Abstract—In this paper, we propose a soft-input soft-output stack equalizer for multiple input multiple output (MIMO) frequency selective fading channels. In the literature, the soft or hard input/output stack algorithms to equalize single or multiple antenna time-invariant intersymbol interference (ISI) channels exist. After some modifications of the original sequential decoding metric, we show that the soft-input soft-output stack algorithm can be used at the receiver of the coded systems over frequency selective fading channels. Our examples illustrate that the proposed metrics result in promising near-optimum equalizers while offering a complexity independent of the memory of the channel.

I. INTRODUCTION

Recently, it has been shown that the capacity of wireless communication systems increases considerably with the use of multiple antennas at the receiver and the transmitter [1], [2]. A practical means of achieving this capacity limit is the use of space-time coding (STC) techniques. In [3], [4] STC for frequency selective channels are presented. Space time bit interleaved coded modulation with iterative receivers are presented in [5], [6]. The computational complexity of optimally decoding these codes or equalizing MIMO ISI channels, increases exponentially with the number of transmit antennas and the number of the ISI taps. This may be a significant problem for practical systems, motivating the need for reduced complexity channel equalizers/decoders.

With the motivation of reducing the complexity of equalization over single or multiple antenna frequency selective fading channels, we propose a near-optimum soft-input soft-output stack algorithm as an extension of the work in [7]. We show that with some modifications of the original metric, the soft-input soft-output stack equalization can be used at the receiver of the coded systems over single or multiple antenna frequency selective fading channels. The complexity of the algorithm remains low while offering a bit error rate (BER) performance close to that of the optimal BCJR algorithm [8].

Sequential decoding, which works on the code/channel tree instead of the code/channel trellis, is suboptimal. However, at high signal-to-noise ratios (SNRs), it performs almost the same as the maximum-likelihood (ML) decoding. Sequential decoding algorithm can also be used to equalize fixed ISI channels [10], [11].

A soft-input soft-output sequential decoding method utilizing the metric proposed in [12] is presented in [7]. The new suboptimal soft-input soft-output decoding technique can be used in turbo decoding or iterative equalization/decoding as in [13] where the algorithm is only used for fixed ISI (e.g., magnetic recording) channels. A similar soft-input soft-output sequential decoding algorithm, which uses path augmentation and an auxiliary stack is developed in [14] and used to equalize fixed ISI channels in [15].

Most of the previous works on soft-input soft-output sequential decoding algorithms are not applied to frequency selective fading channels due to the difficulty of the metric computation over these channels. In this work, we modify the soft-input soft-output stack algorithm of [7] to make it suitable for equalization of frequency selective fading channels.

This paper is organized as follows. Section II briefly explains the system model. Section III describes the soft-input soft-output stack equalization for frequency selective fading channels. Some numerical examples are presented in Section IV, and finally, the paper is concluded in Section V.

II. SYSTEM MODEL

Fig. 1 shows the block diagram of the transmitter and the receiver. The information sequence is encoded, interleaved and then divided by a serial-to-parallel converter into several data streams. The resulting data streams are then modulated and transmitted through different antennas simultaneously. The channel is modeled as a quasistatic frequency selective MIMO Rayleigh fading channel where the sub-channels fade independently. We note that this is a relatively general system model which includes the single antenna and/or uncoded systems as special cases. At the receiver, both the received signals from multiple receive antennas and the soft information from the decoder are used by the stack equalizer to produce the soft information about the coded bits. Iterative exchange of the soft information through the interleaver and deinterleaver is performed and finally the decoder provides the decoded sequence.

1This research is funded in part by the NSF CAREER Award CCR-9984237.
The received signal $y_{n,k}$ at the receive antenna $n$, at time $k$ ($k = 1, \ldots, K$) is given as

$$y_{n,k} = \sqrt{\rho/(LM)} \sum_{l=0}^{L-1} \sum_{m=1}^{M} h_{m}^{n}(l)u_{m,k-l} + w_{n,k}$$  \hspace{1cm} (1)

where $u_{m,k}$ is the transmitted symbol from antenna $m$, $M$ is the number of transmit antennas, $N$ is the number of receive antennas, and $L$ is the total number of ISI taps. $w_{n,k}$ is the noise sample at the receive antenna $n$, and it is i.i.d. complex Gaussian random variable having zero mean and variance $1/2$ per dimension. $h_{m}^{n}(l)$ is the channel coefficient between transmit antenna $m$ and receive antenna $n$ from the $t^{th}$ ISI tap. Channel coefficients are spatially independent zero mean complex Gaussian random variables and remain constant throughout the frame, i.e., the channel is quasistatic fading. $\rho$ is the average SNR at each receive antenna. Clearly, the case with $M = N = 1$ corresponds to single antenna frequency selective fading channel.

III. SOFT-INPUT SOFT-OUTPUT STACK EQUALIZATION FOR FREQUENCY-SELECTIVE FADING CHANNELS

In the literature, soft/hard input and soft/hard output stack algorithms are proposed to decode time-invariant single or multiple antenna ISI channels. In this section, we first describe the soft-input soft-output sequential decoding algorithm of [7] briefly and then we modify the metric of the algorithm to be able to equalize frequency selective fading channels in a computationally efficient way.

In sequential decoding, the objective is to find a path through the tree describing the code or the channel by using a metric which allows comparison of paths at different depths of the tree [9]. In [7], the soft-input soft-output twin stack decoder is developed based on the conventional stack decoder using the following metric [12],

$$\sum_{j=1}^{c}\log\left(\frac{Pr(y_{j}^{k}|v_{j}^{k})}{Pr(y_{j}^{k})}\right) + \sum_{j=1}^{b}\log(Pr(u_{j}^{k}))$$  \hspace{1cm} (2)

where $y_{j}^{k} = \{y_{j}^{k}\}, j = 1, 2, \ldots, c$ is the received symbol at time $k$, $c$ is the number of output bits from the encoder, $u_{j}^{k}, \ldots, u_{b}^{k}$ are the input bits to the encoder at time $k$, $b$ is the number of input bits to the encoder. The transmission rate is $b/c$. $v_{j}^{k}$ is the output symbol corresponding to $j^{th}$ bit of the encoder at time $k$. The second term in (2) contains the a priori probabilities of the input bits. Since we want to use the stack algorithm to equalize frequency selective fading channels, in the sequel, we drop the index $j$ since $b = c = 1$ for the channel tree. Therefore, $u_{k}$ is the transmitted symbol through the ISI channel, $v_{k}$ is the noiseless ISI channel output depending on the transmitted inputs $u_{k}, u_{k-1}, \ldots, u_{k-(L-1)}$, and $y_{k}$ is the received signal corresponding to $v_{k}$. For ease of tracking, we only provide the metric for single antenna case as the metric expression for MIMO case is straightforward [11].

In order to compute the log-likelihood ratio (LLR) of the transmitted information bits, the BCJR algorithm (without the “gamma” metric, $\gamma$) computation part in [8]) can be utilized [7]. Following the notation in [8], the branch metric $\gamma(y_{k}, m, m') = Pr(y_{k}, S_{k} = m|S_{k-1} = m')$ where $S_{k} = m$ is the state occupied at time $k$, can be written as

$$\gamma(y_{k}, m, m') = Pr(y_{k}|v_{k}) \cdot Pr(S_{k} = m|S_{k-1} = m').$$  \hspace{1cm} (3)

The metric in (2) can be rewritten as

$$\gamma_{f} = \frac{Pr(y_{k}|v_{k})}{Pr(y_{k})} \cdot Pr(S_{k} = m|S_{k-1} = m').$$  \hspace{1cm} (4)

It can be shown that the normalized $\gamma$ values in (3) and (4) are identical for both the BCJR and the soft-input soft-output stack algorithms. Therefore, both $\gamma$ values can be used to decode the convolutional codes [7]. Note that, although the stack algorithm works on the code (or channel) tree, each branch of the code tree corresponds to a transition in the code (or channel) trellis. For details of the soft output computation in the stack decoding algorithm, we refer the reader to [7]. The overall complexity reduction comes from the reduced number of computations for branch metrics ($\gamma$) and other intermediate variables (e.g., $\alpha, \beta$ in [16]) of the BCJR algorithm.

Unlike channel encoders, the number of output symbols for ISI channels depends on the memory of the channel. Therefore, the computation of $Pr(y_{k})$ in the metric (2) requires the computation of $Pr(y_{k}|v_{k})$ for all possible values of $v_{k}$ at each time interval,

$$Pr(y_{k}) = \sum_{v_{k}} Pr(y_{k}|v_{k})Pr(v_{k}).$$  \hspace{1cm} (5)

Due to this computation, the complexity of the soft-input soft-output stack equalizer depends on the memory of the channel (we note that $Pr(v_{k})$ can be taken as constant). For fixed ISI channels, this complexity increase for stack equalizers can be overcome by the preparation of a table for $Pr(y_{k})$ which depends on the limited number of noiseless channel.
outputs, $v_k$. However, for frequency selective fading channels, fading coefficients change in time, and thus, preparation of the $Pr(y_k)$ table at each received frame becomes a more challenging task. We now explain our proposals for adapting the metric in (2) for frequency selective fading channels.

A. First Method: Table Look-up

We first propose the use of a look-up table to obtain an approximate $Pr(y_k)$ value in the metric (2) instead of computing the exact value which requires a high complexity, especially for long channels. In this technique, one can prepare a table for $Pr(y_k)$ for input $y_k$, with the desired resolution depending on the number of channel taps and SNR values. The table can be obtained by using Monte Carlo techniques, in which the averaging is done over randomly generated channel coefficients, noise samples and all possible input combinations. For MIMO channels, we note that the table size increases exponentially with the number of receive antennas. Since the table is prepared only once, the complexity of the stack equalizer is still independent of the number of states in trellis, and no additional real time computations are necessary.

B. Second Method: Constant Bias

To alleviate the problem of computing $Pr(y_k)$, we also propose a new metric by removing the $Pr(y_k)$ term from the original metric shown in (2). We note that this operation does not affect the generation of the soft output. This is explained as follows. The log likelihood ratio of the input bit $u_k$ at time $k$ can be written as

$$L(u_k) = \log \left( \frac{Pr(u_k = +1|y)}{Pr(u_k = -1|y)} \right)$$  \hspace{1cm} (6)

where $y$ is the received sequence. In the modified BCJR algorithm [16], the LLR is written as

$$L(u_k) = \log \frac{\sum_{S^+} Pr(y_k, S_k = m, S_{k-1} = m')/Pr(y)}{\sum_{S^-} Pr(y_k, S_k = m, S_{k-1} = m')/Pr(y)}$$  \hspace{1cm} (7)

where $S^+$ and $S^-$ show the transitions for input bits +1 and -1 respectively. In [16], during the design of the algorithm, the $Pr(y_k)$ in $Pr(y)$ term is canceled. Therefore, removing the $Pr(y_k)$ term in the metric of the soft-input soft-output stack algorithm does not introduce any problems on the computation of the soft output values. With the metric without $Pr(y_k)$ term in (2), we have found that the soft-input soft-output stack equalizer performs very close to the maximum a-posteriori (MAP) equalizer while the complexity becomes comparable to that of the MAP equalizer. We note that in order for the sequential decoding algorithms to work, the metrics of the correct paths should increase and those of the incorrect paths should decrease. Therefore, it can be inferred that the removal of the $Pr(y_k)$ term requires addition of a bias term independent of the number of branches in the channel trellis.

In [17], using the general properties of the Fano metric, a new metric which consists of the Euclidean distance term, $\log(Pr(y_k|v_k))$, and a bias term depending on the SNR, is developed for using sequential decoding algorithm as equalizer. Similarly, in [18], a new metric consisting of a constant bias and Euclidean distance is used to decrease computational complexity. After examining these metrics, we propose to use

$$\log(Pr(y_k|v_k)) + \log(Pr(u_k)) + B$$  \hspace{1cm} (8)

as the metric for the soft-input soft-output stack algorithm over frequency selective fading channels. After extensive simulations with different $B$ values, such as, a multiple of noise variance or expected value of the $\log(Pr(y_k))$, we found that a constant value for $B$ performs the best in terms of providing near-optimum BER performance while having much lower computational complexity than optimal BCJR algorithm as we show in Section IV.

The soft-input soft-output stack equalization with any of the proposed metrics can be used for MIMO systems with minor changes on the channel trellis, $\gamma$ and LLR computations. For example, when 2 transmit and 2 receive antennas are used, the $\gamma(Y_k, m, m')$ for received samples $Y_k = [y_{1,k} y_{2,k}]$ at time $k$ is given as

$$Pr(Y_k|v_k) \cdot Pr(S_k = m|S_{k-1} = m')$$  \hspace{1cm} (9)

which can also be written as

$$Pr(u_{1,k}, u_{2,k}) \cdot Pr(y_{1,k}|S_k, S_{k-1}) \cdot Pr(y_{2,k}|S_k, S_{k-1})$$

$$\approx Pr(y_{1,k}|v_k) \cdot Pr(y_{2,k}|v_k) \cdot Pr(u_{1,k}) \cdot Pr(u_{2,k})$$

where the last approximation holds because the received signals from two receive antennas are independent, and the coded bits are interleaved. The LLR for the coded bit transmitted from antenna 1, $L(u_{1,k})$, is

$$\log \frac{Pr(u_{1,k} = +1, u_{2,k} = -1|Y_k) + Pr(u_{1,k} = +1, u_{2,k} = +1|Y_k)}{Pr(u_{1,k} = -1, u_{2,k} = -1|Y_k) + Pr(u_{1,k} = -1, u_{2,k} = +1|Y_k)}.$$  \hspace{1cm} (10)

The generalization of the LLR and the metric computation for $M \times N$ MIMO system with any signal constellation is straightforward.

C. Complexity

We define the complexity as the average number of extended branches (or the number of metrics computed) per decoded bit. For MIMO systems with binary phase shift keying (BPSK) signalling, the number of branches coming out of any node in the channel tree or from any state in channel trellis is $2^M$. The number of states in channel trellis at any time instant is $2^{(L-1)M}$ where $L$ is the number of ISI taps. At any section of the channel trellis, the total number of branches is $2^M \cdot 2^{(L-1)M} = 2^{ML}$. Since we transmit $M$ bits, the complexity of the optimal MAP equalizer per decoded bit is $2^{ML}$, which makes it impractical for large number of ISI taps and transmit antennas.

Contrary to the optimal MAP equalizer, at high SNR values, the complexity of the soft-input soft-output stack equalizer with the proposed metrics is $2^M$ which is independent of the channel memory. That represents a considerable reduction of
the complexity compared to that of the MAP equalizer. For this reason, the stack equalizer with the proposed metrics is quite appealing for bit interleaved coded systems over MIMO frequency selective fading channels. Obviously, using large signal constellations to obtain higher data rates will result in a higher complexity which motivates the use of the stack equalizer further since the complexity reduction will be larger.

IV. Examples

In this section, we present the performance of the proposed methods using several examples and compare it with the MAP equalizer (using the BCJR algorithm). In the simulations, quasistatic fading channel model with a frame length of 100 BPSK symbols is used. Variances of the fading coefficients for all the taps are assumed to be identical (uniform power profile). Random interleavers are used in iterative equalization/decoding cases. For coded systems, the encoder uses (5,7) (in octal notation) non-systematic convolutional code and the decoder utilizes the BCJR algorithm. In the simulations the average complexity per decoded bit is depicted. Since the performance results for single and multiple antenna cases are quite similar, we provide only single antenna examples for the table look-up method and only MIMO case results for the constant bias method.

A. Table Look-up Method

Fig. 2 shows the BER performance of the soft-input soft-output stack equalizer for uncoded transmission over quasistatic frequency selective fading channel having 6 taps with a uniform power profile. $YRES$ in the figure shows the resolution used in table and $YMAX$ shows the maximum value of the received signal magnitude, $|y_k|$. Fig. 3 shows the computational complexity for this case. We observe that the table look-up method performs close to the original (exact $Pr(y_k)$) metric case while requiring much less computation compared to the MAP equalizer.

B. Constant Bias Method

Fig. 4 shows the BER performance and Fig. 5 shows the computational complexity of soft-input soft-output stack equalizer for bit interleaved coded transmission over MIMO frequency selective quasistatic fading channel having 4 taps, with 2 transmit and 2 receive antennas. Increasing the bias value $B$ increases the BER and decreases the complexity. At a BER of $10^{-4}$, the stack equalizer with $B = 3$ computes only 2 metrics for each decoded bit while its performance is 2 dB worse than that of the MAP equalizer which computes 128 metrics for each decoded bit. This performance loss is due to the reduced reliability of the soft output values from the proposed suboptimal algorithm. Furthermore, at each iteration, the complexity decreases for the proposed equalizer which is another advantage over MAP equalization where the complexity is constant at each iteration. Hence, at high SNRs and in subsequent iterations, the receiver using stack equalizer
can obtain the output much faster compared to the receiver using the MAP equalizer. We also note that although not shown, the bias value, $B$ remains almost the same for different lengths of the channel for reasonable BER performance and complexity.

V. CONCLUSIONS

We showed that the soft-input soft-output stack equalizer with the two proposed metrics can be used as a low complexity, near-optimum equalizer over frequency selective fading channels. The table look-up method provides a similar performance with the case using the original metric with a much lower complexity while requiring a reasonable memory space. In the second method, the new metric uses a constant bias, thus, does not require preparation of a table. Unlike the optimal MAP equalizer, the complexity of the stack equalizer with the proposed metrics does not depend on the memory of the channel which makes the soft-input soft-output stack equalizer quite desirable for channels having long delay spreads. Furthermore, we also showed that the modified soft-input soft-output stack equalization methods are effective low complexity techniques for coded transmission schemes over MIMO frequency selective fading channels where the complexity of optimal equalization increases exponentially with the number of transmit antennas and the ISI taps. The simulation results also show that the complexity of the new equalizer decreases with subsequent iterations which is another advantage over the MAP equalizer.

REFERENCES