A recurrent neural network for optimal real-time price in smart grid

Xing He a,⁎, Tingwen Huang b, Chuandong Li a, Hangjun Che a, Zhaoyang Dong c

a School of Electronics and Information Engineering, Southwest University, Chongqing 400715, PR China
b Department of Mathematics, Texas A&M University at Qatar, Doha, P.O.Box 23874, Qatar
c School of Electrical and Information Engineering, The University of Sydney, Sydney 2006, Australia

ARTICLE INFO

Article history:
Received 30 April 2014
Received in revised form
28 June 2014
Accepted 9 August 2014
Communicated by He Huang
Available online 19 August 2014

Keywords:
Recurrent neural networks
Smart grid
Differential inclusion

ABSTRACT

Recently, some algorithms have been proposed for optimal real-time price in smart grid based on optimization theory. In this paper, a recurrent neural network modeled by means of a differential inclusion is proposed for solving this problem. Compared with the existing algorithms, recurrent neural network as parallel computational models for real-time optimization and applications have received substantial attention in the literature. Our model has the least number of state variables and simple structure. Using nonsmooth analysis, the theory of differential inclusions and Lyapunov-like method, the equilibrium point of the proposed neural networks can converge to an optimal solution of optimal real-time price under certain conditions. Finally, simulation results on two numerical examples show the effectiveness and performance of the proposed neural network.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

Recently, smart grid has received much attention, which is a vision of the future power electrical grid, and it uses information and communication technology to gather and act on information about the behaviors of suppliers and consumers, in an automated fashion to improve the efficiency, reliability, economics, and sustainability of the production and distribution of electricity. Due to the importance of smart grid, more and more researchers focus this subject from various background. In [4], McArthur et al. have proposed multi-agent system to control elements of smart grid, most notably for large area power networks. Wireless sensor networks were proposed to play a key role in the extension of the smart grid towards residential premises [5]. In power grid complex network evolutions for the smart grid, Pagani and Aiello [6] investigated how different network topologies and growth models facilitate a more efficient and reliable network.

Demand Side Management (DSM) is a key element of smart grid, which provides support towards smart grid functionalities in various areas such as electricity market control and management, infrastructure construction, and management of decentralized energy resources and electric vehicles. In recent studies on various mechanisms designed for DSM, Samadi et al. [1] proposed a dynamic strategy, which is an effective approach that aims to reduce the peak demand, and the distributed sub-gradient algorithm was proposed for solving this model. Since the significant study, many similar models are proposed for discussing the real-time price in DSM. For instance, based on [2], Tarasak considered uncertainty optimization constraints. In [3], Gatsis and Giannakis considered cooperative multi-residence demand scheduling approach. For these optimization problems, the distributed algorithm based on Lagrangian multiplier is used to solve. However, on one hand, it suffers from a slow convergence or no convergence as the dual function is non-differential. On the other hand, in many applications of DSM optimization problem in smart grid, real-time solutions are often needed. For solving large-scale optimization problem and real-time application, neural networks based on circuit implementation are more competent. In the last two decades, recurrent neural networks for optimization and their engineering applications have been widely discussed [7–21].

In this paper, motivated by the effectiveness and efficiency of neural network optimization method, we apply effective and novel recurrent neural network methodology to optimal real-time price in smart grid, and recurrent neural network is constructed by a nonsmooth penalty approach, which aimed to solve problem in real time. Compared with the existing algorithms for this problem, recurrent neural network as parallel computational models for real-time optimization and applications is more effective, and our model has the least number of state variables and simple structure. Using nonsmooth analysis, the theory of differential inclusions and Lyapunov-like method, the equilibrium point of the proposed neural networks can converge to an optimal solution of optimal real-time price under certain conditions.
The remainder of this paper is organized as follows. In the next section, the preliminaries relevant optimal real-time price strategy and the neural network model formulation are given. In Section 3, the performance of the proposed neural network is analyzed. In Section 4, two numerical examples are simulated to verify the theoretical analysis. Finally, some conclusions are stated in Section 5.

2. Problem formulation and neural network description

In this section, the real-time price model of smart grid is introduced, then a corresponding recurrent neural network is proposed.

Consider a smart power system consisting of a single energy provider, several load subscribers and a regulatory authority. Let Ω and A denote the set of all time slots and the set of all users, respectively. For each user \( i \in A \) and each time slot \( k \in \Omega \), let \( x_i^k \) denote the amount of power consumed by user \( i \) in time slot \( k \). For all users, let \( U(x_i^k, \omega_i^k) \) represent the corresponding utility function, where \( \omega_i^k \) is a parameter which may vary among users and also at different times of the day. For each time slot \( k \in \Omega \), let \( C_i(k) \) indicate the cost of providing \( L_k \) units of energy offered by the energy provider, where \( L_k \) denotes the generation capacity in each time slot \( k \). For the interactions between the users and the energy provider, the price optimal model of smart grid in real time is formulated as follows:

\[
\text{maximize } \sum_{k \in \Omega} \sum_{i \in A} U(x_i^k, \omega_i^k) - \sum_{k \in \Omega} C_i(k)
\]

s.t.

\[
\begin{align*}
M_i^k &\leq x_i^k & \leq M_i^k \\
L_k^\text{min} &\leq L_k & \leq L_k^\text{max} \\
\sum_{i \in A} x_i^k &\leq L_k & \forall k \in \Omega
\end{align*}
\]

(1)

where \( M_i^k \) and \( M_i^k \) denote the minimum and the maximum power consumption of user \( i \), respectively, and we define \( L_k^\text{min} = \sum_{i \in A} M_i^k \) and \( L_k^\text{max} = \sum_{i \in A} M_i^k \). In the following, we will give some assumptions about utility function \( U(x_i^k, \omega_i^k) \) and cost function \( C_i(k) \).

Assumption 1. \( \partial U(x_i^k, \omega_i^k) / \partial \omega_i^k \geq 0 \), which means that users are always interested to consume more power until they reach their maximum consumption level.

Assumption 2. \( \partial^2 U(x_i^k, \omega_i^k) / \partial (\omega_i^k)^2 \leq 0 \), which indicates that the level of satisfaction for users can gradually get saturated.

Assumption 3. \( \partial U(x_i^k, \omega_i^k) / \partial \omega_i^k > 0 \), which means that the customers are categorized for their utility function, and the parameter \( \omega_i^k \) represents the willingness of participating the demand side management.

Assumption 4. \( U(0, \omega) = 0 \), \( \forall \omega > 0 \), which implies that no power consumption brings no benefit.

Various utility functions can be used as long as they satisfy Assumptions 1–4. In this paper, we consider the following utility functions:

\[
U(x, \omega) = \begin{cases} 
\alpha x - \frac{\alpha}{2} x^2, & 0 \leq x \leq \frac{\omega}{\alpha} \\
\frac{\omega^2}{2\alpha}, & x > \frac{\omega}{\alpha}
\end{cases}
\]

and

\[
U(x, \omega) = \begin{cases} 
\alpha x - \frac{\alpha}{4} x^2, & 0 \leq x \leq \frac{\omega}{\alpha} \\
\frac{3\omega^2}{4}, & x > \frac{\omega}{\alpha}
\end{cases}
\]

where \( \alpha > 0 \) is a parameter.

Assumption 5. The cost function \( C_i(k) \) are increasing in the offered energy capacity, and are strictly convex.

In this paper, we consider the following cost functions, which satisfy Assumption 5:

\[
C_i(k) = a_i L_k^2 + b_i L_k + c_i
\]

and

\[
C_i(k) = \begin{cases} 
a_i L_k + b_i, & L_k \leq \delta \\
b_i L_k + c_i, & L_k > \delta
\end{cases}
\]

where \( a_i, b_i, c_i, a_i, b_i, c_i, \delta \) are positive parameters.

To develop the recurrent neural network for solving problem (1) and simplify our discussion, we introduce the following notations:

\[
x = (x_1^1, x_2^2, \ldots, x_n^N, x_1^2, x_2^2, \ldots, x_n^N, \ldots, x_1^N, x_2^N, \ldots, x_n^N)^T,
\]

\[
m = (m_1^1, m_1^2, m_2^2, \ldots, m_1^N, \ldots, m_N^N)^T,
\]

\[
M = (M_1^1, M_1^2, M_2^2, \ldots, M_1^N, \ldots, M_N^N)^T,
\]

\[
L = (L_1, L_2, \ldots, L_K)^T,
\]

\[
L^{\text{min}} = (L_1^{\text{min}}, L_2^{\text{min}}, \ldots, L_K^{\text{min}})^T,
\]

\[
L^{\text{max}} = (L_1^{\text{max}}, L_2^{\text{max}}, \ldots, L_K^{\text{max}})^T.
\]

where \( N \) is the number of the users in smart grid, and \( K \) is the maximize time slot in \( \Omega \).

Then problem (1) can be rewritten as follows:

\[
\text{min } f(x, L)
\]

\[
m \leq x \leq M
\]

\[
L^{\text{min}} \leq L \leq L^{\text{max}}
\]

\[
(L_{K,K} \otimes I_N^{1 \times 1}) x \leq L
\]

(2)

where \( \otimes \) is the Kronecker product, \( I_N = [1, 1, \ldots]^{1 \times N} \), \( L_{K,K} \) denotes the \( m \times m \) identity matrix. and \( f(x, L) = - \sum_{k \in \Omega} \sum_{i \in A} U(x_i^k, \omega_i^k) - \sum_{k \in \Omega} C_i(k) \). Introducing \( y = (x^T, L^T)^T \), problem (1) can also be rewritten as follows:

\[
\text{min } f(y)
\]

\[
y^{\text{max}} - y \geq 0
\]

\[
y - y^{\text{min}} \geq 0
\]

\[
g(y) \geq 0
\]

(3)

where \( y^{\text{max}} = (M^T, (L^{\text{max}})^T)^T \), \( y^{\text{min}} = (m^T, (L^{\text{min}})^T)^T \) and \( g(y) = L - (L_{K,K} \otimes I_N^{1 \times 1}) x \).

To solve problem (3), based on generalized nonlinear programming circuit, the recurrent neural network proposed in [16] is described as the following differential inclusion:

\[
y'(t) \in -sf(y) + \sigma[d(y^{\text{max}} - y) - d(y - y^{\text{min}})] d(y) dy(y)
\]

(4)

where \( y \in \mathbb{R}^{(N+1)K} \) is the state variable, \( sf \) is Clarke’s generalized gradient of \( f \), \( \sigma \) is a parameter, and \( d(\rho) = (d_1(\rho), d_2(\rho), \ldots, d_{N+1}(\rho)) \).
is defined as follows:
\[ d(\rho) = \begin{cases} 
0, & \rho > 0 \\
[1 - 1, 0], & \rho = 0 \\
-1, & \rho < 0 
\end{cases} \]

**Remark 1.** In the literature, some algorithms based on the Lagrangian multiplier are proposed to solve problem (1). Compared with these methods, based on the penalty function, neural network model (4) has the least number of state variables and simple structure. From reference [16], under certain conditions, the equilibrium points of system (4) satisfy the solution of problem (1). In the next section, we will discuss these conditions.

### 3. Theoretical analysis

In this section, the convergence and optimality of the proposed neural network will be studied in detail.

Due to the discontinuity of function \( d(\rho) \), the solution of system (4) is defined in the sense of Filippov, and \( y^* \in R^{(N+1)k} \) is an equilibrium point of system (4) if the following condition is satisfied:

\[ 0 \in F^*(y^*) = -df(y^*) + \sigma(d(y^{\max} - y^*) - d(y^* - y^{\min}) - d(g(y^*)i\bar{a}(y^*))) \]

and we say that system (4) is said to be globally convergent to an equilibrium point, if for any solution \( y(t) \) with the initial conditions \( y_0 \), there exists an equilibrium \( y^* \) such that \( \lim_{t \to +\infty} y(t) = y^* \). From Assumptions 1-4, we obtain that \( U(x^0, \alpha^0) \) is a concave function. Therefore, due to the convexity of \( f(y) \), the solution of neural network (4) has a unique solution. Let \( S \) denote the constraint set of problem (3), and \( \Phi \) denotes the set of global minimizers of problem (3).

**Theorem 1.** For any initial point \( y_0 \in R^{(N+1)k} \), the proposed neural network is globally convergent to an equilibrium point \( y^* \).

**Proof.** Consider the following energy function:

\[
E(y) = f(y) + \sigma \sum_{i=1}^{(N+1)k} \left\{ \int_0^{y^{\max} - y^*} d(\rho) \, d\rho + \int_0^{y^* - y^{\min}} d(\rho) \, d\rho \right\} + \int_0^{\bar{a}(y)} d(\rho) \, d\rho
\]

Differentiating \( E^*(y) \) along the solution of neural network (4), by choosing \( \xi = -\tilde{y}(t) \in F^*(y^*) \), For a.a. \( t \in [0, +\infty) \), we have

\[
dt E^*(y) = \xi^T \tilde{y} = -||y(t)||^2 \leq 0
\]

Therefore, the proposed neural network is globally stable.

Assume that \( \tilde{y} \) is an interior point of \( S \), consider a large sphere \( B(\tilde{y}, r) = \{y \in R^{(N+1)k} : \|y - \tilde{y}\|_2 \leq r \} \), such that \( S \subset B(\tilde{y}, r) \). To study the performance of neural network (4) for optimal real-time price in smart grid, we have the following results.

**Theorem 2.** If \( \sigma > h_1(r)h_2(r) \), given any \( y_0 \in B(\tilde{y}, r) \), there is a unique solution \( y(t) \) from \( y_0 \) at \( t_0 \), and is such that \( y(t) \in B(\tilde{y}, r) \) for \( t \geq 0 \). Moreover, \( \lim_{t \to +\infty} \text{dist}(y(t), \Phi) = 0 \), where

\[
h_1(r) = \max_{y \in B(\tilde{y}, r)} \left\{ \max_{1 \leq j \leq (N+1)k} \|y_j\|_2 \right\} < +\infty,
\]

\[
h_2(r) = \max_{y \in B(\tilde{y}, r)} \left\{ \frac{\|y - \tilde{y}\|_2}{\int_{m+B(y)}} \right\},
\]

\[
f_m = \min\{y^{\max} - \tilde{y}, (\tilde{y} - y^{\min}), (\bar{a}(\tilde{y}))\}.
\]

\[
B(y) = \left\{ \int_0^{y^{\max} - y^*} d(\rho) \, d\rho + \int_0^{y^* - y^{\min}} d(\rho) \, d\rho \right. \
+ \left. \int_0^{\bar{a}(y)} d(\rho) \, d\rho \right\}
\]

**Theorem 3.** Under the condition of Theorem 2, there exists \( \eta_1 > 0 \), such that

\[
\inf_{y \in S \cap \Phi \left\{ \min \|y\|_2 \right\}} > \eta_1
\]

Then, the proposed neural network is convergent to \( \Phi \) in finite time \( T_c \), where

\[
T_c = \inf\{t \geq 0 : y(t) \in \Phi, t \geq t_1\}
\]

and it can be estimated by

\[
T_c \leq \frac{\|\tilde{y}(t)\|}{\min(\eta_1, \eta_2)}
\]

where

\[
\eta_2 = \frac{\sigma - h_1(r)h_2(r)}{h_2(r)}
\]

**Remark 2.** Using nonsmooth analysis, the theory of differential inclusions and Lyapunov-like method, the details of the proof in Theorems 2 and 3 are similar with that of references [12,16]. Therefore, the proof is omitted. In this paper, the objective function of the price optimal model is nonsmooth, and nonsmooth optimization is certainly of interest in application of smart grid. From Theorems 2 and 3, it can be seen that the state variables of neural network (4) are not only convergent to the optimal solution, but also convergent in finite time, which is very important in engineering applications.

### 4. Numerical simulations

In this section, two examples are given to demonstrate the effectiveness of the proposed recurrent neural network for optimal real-time price in smart grid.

**Example 1.** Consider that there are three users in smart power system, and the entire time cycle is divided into 3 time slots representing the peak hours, mid-peak hours and off-peak hours. From 0:00 am to 7:59 am and from 22:00 pm to 23:59 pm are off-peak hours, and from 8:00 am to 11:59 am and from 14:00 pm to 16:59 pm are mid-peak hours, and from 12:00 am to 13:59 pm and from 17:00 pm to 21:59 pm are peak hours. The cost function representing the peak hours, mid-peak hours and off-peak hours. Example 1.

**Example 2.** Consider that there are three users in smart power system, and the entire time cycle is divided into 3 time slots representing the peak hours, mid-peak hours and off-peak hours. From 0:00 am to 7:59 am and from 22:00 pm to 23:59 pm are off-peak hours, and from 8:00 am to 11:59 am and from 14:00 pm to 16:59 pm are mid-peak hours, and from 12:00 am to 13:59 pm and from 17:00 pm to 21:59 pm are peak hours. The cost function representing the peak hours, mid-peak hours and off-peak hours.
Example 2. We assume that there are $N = 200$ users, and the entire time cycle is divided into 24 time slots representing the 24 h of the day. In the off-peak hour, $m_k^i$ and $M_k^i$ are set randomly from interval $[\frac{1}{5}C_{138}^i, \frac{10}{5}C_{138}^i]$ respectively. In the mid-peak hours, $m_k^i$ and $M_k^i$ are selected randomly from interval $[\frac{5}{5}C_{138}^i, \frac{14}{5}C_{138}^i]$ respectively. In the peak hours, $m_k^i$ and $M_k^i$ are given randomly from interval $[\frac{15}{5}C_{138}^i, \frac{20}{5}C_{138}^i]$ and $[\frac{20}{5}C_{138}^i, \frac{45}{5}C_{138}^i]$, respectively. The utility functions $U(x, \omega)$ and cost functions $C_k(L_k)$ are given as follows:

$$U(x, \omega) = \begin{cases} \omega x - \frac{\alpha}{4} x^4, & 0 \leq x \leq \sqrt[3]{\frac{\omega}{\alpha}} \\ \frac{3\omega}{4} \sqrt[3]{\frac{x^4}{\alpha}} & x > \sqrt[3]{\frac{\omega}{\alpha}} \end{cases}$$

and

$$C_k(L_k) = \begin{cases} \alpha L_k + b_L, & L_k \leq \delta \\ \delta L_k + \delta b_L, & L_k > \delta \end{cases}$$

where $\delta = 1000$, $\alpha = 0.5$, $a_k = 0.1$, $b_k = 0.2$, $\omega$ is selected randomly from the interval $[1, 6]$ and $\sigma = 15$. Fig. 3 shows the optimal energy consumption of all users in each slot.

5. Conclusion

Based on the optimization theory, a recurrent neural network has been applied to solve real-time price problem in smart grid. Compared with the existing results in the literature, neural network as parallel computational models for real-time optimization and applications is very effective, and the model has the least number of state variables and simple structure. The theoretical results imply that it satisfies the conditions of a recurrent neural network for optimal real-time price in smart grid. Furthermore, simulation results are used to demonstrate the theoretical results. In the near future, we will try to apply recurrent neural network for solving other DSM optimization problems in smart grid.

Acknowledgments

This work is supported by Fundamental Research Funds for the Central Universities (Project No. XDJK2014C118, SWU114007) and Natural Science Foundation of China (Grant nos: 61403313, 61374078), and also supported by the Natural Science Foundation Project of Chongqing CSTC (Grant no. cstc2014jcyjA40014). This publication was made possible by NPRP Grant no. NPRP 4-1162-1-181 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.
References


Tingwen Huang is a Professor at Texas A&M University at Qatar, Doha, Qatar. He received his B.S. degree from Southwest Normal University (now Southwest University), China, in 1990, his M.S. degree from Sichuan University, China, in 1993, and his Ph.D. degree from Texas A&M University, College Station, Texas, in 2002. He has expertise in chaotic dynamical systems, neural networks, optimization and control, traveling wave phenomena. He has published more than 70 peer reviewed journal papers. He is one of the editors for the 5 volumes of the Proceedings of the 19th International Conference on Neural Information Processing published in Lecture Notes in Computer Science by Springer. Also, he is one of the editors for a book Advances in Intelligent and Soft Computing published by Springer. His research on chaotic dynamical systems received Qatar National Priority Research Project support from Qatar Research Fund. Now, he serves as an editorial board member for four international journals: IEEE Transactions on Neural Networks and Learning Systems, Cognitive Computation, Advances in Artificial Neural Systems, Intelligent Control and Automation.

Chuanlong Li received his B.S. degree in Applied Mathematics from Sichuan University, Chengdu, China, in 1992, and M.S. degree in Operational Research and Control Theory and Ph.D. degree in Computer Software and Theory from Chongqing University, Chongqing, China, in 2001 and in 2005, respectively. He has been a Professor at the College of Computer Science, Chongqing University, Chongqing 400044, China, since 2007, and IEEE Senior member since 2010. From November 2006 to November 2008, he served as a Research Fellow in the Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Hong Kong, China. He has published more than 100 journal papers. His current research interest covers computational intelligence, neural networks, memristive systems, chaos control and synchronization, and impulsive dynamical systems.

Hangjun Che received the B.E. degree in electronic and information engineering form Chongqing University of Posts and Telecommunications, Chongqing, China, in 2013. Now he is studying for master’s degree at University of Sydney, Australia. He was previously Ausgrid Chair Professor and Director of the Centre for Intelligent Electricity Networks (CIEN), the University of Newcastle, Australia. His research interest includes smart grid, power system planning, power system security, load modeling, renewable energy systems, electricity market, and computational intelligence and its application in power engineering. He is an editor of the IEEE Transactions on Smart Grid and IEEE Power Engineering Letters.

Zhaoyang Dong received the Ph.D. degree from the University of Sydney, Australia, in 1999. He is now a Professor and Head of School at the University of Sydney, Australia. He was previously Ausgrid Chair Professor and Director of the Centre for Intelligent Electricity Networks (CIEN), the University of Newcastle, Australia. His research interest includes smart grid, power system planning, power system security, load modeling, renewable energy systems, electricity market, and computational intelligence and its application in power engineering. He is an editor of the IEEE Transactions on Smart Grid and IEEE Power Engineering Letters.