Impulsive effects on stability of high-order BAM neural networks with time delays

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1. Introduction

Artificial Neural Networks (ANN) have attracted increasing attention due to its great potential in applications such as pattern recognition, signal and image processing, associate memory, and combinatorial optimization [1–3,9,18]. Such neural networks characterized by first-order interactions are shown to have intrinsic limitations such as limited capacity when used in pattern recognition (see, e.g., [3–6]). Also the dilemma of optimization problems that can be solved by using such neural models is limited. Accordingly, many investigators have sought (see [3,7] and the references therein) to use neural networks with higher order structures. Since Narendra and Parthasarathy [8] have carried out a research for applying neural networks to system identification and control of nonlinear systems, specially high-order neural networks due to their excellent approximation capabilities, using fewer units than first-order ones, makes them flexible and robust when faced with new and/or noisy data patterns [9]. Besides, the proposed high-order neural networks for efficient invariant object recognition achieve significant reductions in computation times and memory requirements [10]. Furthermore, several authors have demonstrated the feasibility of using these architectures in many other applications such as function approximation [11,12], grammatical inference [13] and target detection [14].

Bidirectional associative memory (BAM) neural networks were first proposed by Kosko [15,16]. The circuit diagram and connection pattern implementing the delayed BAM networks can be found in Cao and Wang [17]. It should be noted that stability analysis of neural networks is significant for the practical design and application. In solving combinatorial optimization problems, the neural networks must be designed to have a unique equilibrium point, and this point must be globally asymptotically stable to avoid the risk of being trapped in local minima and converge in real time [18]. Thus, it is important to determine the stability for neural networks.

Several literatures [19–24] for stability of first-order delayed BAM neural networks have been proposed. Cao et al. [25] have investigated exponential stability of high-order BAM neural networks. On the other hand, the evolutionary processes of neural networks including BAM networks are usually characterized by abrupt changes at certain time in axonal transmission. Such impulsive changes may impact a great influence on stability of neural networks [26–29]. Daniel et al. [30] have first studied global exponential stability of impulsive high-order BAM neural networks with delays. Most of these literatures rely on Lyapunov–Krasovskii functional and involve Halanay inequality with impulse. Theoretical works by these researchers have just focused on destabilizing effects of impulses in delayed BAM neural networks. Given that impulse-free continue component of BAM neural networks is unstable, this method will more likely fail or cause difficulty in use. The effect of...
impulse, however, is two-folded. Yang and Chua [31] have proven that not only can impulses destabilize system when it is stable, but also they can stabilize it when it is not stable. This property has been widely used to stabilize and synchronize chaotic systems. Liu and Wang [28] have extended this property to study impulsive stabilization of high-order Hopfield neural networks with small time delays.

In this paper, we employ Lyapunov–Razumikhin technique to investigate the effect of impulse on stability of high-order BAM neural networks with time delays. Compared with above methods, Lyapunov–Razumikhin technique has its advantage that, to deal with time delays, its Lyapunov function is unnecessary to be decreasing on the whole state space. This means that the impulse-free high-order BAM neural networks can be unstable. And we will show that the oscillating high-order BAM neural networks can be forced to converge by the impulsive effect. It is worth noting that we do not require that the impulse interval is large than time delay.

The rest of this paper is organized as follows. In Section 2, we describe the impulsive high-order BAM neural networks with time delays and present some necessary preliminaries. In Section 3, two preservation conditions of global exponential stability for such model under impulsive perturbation are developed. In Section 4, we discuss conditions of impulsive stabilization for high-order BAM neural networks and estimate the feasible upper bound of impulse strength. Several examples are finally presented in Section 5 to illustrate the application of theoretical results, which is followed by conclusions in Section 6.

2. Preliminaries

In this paper, we consider the following impulsive high-order BAM neural networks with time delays:

\[
\begin{align*}
\frac{dx(t)}{dt} &= -a_i x(t) + \sum_{j=1}^{m} b_{ji} f_j(y(t-\tau(t))) \\
&\quad + \sum_{j=1}^{m} \sum_{l=1}^{n} b_{lj} g_l(y(t-\tau(t))) f_j(y(t-\tau(t))), \quad t \neq t_k, \\
\Delta x(t_k) &= e_i x(t_k^-) + \sum_{j=1}^{m} w_j h_j(y(t_k^-)), \quad t = t_k, \\
\frac{dy(t)}{dt} &= -d_i y(t) + \sum_{j=1}^{n} c_{ji} g_j(x(t-\sigma(t))) \\
&\quad + \sum_{j=1}^{n} \sum_{l=1}^{m} c_{lj} h_l(g_j(x(t-\sigma(t)))) g_l(y(t-\sigma(t))), \quad t \neq t_k, \\
\Delta y(t_k) &= r_j y(t_k^-) + \sum_{l=1}^{n} u_{lj} z_l(x(t_k^-)) \\
&\quad + \sum_{l=1}^{n} \sum_{j=1}^{m} u_{lj} z_j(x(t_k^-)) z_l(x(t_k^-)), \quad t = t_k,
\end{align*}
\]

where \( t > 0; i = 1,2,\ldots, n; j = 1,2,\ldots, m; k = 1,2,\ldots \). The time sequence \( \{ t_k \} \) satisfies \( 0 < t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots \) and \( \lim_{k \to +\infty} t_k = +\infty \).

\[
\begin{align*}
\Delta x(t_k) &= x(t_k^-) - x(t_k^-), \\
\Delta y(t_k) &= y(t_k^-) - y(t_k^-),
\end{align*}
\]

\( x_i(t), y_j(t) \) denote the potential of the cell \( i \) and \( j \) at time \( t \), respectively. \( a_i \) and \( d_j \) are positive constants, which are the rate of isolation of cells \( i \) and \( j \) from the other states and inputs, respectively. \( b_{ij}, c_{ij}, w_{ij}, u_{ij}, b_{ji}, c_{ji}, W_{ij} \) and \( u_{ji} \) are the first- and second-order connection weights of the neural network, respectively. \( r(t) \) and \( \sigma(t) \) are the transmission delays of the neuron; both of them are continuous functions which satisfy \( 0 \leq r(t) \leq r_{max}, 0 \leq \sigma(t) \leq \sigma_{max} \), and \( t = \max \{ r_{max}, \sigma_{max} \} \).

We assume throughout that the neuron activation functions \( f_j(\cdot), g_j(\cdot), g_j(\cdot) \) and \( z_l(\cdot) \) are continuously differentiable and satisfy the following hypotheses:

[H1]. There exist positive numbers \( M_j, M_j^2, N_j^p, N_j^l \) such that
\[
|f_j(y)| \leq M_j, \quad |g_j(y)| \leq M_j^2, \quad |g_j(x)| \leq N_j^p \quad \text{and} \quad |z_l(x)| \leq N_j^l, 
\]
for all \( x, y \in R \).

[H2]. \( f_j(0) = -g_j(0) = g_j(0) = 0, \quad (i = 1,2,\ldots, n; j = 1,2,\ldots, m) \).

[H3]. There exist positive numbers \( K_j, K_j^2, L_j^p \) and \( L_j^l \) such that
\[
|f_j(x) - f_j(y)| \leq K_j |x - y|, \quad |g_j(x) - g_j(y)| \leq K_j^2 |x - y|; 
\]
\[
|g_j(x)| - L_j^p |x - y| \leq K_j |x - y| 
\]
for all \( x, y \in R \) (i = 1,2, ...; n; j = 1,2, ...; m).

If we denote
\[
\begin{align*}
\Delta x(t) &= [\Delta x_1(t),\Delta x_2(t),\ldots,\Delta x_n(t)]^T; \\
\Delta y(t) &= [\Delta y_1(t),\Delta y_2(t),\ldots,\Delta y_m(t)]^T; \\
\Delta f_j(t) &= [\Delta f_{j1}(t),\Delta f_{j2}(t),\ldots,\Delta f_{jm}(t)]^T; \\
\Delta g_j(t) &= [\Delta g_{j1}(t),\Delta g_{j2}(t),\ldots,\Delta g_{jm}(t)]^T; \\
\Delta z_l(t) &= [\Delta z_{l1}(t),\Delta z_{l2}(t),\ldots,\Delta z_{lm}(t)]^T; \\
\Delta h(t) &= [\Delta h_1(t),\Delta h_2(t),\ldots,\Delta h_m(t)]^T; \\
\Delta h(t) &= [\Delta h_1(t),\Delta h_2(t),\ldots,\Delta h_m(t)]^T; \\
\Delta \Theta(t) &= [\Delta \Theta_1(t),\Delta \Theta_2(t),\ldots,\Delta \Theta_m(t)]^T; \\
\Delta \Gamma(t) &= [\Delta \Gamma_1(t),\Delta \Gamma_2(t),\ldots,\Delta \Gamma_m(t)]^T; \\
\Delta \Psi(t) &= [\Delta \Psi_1(t),\Delta \Psi_2(t),\ldots,\Delta \Psi_m(t)]^T; \\
\Delta \Psi(t) &= [\Delta \Psi_1(t),\Delta \Psi_2(t),\ldots,\Delta \Psi_m(t)]^T.
\end{align*}
\]
then, system (1) can be rewritten in the following vector–matrix form:

\[
\frac{dx(t)}{dt} = -Ax(t) + Bf_j(y(t-\tau(t))) + \Pi_j^T \Theta_j g_j(x(t-\sigma(t))), \quad t \neq t_k, \\
\Delta x(t_k) = \xi x(t_k^-), \quad t = t_k,
\]

\[
\begin{align*}
\frac{dy(t)}{dt} &= -Ay(t) + Cg_j(x(t-\sigma(t))) + \Theta_j^T \Sigma_j \Psi_j \xi y(t), \quad t \neq t_k, \\
\Delta y(t_k) &= \mu_j y(t_k^-) + U_j \zeta(x(t_k^-)) + \Theta_j^T \Sigma_j \Psi_j \xi y(t_k^-), \quad t = t_k,
\end{align*}
\]

\[
\begin{align*}
\Delta x(t_k) &= [\Delta x_1(t_k),\Delta x_2(t_k),\ldots,\Delta x_n(t_k)]^T, \quad x(t_k^-) = \lim_{t \to t_k^-} x(t), \\
\Delta y(t_k) &= [\Delta y_1(t_k),\Delta y_2(t_k),\ldots,\Delta y_m(t_k)]^T, \quad y(t_k^-) = \lim_{t \to t_k^-} y(t).
\end{align*}
\]
\[ L^2 = \text{diag}(L_1^2, L_2^2, ..., L_n^2). \]

For any given \( x \in \mathbb{R}^n \), its norm is defined as \( |x| = \sqrt{x^T A x} \). \( A \) and \( A^{-1} \) denote the transpose and inverse of the matrix \( A \), respectively. \( A > 0 \) means that square matrix \( A \) is real symmetric and positive definite. \( \lambda_{\text{max}}(A) \) and \( \lambda_{\text{min}}(A) \) represent the maximum and minimum eigenvalues of matrix \( A \), respectively.

The initial conditions associated with system (1) are of the following equations:

\[ x_i(t_0) = \varphi_i(t_0), \quad y_i(t_0) = \psi_i(t_0), \quad t_0 - \tau \leq t \leq t_0 \]

in which \( \varphi_i(t), \psi_i(t) \) \( (i = 1, 2, ..., n; j = 1, 2, ..., m) \) are continuous functions.

Their norms are defined, respectively, by

\[ ||\varphi_i||_{t_0} = \sup_{t_0 - \tau \leq s \leq t_0} |\varphi_i(s)| \quad \text{and} \quad ||\psi_i||_{t_0} = \sup_{t_0 - \tau \leq s \leq t_0} |\psi_i(s)|. \]

**Definition 1.** Suppose \( x \) is any negative number, then \( |x| \) means the minimum which is less than or equal to the absolute value of \( x \).

Before we give our main results, we also need the following lemmas.

**Lemma 1.** (Sanchez [32]). Suppose \( W, U \) are any matrices, \( \epsilon \) is a positive number and matrix \( D > 0 \), then \( W^T U + U^T W \leq \epsilon W^T D W + \epsilon^{-1} U^T D^{-1} U \).

**Lemma 2.** (Schur complement (Boyd et al. [33])). The following LMIs:

\[
\begin{bmatrix}
Q(x) & S(x) \\
S^T(x) & R(x)
\end{bmatrix} > 0,
\]

where \( Q(x) = Q^T(x), R(x) = R^T(x), \) and \( S(x) \) depend affinely on \( x \), is equivalent to \( R(x) > 0, Q(x) - S(x) K^{-1}(x) S^T(x) > 0 \).

3. Preservation of global exponential stability

In this section, it will be shown that, under certain conditions, system (1) has a unique equilibrium point which is globally functionally stable in spite of the existence of impulse perturbation.

**Theorem 3.1.** Under assumptions (H1)–(H3), the equilibrium point of system (1) is unique and globally exponentially stable if the following conditions are satisfied:

\[ a - \frac{q b}{2} \leq \eta \lambda, \]

for any \( q \) satisfying following inequality:

\[ \ln \eta \geq \lambda \tau + (1 + \eta) \lambda x, \]

where \( \lambda > 0, \tau > 0 \) and \( \eta < 0 \) are constants, and

\[ a = \max(\lambda_{x_1}, \lambda_{x_2}), \quad b = \min(\lambda_{x_2}, \lambda_{y_2}) \]

with \( \lambda_{x_1} = \lambda_{\text{max}}(Q_{x_1}), \lambda_{x_2} = \lambda_{\text{min}}(Q_{x_2}), \lambda_{y_1} = \lambda_{\text{max}}(Q_{y_1}), \) and \( \lambda_{y_2} = \lambda_{\text{min}}(Q_{y_2}) \).

(iii) \( \lambda_{y_1} \leq \max \left\{ \left[ ||I + F|| + \max_{1 \leq i \leq n} ||L_i^2|| \right] \left[ ||W|| + ||A_2^2|| ||I_2|| \right], \quad \left[ ||I + R|| + \max_{1 \leq j \leq m} ||K_j^2|| ||U|| + ||\Theta_2^2|| ||\Sigma_2|| \right] \right\} \).

(iv) \( t_{k+1} - t_k \leq \xi \), \( \ln \lambda_j \leq -(1 + \eta + \lambda) \xi (t_{k+1} - t_k) \) \( (k = 1, 2, ...) \).

**Proof.** Let \( \|\varphi\|^2 = ||\varphi||^2 + ||\psi||^2 \) and choose \( M > 1 \) such that

\[ e^{(1 + \eta) \xi (t_{k+1} - t_k)} \leq M \leq e^{(1 + \eta + \lambda) \xi (t_{k+1} - t_k)}. \]

Then

\[ 0 < ||\varphi||^2 < ||\varphi||^2 e^{(1 + \eta + \lambda) \xi (t_{k+1} - t_k)} \leq M ||\varphi||^2 e^{(1 + \eta + \lambda) \xi (t_{k+1} - t_k)}. \]

Letting \( V(t) = \|x(t)\|^2 + \|y(t)\|^2 \), we shall show that

\[ V(t) \leq M \|\varphi\| e^{-\xi(t_{k+1} - t_k)}, \quad t \in [t_{k+1}, t_k), \quad k = 1, 2, ..., \]

Firstly, we shall prove that

\[ V(t) \leq M \|\varphi\| e^{-\xi(t_{k+1} - t_k)}, \quad t \in [t_{k+1}, t_k). \]

From (6), one observes that, for \( t \in [t_0, t_1) \),

\[ V(t) \leq ||\varphi||^2 \leq M \|\varphi\|^2 e^{-\xi(t_{k+1} - t_k)}. \]

If (8) is not true, then there must exist some \( \tau \in [t_0, t_1) \) such that

\[ V(t) > M \|\varphi\|^2 e^{-\xi(t_{k+1} - t_k)} \geq \|\varphi\|^2 e^{(1 + \eta + \lambda) \xi (t_{k+1} - t_k)} > \|\varphi\|^2 \]

\[ \geq V(t_0 + \delta), \quad \delta \in [-\tau, 0]. \]

This implies that there exists some \( t^* \in [t_0, t_1) \) such that

\[ V(t^*) = \|\varphi\|^2, \]

and

\[ V(t^*) \leq V(t) \leq V(t^*), \quad t^* \leq t \leq t^*. \]

Then, for any \( t \in [t^*, t^*] \),

\[ V(t') \leq M \|\varphi\| e^{(1 + \eta + \lambda) \xi (t_{k+1} - t_k)}, \quad \xi (t_{k+1} - t_k). \]

Therefore, the derivative of \( V(t) \) belonging to the solution of (3) on \( t \in [t^*, t^*] \) is calculated and estimated as following formulary:

\[ D^+ V(t) = 2x^T(t)BF(y(t - \sigma(t))) + 2y^T(t)[Dy(t) + Cg(x(t - \sigma(t))) + \Theta_1^T \Sigma_1 g(x(t - \sigma(t)))]. \]

From Lemma 1, we have

\[ 2x^T(t)BF(y(t - \sigma(t))) \leq \lambda x^T(t) B^2 x(t) + y^T(t)[y(t - \sigma(t))] P_{df}(y(t - \sigma(t))), \]

\[ 2y^T(t)[Dy(t) + Cg(x(t - \sigma(t))) + \Theta_1^T \Sigma_1 g(x(t - \sigma(t)))]. \]

\[ 2x^T(t) \Theta_1^T \Sigma_1 g(x(t - \sigma(t))) \leq \lambda x^T(t) B^2 x(t) + y^T(t)[y(t - \sigma(t))] P_{df}(y(t - \sigma(t))), \]

\[ 2y^T(t)[Dy(t) + Cg(x(t - \sigma(t))) + \Theta_1^T \Sigma_1 g(x(t - \sigma(t)))]. \]
From $T^2(T_1 = |f(y(t-\tau(t))|)^2 I$ and $\|f(y(t-\tau(t)))\|^2 \leq \sum_{j=1}^{\infty} N^2_j$, it follows that $x^2(t)T^2 / \sum_{j=1}^{\infty} N^2_j \leq M^2 / \sum_{j=1}^{\infty} N^2_j$.

From $\Theta_1 \Theta_1 = \|g(x(t-\sigma(t)))\|^2 I$ and $\|g(x(t-\sigma(t)))\|^2 \leq \sum_{i=1}^{n} (N^2_i \leq N^2_N$, it follows that $y^2(t) / \Theta_1 \Theta_1 \leq N^2_N / \sum_{j=1}^{\infty} N^2_j$. (15b)

Substituting (14) and (15) into (13), and from condition (i), (ii), we obtain

\[
D^+ V(t) \leq x^2(t) A - AT^2 + BP^2 x^2(t) + C x^2(t) / \sum_{j=1}^{\infty} N^2_j \|x(t)\|^2 + y^2(t) \|y(t-\tau(t))\|^2 P + e_1 / \sum_{j=1}^{\infty} N^2_j \|y(t-\tau(t))\|^2 + e^T(t-\sigma(t)) / \sum_{j=1}^{\infty} N^2_j \|y(t-\tau(t))\|^2 + e^T(t-\sigma(t)) / \sum_{j=1}^{\infty} N^2_j \|y(t-\tau(t))\|^2 - \lambda e_2 \|x(t-\sigma(t))\|^2 - \lambda y_1 \|y(t-\tau(t))\|^2 + \alpha \|x(t-\sigma(t))\|^2 - \gamma \|y(t-\tau(t))\|^2. \tag{16}
\]

Substituting (12) to the above formula yields

\[
D^+ V(t) \leq (a-qb) V(t) \quad \text{for } t \in [\tau^*, \tau^*].
\]

We also know that $D^+ V(t) < 0$, for $t \in [\tau^*, \tau^*]$. This implies that $V(t^*) > V(t^*)$.

i.e.,

\[
\|\phi\|^2 > M \|\phi\|^2 e^{-\lambda(t_1-t_0)}.
\]

which contradicts (6). Hence, (8) holds and the (7) is true for $k = 1$. Now, we assume that (7) holds for $k = 1, 2, ..., m$, i.e.,

$V(t) \leq M \|\phi\|^2 e^{-\lambda(t_1-t_0)}$, \quad $t \in [t_{k-1}, t_k)$, \quad $k = 1, 2, ..., m$.

Next, we shall show that (7) holds for $k = m+1$, i.e.,

$V(t) \leq M \|\phi\|^2 e^{-\lambda(t_1-t_0)}$, \quad $t \in [t_m, t_{m+1})$. \tag{18}

Similarly, we also suppose (18) is not true for a contradiction. We define

$\bar{t} = \inf \{t \in [t_m, t_{m+1}) | V(t) > M \|\phi\|^2 e^{-\lambda(t_1-t_0)} \}.

Note that

\[
V(t_m) = \|x(t_m) + \Delta x(t_m)\|^2 + \|y(t_m) + \Delta y(t_m)\|^2 \leq \|x(t_0) + \Delta x(t_0)\|^2 + \|y(t_0) + \Delta y(t_0)\|^2 + \|x(t) + \Delta x(t) + R\|^2 \|x(t) + \Delta x(t)\|^2 + \|y(t) + \Delta y(t) + R\|^2 \|y(t) + \Delta y(t)\|^2 \leq \|x(t_0) + \Delta x(t_0)\|^2 + \|y(t_0) + \Delta y(t_0)\|^2 + \max_{1 \leq n \leq m} (K^2_n) \|x(t) + \Delta x(t)\|^2 + \|y(t) + \Delta y(t)\|^2 + \max_{1 \leq n \leq m} (K^2_n) \|x(t) + \Delta x(t)\|^2 + \|y(t) + \Delta y(t)\|^2. \tag{19}
\]

From conditions (iii) and (iv), we obtain

\[
V(t_m) \leq \int_{t_m}^{t} \frac{dV(t)}{dt} = \int_{t_m}^{t} (M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} - e^{-\gamma(t-\tau(t))} e^{-\lambda(t_1-t_0)} M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} - e^{-\gamma(t-\tau(t))} e^{-\lambda(t_1-t_0)} M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} + M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))}) dt.
\]

Hence $\bar{t} \neq t_m$. By the continuity of $V(t)$ in the interval $[t_m, t_{m+1})$, we have

\[
V(\bar{t}) = M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))}.
\]

and

\[
V(t) \leq M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))}, \quad t \in [t_m, \bar{t}]. \tag{21}
\]

This implies that there exists some $t^* \in (t_m, \bar{t})$ such that

\[
V(t^*) = \int_{t_m}^{t^*} \frac{dV(t)}{dt} = \int_{t_m}^{t^*} (M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} - e^{-\gamma(t-\tau(t))} e^{-\lambda(t_1-t_0)} M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} - e^{-\gamma(t-\tau(t))} e^{-\lambda(t_1-t_0)} M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} + M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))}) dt.
\]

Then, for $t \in [t^*, \bar{t}]$ and $s \in (-\tau, 0)$, we know that (see Remark 1)

\[
V(t+s) \leq e^{-\gamma(s)} e^{-\lambda(t_1-t_0)} M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} \leq e^{-\gamma(s+|t_1-t_0|) |m|} V(t^*) \leq q V(t), \quad t \in [t^*, \bar{t}]. \tag{23}
\]

Then, arguing similarly as before, we have $D^+ V(t) < 0$ for $t \in [t^*, \bar{t}]$. Note that

\[
V(\bar{t}) \leq V(t^*) = \int_{t_m}^{t^*} \frac{dV(t)}{dt} = \int_{t_m}^{t^*} (M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} - e^{-\gamma(t-\tau(t))} e^{-\lambda(t_1-t_0)} M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} - e^{-\gamma(t-\tau(t))} e^{-\lambda(t_1-t_0)} M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} + M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))}) dt.
\]

which is a contradiction. This implies the assumption is not true, and hence, (18) holds. Therefore, by some mathematical induction, we obtain (7) holds for any $k \in Z^+$. That is

\[
V(t) \leq M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))}, \quad t \in [t_k, t_k), \quad k = 1, 2, ..., \tag{24}
\]

i.e.,

\[
|x(t)|^2 + |y(t)|^2 \leq M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))}, \quad t \in [t_k, t_k), \quad k = 1, 2, ... .
\]

The proof is thus completed. \Box

Remark 1. For $t \in [t^*, \bar{t}]$ and $s \in (-\tau, 0)$, when $t+s \in [t_m, \bar{t}]$, from (21), we obtain

\[
V(t+s) \leq M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} \leq e^{t \gamma} e^{-\lambda(t_1-t_0)} M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))}. \tag{25}
\]

When $t+s \in [t_0 \tau, t_m)$, from (11), we obtain

\[
V(t+s) \leq M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} \leq M \|\phi\|^2 e^{-\lambda(t_1-t_0)} e^{-\gamma(t-\tau(t))} e^{t \gamma}. \tag{26}
\]

Remark 2. Conditions (i) and (ii) in Theorem 3.1 are to ensure the global exponential stability of impulse-free system. Condition (iii) in Theorem 3.1 is to characterize quantitatively the impulse strength. And the last condition in Theorem 3.1 characterize the aggregated effects of impulse strength, time delay and exponential convergence rate of impulse-free subsystem, from which one can estimate the feasible upper bound of impulse strength.

For computational consideration, we give an easy-verified version of Theorem 3.1.
Theorem 3.2. Under assumptions (H1)–(H3), the equilibrium point of system (1) is unique and globally exponentially stable if the following conditions are satisfied:

(i) \(-a + qb \leq \eta \lambda,\) 
\begin{equation}
\eta q \geq \lambda + (1 + [\eta] + \eta) \lambda, \tag{25a}
\end{equation}
for any \(q\) satisfying following inequality:
\begin{equation}
\ln q \geq \lambda + (1 + [\eta] + \eta) \lambda, \tag{25b}
\end{equation}
where \(\lambda > 0, \alpha > 0\) and \(\eta < 0\) are constants. And
\begin{align*}
a &= \min \left\{ \min_{1 \leq i \leq n} a_i, \min_{1 \leq j \leq m} d_j \right\}, \\
b &= \max \left\{ \max_{1 \leq i \leq n} \left( \sum_{i=1}^{n} |c_{ij}| + \sum_{i=1}^{n} |c_{ij}| N_{f_i}^k \right) L_i^k \right\}, \\
\max_{1 \leq j \leq m} \left\{ \sum_{i=1}^{n} |b_{ij}| + \sum_{i=1}^{n} |b_{ij}| M_{f_i}^k \right\}. 
\end{align*}
(ii) \(J_k \geq \max_{1 \leq i \leq n} \left( 1 + e_i \right) + \max_{1 \leq i \leq n} \left( \sum_{i=1}^{n} |u_{ji}| + \sum_{i=1}^{n} |u_{ji}| N_{f_i}^k \right) L_i^k \),
\begin{align*}
\max_{1 \leq j \leq m} \left\{ \sum_{i=1}^{n} |w_{ji}| + \sum_{i=1}^{n} |w_{ji}| M_{f_i}^k \right\}.
\end{align*}
(iii) \(t_{k+1} - t_k \leq \alpha,\) and \(\ln J_k \leq -(1 + \eta) \lambda(t_{k+1} - t_k) \quad (k = 1, 2, \ldots).\)

Proof. Let \(V(t) = \sum_{i=1}^{n} |x_i(t)| + \sum_{i=1}^{n} |y_i(t)|.\) The proof is similar to that of Theorem 3.1, and therefore, we omit it here. \(\Box\)

Remark 3. It is worthy noting that Condition (i) in this theorem implies the first two conditions in Theorem 3.1.

4. Impulsive stabilization

This section deals with impulsive stabilization for the case that the corresponding impulse-free subsystem is not asymptotically stable, and shall establish two sufficient conditions ensuring stabilization to estimate the feasible upper bound of impulse strength.

Theorem 4.1. Under assumptions (H1)–(H3), the equilibrium point of system (1) is unique and globally exponentially stable if the following conditions are satisfied:

(i) There exists positive diagonal matrices \(P_a\) and \(P_p\), positive scalars \(\epsilon_a\) and \(\epsilon_y\) such that
\begin{align*}
\Omega_{a1} &= -A - A^T + BP_a^{-1} B^T + \epsilon_a^{-1} M_{\sum_{a}}^k, \\
\Omega_{a2} &= L_i^k (P_a - \epsilon_a \epsilon_y \sum_{l=1} P_l^k)^2, \\
\Omega_{a1} &= -B - B^T + CP_p^{-1} C^T + \epsilon_y^{-1} M_{\sum_{p}}^k, \\
\Omega_{a2} &= K_i^T (P_a - \epsilon_a \epsilon_y \sum_{l=1} P_l^k) K_i^T, \\
0 &\leq a - qb \leq \eta \lambda, \\
\ln q &\geq \lambda + (1 + \eta) \lambda, \\
\text{where } &\lambda > 0, \alpha > 0, \eta > 0 \text{ are constants, and }
\end{align*}
\begin{align*}
a &= \max \left\{ \lambda, \lambda_1 \right\}, \\
b &= \min \left\{ \lambda, \lambda_2 \right\}, \\
\lambda_1 &= \lambda_{\max}(Q_{a1}), \\
\lambda_2 &= \lambda_{\min}(Q_{a2}), \\
\lambda_3 &= \lambda_{\max}(Q_{a3}), \\
\lambda_4 &= \lambda_{\min}(Q_{a4}), \\
\lambda_5 &= \lambda_{\max}(Q_{a5}), \\
\lambda_6 &= \lambda_{\min}(Q_{a6}).
\end{align*}
(ii) \(J_k \geq \max_{1 \leq i \leq n} \left( (1 + E_i) + \max_{1 \leq j \leq m} |U_i|^2 (|W_i|^2 + ||G_i^2|| ||P_i^2||)^2 \right),\)
\begin{equation}
\left( (1 + R_i) + \max_{1 \leq j \leq m} |K_i^2|^2 (||U||^2 + ||\Theta_i^2|| ||S_i^2||)^2 \right).
\tag{28}
\end{equation}
(iii) \(t_{k+1} - t_k \leq \alpha,\) and \(\ln J_k \leq -(1 + \eta) \lambda(t_{k+1} - t_k) \quad (k = 1, 2, \ldots).\)

Proof. Let \(V(t) = \sum_{i=1}^{n} |x_i(t)| + \sum_{i=1}^{n} |y_i(t)|.\) The proof is similar to that of Theorem 3.1, and therefore, we omit it here. \(\Box\)

Note that, for \(t \in [t_0, \zeta_0],\)
\begin{equation}
V(t) \leq \left| \epsilon_i \right| \leq M \left| \phi_i \right| e^{-\gamma(t-\zeta_0)}.
\tag{29}
\end{equation}
This implies \(0 < \left| \epsilon_i \right| \leq M \left| \phi_i \right| e^{-\gamma(t-\zeta_0)} \quad (t \in [t_0, \zeta_0]).\)

Similarly, we firstly prove that \(V(t) \leq M \left| \phi_i \right| e^{-\gamma(t-\zeta_0)} \quad (t \in [t_0, \zeta_0]).\)
\begin{equation}
\text{This implies that there exists some } t' \in (t_0, \zeta_0) \text{ such that } \tag{30}
\end{equation}
\begin{equation}
V(t') = M \left| \phi_i \right| e^{-\gamma(t-\zeta_0)},
\tag{31}
\end{equation}
and \(V(t') \leq V(t) \leq V(t), \quad t' \leq t \leq t'.\)
\begin{equation}
\text{Then, for any } t \in [t', t'], \tag{32}
\end{equation}
\begin{equation}
V(t) \leq M \left| \phi_i \right| e^{-\gamma(t-\zeta_0)} \quad (t \in [t', t']).
\tag{33}
\end{equation}
Similarly, the derivative of \(V(t)\) belonging to the solution of (3) on \(t \in [t', t']\) is calculated and estimated as follows:
\begin{equation}
D^+ V(t) \leq \left| \epsilon_i \right|^2 (\epsilon_i^{-1} e^{-\gamma(t-\zeta_0)} < M \left| \phi_i \right| e^{-\gamma(t-\zeta_0)} = V(t'), \tag{34}
\end{equation}
\begin{equation}
V(t') \leq V(t') e^{-\gamma(t'-t')} < \left| \epsilon_i \right|^2 e^{-\gamma(t-\zeta_0)} < M \left| \phi_i \right|^2 e^{-\gamma(t-\zeta_0)} = V(t'),
\tag{35}
\end{equation}
which is a contradiction. Hence, (30) holds and the (29) is true for \(k = 1, \ldots, m,\)
\begin{equation}
V(t) \leq M \left| \phi_i \right|^2 e^{-\gamma(t-\zeta_0)} \quad (t \in [t_0, \zeta_0]), \quad k = 1, 2, \ldots, m.
\tag{36}
\end{equation}
In the sequel, we shall show that (29) holds for \( k = m + 1 \), i.e.,
\[
V(t) \leq M \| \phi \|^2 e^{-\beta(t-t_0)}, \quad t \in [t_m, t_{m+1}).
\]
Similarly, we also suppose that (36) is not true for a contradiction. We define
\[
\tilde{t} = \inf(t \in [t_m, t_{m+1}) | V(t) > M \| \phi \|^2 e^{-\beta(t-t_0)}).
\]
Note also that
\[
V(t_m) = \left| x(t_m) + \Delta x(t_m) \right|^2 + \left| y(t_m) + \Delta y(t_m) \right|^2 \\
\leq \left| \left[ I + \frac{1}{C_1} \right] \left\{ |x| + \left| | \alpha_1^T \right| |U| + \left| \alpha_2^T \right| |Z_2| \right\} \right|^2 \\
+ \left[ \left| \frac{1}{C_3} \right| + \frac{1}{C_3} \left( \left| F \right| + \left| \alpha_3^T \right| |Z_1| \right) \right]^2 |y(t_k)|^2. \tag{37}
\]
From conditions (iii) and (iv), we obtain
\[
V(t_m) \leq \sum_{l=1}^{m} V(t_l) \\
< \sum_{l=1}^{m} M \| \phi \|^2 e^{-\beta(t_l-t_0)} \quad \text{for} \quad t \in \left[t_m, \tilde{t} \right).
\]
This implies that there exists some \( t^{**} \in (t_m, \tilde{t}) \) such that
\[
V(t^{**}) = \sum_{l=1}^{m} M \| \phi \|^2 e^{-\beta(t_l-t_0)},
\]
and
\[
V(t^{**}) \leq V(t) \leq V(\tilde{t}), \quad t \in [t^{**}, \tilde{t}]. \tag{39}
\]
Then, for \( t \in [t^{**}, \tilde{t}] \) and \( s \in [-\tau, 0] \), we know that
\[
V(t+s) \leq e^{\beta s} \sum_{l=1}^{m} M \| \phi \|^2 e^{-\beta(t_l-t_0)} \leq V(t^{**}) < qV(t), \quad t \in [t^{**}, \tilde{t}]. \tag{40}
\]
Arguing similarly as before, we have \( D^+ V(t) \leq \eta \lambda V(t) \) for \( t \in [t^{**}, \tilde{t}] \).

Since
\[
V(\tilde{t}) \leq V(t^{**}) e^{\beta(t-t^{**})} \\
= \sum_{l=1}^{m} M \| \phi \|^2 e^{\beta(t_l-t_0)} e^{-\beta(t_l-t_{l-1})} e^{\beta(t_{l-1}-t^{**})} \\
= \sum_{l=1}^{m} M \| \phi \|^2 e^{\beta(t_l-t_0)} e^{-\beta(t_l-t_{l-1})} e^{\beta(t_{l-1}-t^{**})} \quad \text{for} \quad t \in [t_{k}, t_{k+1}), \quad k = 1, 2, \ldots,
\]
which is a contradiction. This implies the assumption is not true, and hence, (30) holds.

Therefore, by some mathematical induction, we obtain (29) holds for any \( k \in \mathbb{Z}^+ \), so we obtain
\[
V(t) \leq M \| \phi \|^2 e^{-\beta(t-t_0)}, \quad t \in [t_{k-1}, t_k), \quad k = 1, 2, \ldots,
\]
and, \( x(t) \|^2 + \left| y(t) \right|^2 \leq M \| \phi \|^2 e^{-\beta(t-t_0)}, \quad t \in [t_{k-1}, t_k), \quad k = 1, 2, \ldots
\]
The proof is completed. \( \square \)

**Remark 4.** Conditions (i) and (ii) in Theorem 4.1 are to characterize the dynamical property of impulse-free system, from which one can observe that the impulse-free subsystem might be unstable. Condition (iii) in Theorem 4.1 is to characterize quantitatively the impulse strength. And the last condition in Theorem 4.1 characterize the aggregated effects of impulse strength, time delay and exponential convergence rate of impulse-free subsystem, from which one can estimate the feasible upper bound of impulse strength.

**Remark 5.** In Theorem 3.1, we assume that the impulse-free subsystem is globally exponentially stable, and establish two sufficient conditions to ensure that the impulse system can preserve the dynamical properties of the crisp system. But Theorem 4.1 is just the opposite. In Theorem 4.1, we present a sufficient condition for impulsive stabilization of crisp system.

Also, for computational consideration, we give an easy-verified version of Theorem 4.1.

**Theorem 4.2.** Under assumptions (H1)–(H3), the equilibrium point of system (1) is unique and globally exponentially stable if the following conditions are satisfied:

\[
(i) \quad 0 \leq -a + qb \leq \eta \lambda, \tag{43a}
\]
for any \( q \) satisfying following inequality:
\[
\ln q > \lambda (1 + \eta) \lambda x, \tag{43b}
\]
where \( \lambda > 0, \quad x > 0 \) and \( \eta > 0 \) are constants. And
\[
a = \min \left\{ \frac{1}{1 \leq i \leq n} a_i, \frac{1}{1 \leq j \leq m} d_j \right\},
\]
\[
b = \max \left\{ \frac{1}{1 \leq i \leq n} \sum_{j=1}^{m} |c_{ij}| + \frac{1}{1 \leq j \leq m} |c_{ij}| M_j |K_j^T| \right\},
\]
\[
(\text{ii}) \quad f_k = \max \left\{ \frac{1}{1 \leq i \leq n} \left( |u_i| + \frac{1}{1 \leq j \leq m} |w_{ij}| M_j |K_j^T| \right) \right\},
\]
\[
\left| u_k - f_k \right| + \left| \frac{1}{1 \leq j \leq m} \sum_{k=1}^{n} |w_{ij}| M_j |K_j^T| \right| \leq \lambda x - \ln q \leq 0.
\]

**Remark 6.** When the impulse-free subsystem is not asymptotically stable, it can be stabilized by some appropriate impulse. Liu [28] have obtained the relation between the measure of strength of the impulse and the upper bound of the impulsive interval, \( b_k = e^{-\frac{\lambda x}{q}} \).

If we choose some appropriate parameters for \( \lambda \) and \( \varphi \), we can improve the accuracy via the following optimization problem:

\[
\begin{align*}
\text{max } & q \\
\text{s.t. } & a - q b - \eta \lambda \varphi \leq 0 \\
& q b - a \leq 0 \\
& \lambda (1 + \eta) \lambda x - \ln q \leq 0 \\
& q > 0, \quad \eta > 0.
\end{align*}
\]

We can obtain the maximum value of \( \eta \), which is restricted by \( q \). Our results show a more accuracy estimation for the strength (see Example 3): \( f_k < e^{-\lambda x/q} \).
5. Illustrative examples

In this section, we discuss some examples to illustrate theorems of two sections.

Example 1. Consider the impulsive high-order BAM shown as follows:

\[
\frac{dx_i(t)}{dt} = -a_i x_i(t) + \sum_{j=1}^{2} b_{ij} f_j(y_j(t-\tau(t))) + \sum_{j=1}^{2} b_{ij} f_j(y_j(t-\tau(t))), \quad t \neq t_k, \tag{45a}
\]

\[
\Delta x_i(t_k) = c_i x_i(t_k^-) + \sum_{j=1}^{2} w_{ij} h_j(y_j(t_k^-)) + \sum_{j=1}^{2} w_{ij} h_j(y_j(t_k^-)), \quad t = t_k, \tag{45b}
\]

\[
\frac{dy_j(t)}{dt} = -d_j y_j(t) + \sum_{i=1}^{2} c_{ij} g_i(x_i(t-\sigma(t))) + \sum_{i=1}^{2} c_{ij} g_i(x_i(t-\sigma(t))), \quad t \neq t_k, \tag{45c}
\]

\[
\Delta y_j(t_k) = r_j y_j(t_k^-) + \sum_{i=1}^{2} u_{ij} z_i(x_i(t_k^-)) + \sum_{i=1}^{2} u_{ij} z_i(x_i(t_k^-)), \quad t = t_k, \tag{45d}
\]

where

\[
A = \begin{bmatrix} 0.52 & 0.27 \\ 0.2 & -0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & -0.3 \\ 0.1 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.9307 & -0.7911 \\ 0.8623 & -0.8956 \end{bmatrix},
\]

\[
B_2 = \begin{bmatrix} 0.6136 & -0.9123 \\ 0.9013 & -0.9078 \end{bmatrix}, \quad E = \begin{bmatrix} -1.1201 \\ -1.0980 \end{bmatrix},
\]

\[
W = \begin{bmatrix} 0.1001 & -0.0988 \\ 0.2012 & 0.1035 \end{bmatrix}, \quad W_1 = \begin{bmatrix} -0.2100 & 0.1011 \\ 0.0877 & -0.1522 \end{bmatrix},
\]

\[
W_2 = \begin{bmatrix} 0.0401 & -0.1859 \\ -0.3001 & 0.1412 \end{bmatrix}, \quad D = \begin{bmatrix} 0.21 \\ 0.16 \end{bmatrix},
\]

\[
C = \begin{bmatrix} -0.27 & -0.3014 \\ 0.2899 & 0.3101 \end{bmatrix},
\]

\[
C_1 = \begin{bmatrix} -0.3012 & 0.2931 \\ -0.2893 & -0.3003 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.2100 & 0.1723 \\ 0.1957 & -0.2089 \end{bmatrix},
\]

\[
R = \begin{bmatrix} -1.1001 \\ -0.9859 \end{bmatrix}, \quad U = \begin{bmatrix} 0.1330 & -0.1907 \\ 0.1056 & 0.1078 \end{bmatrix},
\]

\[
U_1 = \begin{bmatrix} -0.2101 & 0.0699 \\ -0.8011 & -0.1020 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.0604 & 0.1302 \\ 0.0988 & -0.2006 \end{bmatrix},
\]

\[
f_1(x) = \tanh(0.95x), \quad f_2(x) = \tanh(0.82x),
\]

\[
g_1(x) = \tanh(0.93x), \quad g_2(x) = \tanh(0.90x),
\]

\[
h_1(x) = \tanh(x), \quad h_2(x) = \tanh(x),
\]

\[
z_1(x) = \tanh(x), \quad z_2(x) = \tanh(x).
\]

Then,

\[
M^f = \text{diag}(1,1), \quad M^h = \text{diag}(1,1), \quad N^f = \text{diag}(1,1), \quad N^h = \text{diag}(1,1),
\]

\[
K^f = \text{diag}(0.95,0.82), \quad K^h = \text{diag}(0.01,0.01), \quad L^f = \text{diag}(0.93,0.90),
\]

\[
L^h = \text{diag}(0.01,0.01), \quad \tau(t) = 2, \quad \sigma(t) = 3.
\]

For Theorem 3.1, it is found that \( P_{s} = \text{diag}(1.0310,1.0310), \)
\( P_{y} = \text{diag}(5.1411,5.1411), \)
\( c_{a} = 1.0501, \quad c_{b} = 1.0501 \) satisfy condition (i) with \( a = -0.8764, b = -0.8390. \) Providing \( \varepsilon = 2, \eta = -0.5 \) and \( \lambda = 0.002, \) then we obtain \( |\eta| = 1, q > 1.0446, 0.0166 < j_k < 0.9946, q > e^{\varepsilon t + (1+|\eta|)j_k} = 1.0121. \) Choose \( j_k = 0.9 \) and \( q = e^{0.03}. \)

The equilibrium point of system (44) is unique and globally exponentially stable, and its convergence rate is 0.001.

For numerical simulation, we select initial functions as

\[
\begin{align*}
\varphi_1(s) &= 2, & s &\in [-2.0), \\
\varphi_2(s) &= -3, & s &\in [-0.2, 0), \\
\psi_1(s) &= 4, & s &\in [-3.0), \\
\psi_2(s) &= -1, & s &\in [-3.0, 0),
\end{align*}
\]

and \( t_k - t_{k-1} = 1 + \sin((\pi/2)k). \)

When the system is running with impulses, its convergence rate accelerates if the strength of impulses is appropriate. As is shown from Figs. 1 and 2, the equilibrium point \((x^*,y^*)=(0,0)\) of system with impulses is globally exponentially stable and converges faster.

![Fig. 1. System without impulses.](image1)

![Fig. 2. Impulsive system with \( t_k - t_{k-1} = 1 + \sin((\pi/2)k). \)](image2)
Example 2. Let us change the parameters of system (45) as a new system
\[
A = \begin{bmatrix} 5.1291 \\ 7.4235 \end{bmatrix}, \quad D = \begin{bmatrix} 3.8261 \\ 4.2324 \end{bmatrix}.
\]
And the others do not change. We obtain \(a=3.8261\) and \(b=3.5907\) with \(q < 1.0656\). Provided \(\xi=2\), \(\tau_k=\tau_{k-1}=2\), \(\eta=-0.5\) and \(\lambda=0.002\), choose \(j_k=0.9\) and \(q=e^{0.03}\) with \(|\eta|=1\). It is found that \(J_k < 0.9946\), \(q > 1.0121\) and \(J_k \geq 0.001\), see Figs. 3 and 4.

Remark 7. Obviously, some parameters of system (45) do not satisfy the Theorem 3.2. The conditions of Theorem 3.2 are much stricter than Theorem 3.1. It can ensure the feasibility of condition (ii)–(iv) with a large convergence rate.

Example 3. Let us consider the high-order BAM with unstable system
\[
\frac{dx_i(t)}{dt} = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} y_j(t-\tau(t)) + \sum_{j=1}^{m} c_{ij} e_i(t-\sigma(t)), \quad t \neq \tau_k,
\]
\[
\Delta x_i(t_k) = e_i(t_k), \quad t = \tau_k,
\]
\[
\frac{dy_j(t)}{dt} = -d_j y_j(t) + \sum_{i=1}^{n} c_{ij} e_i(t-\sigma(t)) + \sum_{i=1}^{m} c_{ij} e_i(t-\sigma(t)), \quad t \neq \tau_k,
\]
\[
\Delta y_j(t_k) = e_j(t_k), \quad t = \tau_k.
\]

It easy to check the conditions (ii)–(iv) in Theorem 3.2 are satisfied. Therefore, the equilibrium point of the new one is unique and globally exponentially stable and its convergence rate is 0.001, see Figs. 3 and 4.

![Fig. 3. System without impulses.](image)

![Fig. 4. Impulsive system with \(\tau_k=\tau_{k-1}=2\).](image)
\[
\begin{align*}
\psi_1(s) &= 4, \quad s \in [-5, 0), \\
\psi_2(s) &= -1, \quad s \in [-5, 0), \\
\psi_3(s) &= -4, \quad s \in [-5, 0).
\end{align*}
\]

For Theorem 4.1 and (44), it is found that
\[
P_x = \text{diag}(10.7878, 10.7878, 10.7878), \quad P_y = \text{diag}(3.3000, 3.3000, 3.3000),
\]
\[
\begin{align*}
\psi_1(s) &= \frac{1}{C_0}, \quad s = 0, \\
\psi_2(s) &= \frac{1}{C_0}, \quad s = 0, \\
\psi_3(s) &= \frac{1}{C_0}.
\end{align*}
\]

The results in [25] could be applied to analyze impulsive stabilization of system (46). With \(\alpha = 1\), \(\lambda = 0.01\) and \(q = 1.0202\), \(J_k \leq e^{-2(\alpha\lambda(t_k - t_{k-1}))} = 0.9802\). We obtain \(E, R \in \{-1.9802, -0.0198\}\). However, system (46) cannot be stabilized by some strength of impulses in this interval. For example, if we choose \(J_k = 0.9\), with \(E = (-0.1, -0.1, -0.1), R = (-0.1, -0.1, -0.1)\), and \(t_k - t_{k-1} = 1\), the conditions in [28] are satisfied. However, system (46) cannot be stabilized to the original, see Fig. 7 for more details.

6. Conclusion

In this paper, the effect of impulse on stability of high-order BAM neural networks with time-varying delays has been investigated. By employing the Lyapunov–Razumikhin technique, two sufficient conditions are obtained to preserve global exponential stability for neural networks under impulse perturbation. Furthermore, another two conditions for stabilizing high-order BAM neural networks have been derived via the effect of impulses. Our results show that the impulsive interval has a powerful influence on the strength of impulse. In other words, it is an artfully indirect effect on stability. And then we improve the accuracy for estimating the strength of impulse. All of these criteria are easy to verify. And the methods derived in this paper can be used to analyze and design some other high-order artificial neural networks.

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