - (\delta^{p2} + \delta^c)^2(1 - s)}, \\
\theta_{LH} = \frac{2\delta^p[2 - s - \delta^c(2 - s)] - (\delta^{p2} + \delta^c)(1 - s)}{2\delta^p[4 - 2s - \delta^c(3 - s)] - (\delta^{p2} + \delta^c)(1 - s)}, \\
\theta_{L0} = \frac{(1 - \delta^c)(\delta^c + 3\delta^p)(1 - s)}{2\delta^p[4 - 2s - \delta^c(3 - s)] - (\delta^{p2} + \delta^c)(1 - s)}.
\]

Next, notice that, if \(s\) is sufficiently small, the firm may prefer releasing both products simultaneously. As products are identical, the firm sets \(p_L = p_H = p\). Consumers will either buy one good (that we arbitrarily denote as \(L\)), or both. The firm then offers the pattern \(L/B\) to maximize:

\[
\pi_{LB}^{SIM} = 2p(1 - \theta_{LB}) + p(\theta_{LB} - \theta_{L0}).
\]
The price that solves this problem is \( p = (1 - s)u/(2 - s) \), and the firm obtains \( \pi_{LB}^{SIM} = (1 - s)u/(2 - s) \). In this region, that we call \( D \), indifferent types are \( \theta_{LB} = \frac{1}{2 - s} \) and \( \theta_{L0} = \frac{1 - s}{2 - s} \). It can now be computed that the value of \( \pi_{LB}^{SIM} = \pi_{LB}^{SEQ} \) is

\[
\hat{s} = \frac{(\delta^2 - 4\delta^p + 3\delta^c\delta^p)}{(\delta^2 - 3\delta^p + 2\delta^c\delta^p)} + \frac{(1 - \delta^c)[(1 - \delta^p)\delta^p(4\delta^p - (\delta^p + \delta^c)^2)]^{1/2}}{(1 - \delta^p)(\delta^2 - 3\delta^p + 2\delta^c\delta^p)}.
\]

Therefore, region \( B \) is in the interval \( \hat{s} < s \leq s^1 \), and region \( D \) is in the interval \( 0 < s \leq \hat{s} \).

Indeed, this is the equilibrium when \( \theta_{HB} > \theta_{LB} \) in region \( B \), which happens when the ratio \( \delta^c/\delta^p \) is high enough.

Instead, when the ratio \( \delta^c/\delta^p \) is small enough, at the prices in (10), there is a \( s \) in region \( B \) that equals \( \theta_{HB} = \theta_{LB} \):

\[
s^2 = \frac{(\delta^p - \delta^c)(1 + \delta^p)}{\delta^p(2 + \delta^p) - 2\delta^p\delta^c - \delta^2}.
\]

This implies that for \( s \leq s^2 \) the second marginal buyer will buy \( B \) instead of \( H \), since now \( \theta_{HB} < \theta_{LB} \), and no consumer buys \( H \) alone. Thus, for \( s \leq s^2 \), the firm offers sequentially the pattern \( L/B \) and sets \( p_L = \frac{(1-s)\mu}{1-\delta^c}\) to make sure that \( \theta_{HB} = \theta_{LB} \). In particular, the firm chooses \( p_H \) and \( p_L \) to maximize:

\[
\pi_{LB}^{SEQ} = (p_H + \delta^c p_L)(1 - \theta_{LB}) + \delta^p p_L(\theta_{LB} - \theta_{L0}).
\]

The prices and the associated profits are:

\[
p_L = \frac{(1-s)[1 + \delta^p(1-s) - s\delta^c]}{[1 + \delta^p(1-s)^2 - s\delta^c]}; \quad p_H = \frac{(1 - \delta^c s)[1 + \delta^p(1-s) - s\delta^c]}{[1 + \delta^p(1-s)^2 - s\delta^c]};
\]

\[
\pi_{LB}^{SEQ} = \frac{[1 + \delta^p(1-s) - s\delta^c]^2 u}{[1 + \delta^p(1-s)^2 - s\delta^c]^2 4}.
\]

In this region, called \( C \), the indifferent types are \( \theta_{LB} = \frac{\delta^p(s-1) + \delta^c s - 1}{2[1 + \delta^p(1-s)^2 - s\delta^c]} \) and \( \theta_{L0} = \frac{(s-1)(\delta^p(s-1) + \delta^c s - 1)}{2[1 + \delta^p(1-s)^2 - s\delta^c]} \).

Finally, we call \( s^3 \) the value of \( s \) that equals \( \pi_{LB}^{SIM} = \pi_{LB}^{SEQ} \), which satisfies:

\[
(s^3 - 1)[4(1 + \delta^p s^3 - 1)^2 - \delta^c s^3] - (s^3 - 2)[\delta^p(s^3 - 1) + \delta^c s^3 - 1]^2 = 0.
\]

Taking this into account, region \( C \) is in the interval \( s^3 < s \leq s^2 \) and region \( D \) is in the interval \( 0 < s \leq s^3 \). The following figure plots the various regions, when \( \delta^p = 0.9 \) and \( \delta^c = 0.7 \), and thus also shows that they are all non-empty. Q.E.D.
Proof of Proposition 5. Consumers can be potentially split between different combinations of products. $\theta_{LH} = (p_H - dp_L)/(u_H - du_L)$ is the consumer that is indifferent between watching the movie through a DVD and watching it in a theater, $\theta_{HB} = dp_L/(u_B - u_H)$ is the consumer that is indifferent between watching the movie in a theater and also buying a DVD on top, and $\theta_{LB} = p_H/(u_H - du_L)$ is the consumer that is indifferent between buying a DVD and also going to the theater in addition. Moreover, $\theta_{i0} = p_i/u_i$ is the consumer that is indifferent between buying the version $i = \{H, L\}$ at the price $p_i = \{p_H, p_L\}$ and not buying anything. Finally, $\theta_{B0} = (p_L + p_H)/u_B$ is the consumer that is indifferent between buying both versions and not buying anything.

When $s = 0$ and the pattern offered by firms is $L/B$, the problem of the exhibitor at $t_0$ is $\max_{p_H} \pi_{LB}^e = (p_H - a)(1 - \theta_{LB})$ and the problem of the distributor is $\max_{p_L} \pi_{LB}^d = dp_L(1 - \theta_{L0}) + a(1 - \theta_{LB})$. Computing the Nash equilibrium in prices and, after substituting...
them in the profits, yields:
\[
\pi^d_{LB} = \frac{2au_H + du_Lu_H - 2a^2}{4u_H}; \quad \pi^e_{LB} = \frac{(u_H - a)^2}{4u_H}.
\]

At time \( t-1 \), the length of the video window and the rental price are determined in a Nash bargaining, where the outside option for the exhibitor is zero, while the outside option for the distributor is to sell \( L \) alone and derive a profit \( u_L/4 \). Thus the pair of firms solve the following maximization problem:
\[
\max_{a,d} \Omega = \left( \frac{\pi^d_{LB} - \frac{u_L}{4}}{\pi^e_{LB}} \right)^\alpha \left( \pi^e_{LB} \right)^{1-\alpha},
\]
where \( 0 < \alpha < 1 \) is the degree of bargaining power of the distributor and \( 1 - \alpha \) is that of the exhibitor.

First, notice that \( \pi^d_{LB} \) depends positively on \( d \), while \( \pi^e_{LB} \) is independent of it. Thus \( d \) is set at its highest possible value, \( d = 1 \). Given this value, the access charge that maximizes \( \Omega \) is simply \( a = \alpha u_H/2 \). With this result we can obtain the retail prices of the proposition.

As goods are independent (\( s = 0 \)) it is clear that these are the maximum profits that the parties can obtain with a linear retail price and \( L/B \) is indeed the optimal pattern. Q.E.D.

**Proof of Proposition 6.** Consider that \( s = 1 \) and firms offer the pattern \( L/H \). At time \( t_0 \) the exhibitor sets \( p_H \) to maximize \( \pi^e_{LH} = (p_H - a)(1 - \theta_{LH}) \) and the distributor sets \( p_L \) to maximize \( \pi^d_{LH} = dp_L(\theta_{LH} - \theta_{L0}) + a(1 - \theta_{LH}) \). Solving these problems and substituting the prices in the profit functions yields:
\[
\pi^d_{LH} = \frac{(u_H - a)[du_L(u_H - du_L) + a(du_L + 8u_H)]}{(4u_H - du_L)^2}, \quad \pi^e_{LH} = \frac{4(u_H - a)^2(u_H - du_L)}{(4u_H - du_L)^2}.
\]

As in Proposition 4, contract terms are established at time \( t-1 \) by maximizing the following expression:
\[
\max_{a,d} \Omega = \left( \frac{\pi^d_{LB} - \frac{u_L}{4}}{\pi^e_{LB}} \right)^\alpha \left( \pi^e_{LB} \right)^{1-\alpha}. \tag{11}
\]

Assume first there is an interior solution with \( 0 < d < 1 \). This solution is characterized by the following FOCs:
\[
\frac{\alpha}{1 - \alpha} \frac{\pi^e_{LB}}{\pi^d_{LB} - \frac{u_L}{4}} = -\frac{\partial \pi^e_{LB}/\partial a}{\partial \left( \pi^d_{LB} - \frac{u_L}{4} \right)/\partial a}, \tag{12}
\]
\[
\frac{\alpha}{1 - \alpha} \frac{\pi^e_{LB}}{\pi^d_{LB} - \frac{u_L}{4}} = -\frac{\partial \pi^e_{LB}/\partial d}{\partial \left( \pi^d_{LB} - \frac{u_L}{4} \right)/\partial d}. \tag{13}
\]
Taking the ratio, the FOCs simplify to:

\[ \frac{\partial \pi_{LB}^*}{\partial a} / \partial (\pi_{LB}^* - \frac{u_H}{k}) = \frac{\partial \pi_{LB}^*}{\partial d} / \partial d' \]

which, after substitutions, results in the following rental charge:

\[ a = \frac{(2k^2 + 8d^*k - d^{*2})u_L}{18k}, \tag{14} \]

where \( k = u_H/u_L > 1 \) and where \( d^* \) is the solution of the following equation, obtained after substituting (14) into (13):

\[ d^{*3} + 9(2 - \alpha)d^{*2}k + 6(3\alpha - 14)d^{*2}k^2 + k^2[8k(9\alpha - 2) + 81(1 - \alpha)] = 0. \tag{15} \]

We do not report the explicit value for \( d^* \) as this involves a long expression, but this solution takes values in the appropriate interval \([0, 1]\) when parameters \( k \) and \( \alpha \) are in a relevant range (see also Figure 3 in the text for a plot of \( d^* \)).

We now discuss what happens when an interior solution does not exist. Take first the corner solution \( d = 1 \) (both versions are released simultaneously). In this case, the rental price that maximizes \( \Omega \) is obtained only from (12), which gives:

\[ a_1 = \frac{2 + 16k^2 - (4k - 1)\alpha + 2ak - [16k(k - 1)(1 - \alpha) + (\alpha + 2ak)^2]^{1/2}}{4 + 32k}u_L. \]

To define the range of validity of this solution, substitute \( d^* = 1 \) into (15), to get

\[ (k - 1)u_L^2[8(9\alpha - 2)k^2 + (9\alpha - 19)k - 1]. \]

Indeed, this expression is positive for any value of \( \alpha > \alpha_1 = \frac{16k^2 + 19k + 1}{4k^2 + 9k} \). This implies that for \( \alpha > \alpha_1 \) we are in a corner solution where \( d = 1 \) and \( a = a_1 \). As \( k = u_H/u_L > 1 \) and \( \alpha_1 \) decreases in \( k \), then \( \alpha_1 \) takes values from 2/9 (when \( k \to \infty \)) to 4/9 (when \( k \to 1 \)).

Take now the corner solution \( d = 0 \). This means that version \( L \) is not released. In this case, the rental price that maximizes \( \Omega \) is obtained only from (12), which gives:

\[ a_2 = \frac{[(2 + \alpha)k - k^{1/2}[k(2 - \alpha)^2 - 8(1 - \alpha)]^{1/2}u_L}{4}. \tag{16} \]

To define the range of validity of this solution, substitute \( d^* = 0 \) into (13), to get

\[ k^2[81 - 16k + 9\alpha(8k - 9)]. \]

35
This expression is always positive for $\alpha > 2/9$, hence this corner solution cannot exist in this range of values. Instead, for lower values of $\alpha < \alpha_2 = \frac{16k-81}{72k-81}$ the corner solution can arise if $k$ is sufficiently high. For $k < 81/16$, it is impossible for $\alpha_2$ to take admissible values. Therefore, $\alpha_2 > 0$ for $k > \frac{81}{16}$ and $\alpha_2 = 0$ otherwise. As $\alpha_2$ is increasing in $k$ then it takes values from 0 (when $k \to \frac{81}{16}$) to $2/9$ (when $k \to \infty$). **Q.E.D.**

**References**


