Adaptive NN control for a class of stochastic nonlinear systems with unmodeled dynamics using DSC technique

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1. Introduction

It is well known that stochastic disturbances often exist in many practical systems. Their existence is a source of instability of the control systems, thus, the investigations on stochastic systems have received considerable attention in recent years. Efforts toward stabilization of stochastic nonlinear systems have been initiated in the works [1–8]. Recently, Pan and Basar [9] first derived a backstepping design for the stochastic nonlinear strict-feedback system motivated by a risk-sensitive cost criterion. Since then, many interesting control schemes have been proposed by using the well-known backstepping technique for different stochastic systems. For example, to solve the unmeasured states and the unmodeled dynamics, several different adaptive output feedback controllers were developed for strict-feedback stochastic nonlinear systems [10–12]. In recent years, the input-to-state stability (ISS) property has quickly become a foundational concept in nonlinear feedback controller design and analysis since it was firstly formulated in [13]. By combining the idea of nonlinear gain with the ISS property, several novel versions of nonlinear small-gain theorems are established. For example, Jiang [14] and Jiang et al. [15] constructed the recursive robust controller by applying the small-gain theorems to nonlinear robust stabilization for a class of nonlinear systems with nonlinear uncertainties and unmodeled dynamics. Liu [16] presented a unifying solution to the stochastic output feedback stabilization for stochastic nonlinear systems with stochastic input-to-state stable (SISS) nonlinear unmodeled dynamics systems. For the purpose of removing the additional stability margin condition imposed on the unmodeled dynamics, Wu [10] applied the stochastic small-gain theorem and backstepping design technique in the stochastic nonlinear systems with uncertain nonlinear functions and unmodeled dynamics. It should be pointed out that the above mentioned results are only suitable for those nonlinear systems with known nonlinear functions.

Recently, the neural networks have been proved to be a very useful tool for solving the control problem of uncertain systems. For example, the neural network was used to approximate the agent’s uncertain dynamics in [17]. The adaptive neural network control schemes have been found to be particularly useful for the control of nonlinear uncertain systems with unknown nonlinear functions in practical application. One main advantage of these schemes is that the unknown nonlinear functions can be approximated by the neural networks, and, the stability of the closed-loop system is also guaranteed. For this purpose, Chen [18] and Li [19] introduced the adaptive neural network control schemes to the
output feedback stochastic nonlinear strict-feedback systems. In order to approximate the unknown and desired control input signals instead of the unknown nonlinear functions, Zhou [20] proposed a direct adaptive radial basis function NNs (RBF NNs) control method, which significantly reduced the computation burden. Na [21] proposed an adaptive predictor incorporated with a high-order neural network (HONN) observer to obtain the future system states predictions. A neural-network-based tracking controller was proposed for the manipulator with uncertain kinematics, dynamics and the actuator model by Cheng [22]. More recently, some positive results were gained for classes of uncertain nonlinear MIMO systems with time delays by combination of backstepping technique and NN approximators [23–25].

However, these adaptive NN control approaches did not consider the problem of the unmodeled dynamics. As stated in [26–28], the unmodeled dynamics or dynamical disturbances often exist in many practical nonlinear systems, which are the major sources of resulting in the instability of the control systems. Therefore, to study the stochastic nonlinear systems with consideration of dynamical uncertainties is very important in control theory and applications. For example, adaptive NN control for a class of stochastic nonlinear systems with three types of uncertainties was investigated in paper [29]. However, it still inherited the drawback of the backstepping technique, that is, the problem of ‘explosion of complexity’, which is caused by repeated differentiations of some nonlinear functions, i.e., the virtual controllers designed at each step within the conventional backstepping technique. As a result, the complexity of a controller drastically grows as the order of the system increases.

Recently, for the purpose of overcoming the drawback of backstepping technique, a dynamic surface control (DSC) technique has been proposed to avoid this problem by introducing a first-order low-pass filter at each step of the conventional backstepping design procedure [29]. For example, in [30], by approximating the unknown nonlinear functions with radial basis function (RBF), the dynamic surface control technique was incorporated into the existing neural network based adaptive control design framework. In [31], the dynamic surface control technique was incorporated into the decentralized control for a class of large-scale interconnected stochastic nonlinear system. Tong [32] incorporated the dynamic surface control technique into the first adaptive fuzzy control scheme for a class of stochastic nonlinear strict-feedback systems. Chen [33] combined the dynamic surface control technique with the neural network control technique in the output feedback stochastic nonlinear strict-feedback systems, which overcame the problem of ‘explosion of complexity’.

Inspired by the aforementioned discussions, in this paper we consider the adaptive neural network output feedback stabilization problem, by incorporating the dynamic surface control technique into the adaptive neural network control, for a class of stochastic nonlinear system with unmodeled dynamics. The main contributions lie in the following, (i) this paper only employs one neural network to compensate for all unknown functions depending on the system output, so that the upper bounding functions are allowed to be completely unknown and it simplifies the design procedure and reduces the computation loads; (ii) the proposed adaptive DSC method can overcome the problem of ‘explosion of complexity’ inherent in the conventional backstepping designs and thus it becomes much simpler than the existing adaptive NN backstepping controllers; (iii) With the concept of input-to-state practical stability (ISpS) and nonlinear small–gain theorem extending to the stochastic case, together with the RBF NN technique, an adaptive NN output feedback controller is proposed; (iv) The reduced-order observer is introduced to estimate the unmeasured states.

The rest of the paper is organized as follows. In Section 2 some notations are provided. In Section 3 the preliminaries and problem formulation are presented. The design of reduced-order observer is presented in Section 4. In Section 5 a DSC controller is presented for controlling uncertain stochastic nonlinear system, in this section the stability analysis is also presented. In Section 6 the simulation example is presented, Section 7 contains the conclusion.

2. Notations

The following notations will be used throughout this paper. \( R^+ \) denotes the set of all nonnegative real numbers; \( R^n \) denotes the real \( n \)-dimensional space; \( R^{n \times r} \) denotes the real \( n \times r \) matrix space; for a given vector or matrix \( X, X^T \) denotes its transpose; \( |X| \) denotes the Euclidean norm of a vector \( X \), and the corresponding induced norm for matrix \( X \) is denoted by \( |X|_2 \); when \( X \) is square matrix, the Frobenius norm of \( X \) is defined by \( |X| = \sqrt{Tr(X^T X)} \), where \( Tr(X) \) denotes its trace; \( C \) denotes the set of all functions with continuous \( i \)th partial derivatives; \( K \) denotes the set of all functions: \( R^+ \rightarrow R^+ \), which are continuous, strictly increasing, and vanish at zero; \( K_\infty \) denotes the set of all functions that are of class \( K \) and unbounded; \( KL \) denotes the set of all functions \( \beta(s, t) : R^+ \times R^+ \rightarrow R^+ \), which are of class \( K \) for each fixed \( t \) and decreased to zero as \( t \rightarrow \infty \) for each fixed \( s \).

3. Preliminaries and problem formulation

Consider the following stochastic nonlinear system

\[
\frac{dx}{dt} = f(x, u)dt + g(x, u)dw
\]  

where \( x \in R^m, u \in R^n \) are the state and the input of system, respectively, \( w \) is an \( r \)-dimensional standard Brownian motion defined on the complete probability space \( (\Omega, F, P) \) with \( \Omega \) be a sample space, \( F \) being a \( \sigma \)-field, \( \{F_t\}_{t \geq 0} \) being a filtration, and \( P \) being a probability measure. The Borel measurable functions \( f : R^{m+n} \rightarrow R^m \) and \( g : R^{m+n} \rightarrow R^{m+r} \) are locally Lipschitz in \( (x, u) \in R^{m+n} \) for all \( t > 0 \).

For any given positive function \( V(x) \in C^2(R^m \rightarrow R^+) \), associated with the stochastic differential Eq. (1) we define the differential operator \( \ell \) as follows:

\[
LV(x) = \frac{\partial V}{\partial x} (x, u) + \frac{1}{2} \text{Tr} \left( \frac{\partial^2 V}{\partial x^2} g(x, u) \right)
\]  

(2)

For the purpose of studying the stochastic nonlinear systems with uncertainties, we extend the theory of Input-to-State practically Stable (ISpS) and the corresponding Lyapunov stability criterion [34] to the stochastic nonlinear system (1).

Definition 1. [10] System (1) is ISpS in probability if for any \( \varepsilon > 0 \), there exist a class of KL function \( \rho_x(\cdot, \cdot, \cdot), \) a class of KL function \( \rho_u(\cdot) \) and a constant \( d_\varepsilon \geq 0 \) such that

\[
P\left( |x(t)| < \rho_x(|x_0|), t > \chi(|u|) + d \right) \geq 1 - \varepsilon, \quad \forall t > 0, \quad x_0 \in R^m \setminus \{0\}
\]  

(3)

If \( u \equiv 0 \), system (1) is practically stable in probability.

Remark 1. For the purpose of researching the uncertain stochastic problem, as a nature extension of input-to-state (practically) stability [34,35], the above definition is introduced.

Theorem 1. [10,11] System (1) is ISpS in probability, if there exist some function \( V(x) \), class \( K_\infty \) functions \( \alpha, \pi, \xi \), class \( K \) function \( a \) and constant \( d \geq 0 \), such that

\[
\alpha(|x|) \leq V(x) \leq \pi(|x|)
\]  

(4)

\[
LV(x) \leq -\alpha(|x|) + \xi(|u|) + d
\]  

(5)

If there exist \( C \) function \( V(x) \), and constant \( c > 0, d \geq 0 \), such that

\[
LV(x) \leq -\alpha(|x|) + \xi(|u|) + d, \quad \text{then, system (1) is ISpS in probability.}
\]
Namely, \( \forall \varepsilon > 0 \), there exist a class of KL function \( \beta_\varepsilon (\cdot, \cdot) \), a class of constant \( K \) function \( \gamma (\cdot) \), and a constant \( d_c > 0 \) such that
\[
P(|V(x)| < \beta_\varepsilon (|V(x(0)), t) + \gamma (|u|_{\infty}) + d_c) \geq 1 - \varepsilon
\]
where \( \gamma = \delta (2\gamma (\cdot), d_c = (2d/c)^y) \).

**Theorem 2.** (Stochastic small-gain theorem) [10] The interconnected dynamic system
\[
dx_1 = f_1(x_1, x_2)dt + g_1(x_1, x_2)dw_1
\]
\[
dx_2 = f_2(x_1, x_2)dt + g_2(x_1, x_2)dw_2
\]
where \( x \in \mathbb{R}^{2+n} \) are the states of the system, \( w_1 \in \mathbb{R}^1 \), \( w_2 \in \mathbb{R}^2 \) are the independent standard Wiener process. The Borel measurable functions: \( f_1, f_2, g_1, g_2 \) is locally Lipschitz and bounded. Suppose the \( x_1 \)-system and \( x_2 \)-system of system (6) are ISS in probability with \( x_2 \) as input and \( x_1 \) as state, and \( x_1 \) as input and \( x_2 \) as state, respectively, i.e., for any \( e_1 \) and \( e_2 > 0 \),
\[
P(|x_1(t)| < \beta_e (x_1(0), t) + \gamma (|u|_{\infty}) + d_1) \geq 1 - e_1
\]
\[
P(|x_2(t)| < \beta_e (x_2(0), t) + \gamma (|u|_{\infty}) + d_2) \geq 1 - e_2
\]
where \( \beta_e (\cdot, \cdot) \) is class KL functions, \( \gamma_1 \) is \( K_{\infty} \) function, and constant \( d_1 > 0 \). If there exist nonnegative parameters \( \mu_1, \mu_2, \delta_0 \), such that nonlinear gain functions \( \gamma_1, \gamma_2 \) satisfy
\[
1 + \mu_1 \gamma_1 - (1 + \mu_2) \gamma_2(s) \leq 0, \quad \forall s \geq 0 > 0
\]
Then, system (6) is ISS in probability, i.e., for any \( \varepsilon > 0 \), there exist a class of KL function \( \beta_\varepsilon (\cdot, \cdot) \), and an arbitrarily small constant \( \delta_0 > 0 \), such as
\[
P(|x_1| < \beta_\varepsilon (|x_1|, t) + d_1) \geq 1 - \varepsilon
\]
Consider the systems described by the following stochastic nonlinear systems with unmodelled dynamics:
\[
dz = q_1(z, t)dt + q_2(z, t)dw_0
\]
\[
dx = [x_{i+1} + f_i(y)]dt + g_i(y)dw_i
\]
\[
dx_0 = [u + f_0(y)]dt + g_0(y)dw_0
\]
where \( x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n, u, y \in \mathbb{R} \) are the states, and the input and the measurable output of the systems, respectively; the states \( x_i(t = 2, \ldots, n) \) are unmeasured which will be estimated by using the reduced-order observer; \( z \) denotes the unmodeled dynamics, \( w_i \in \mathbb{R}(i = 0, \ldots, n) \) are independent standard Wiener processes defined on the complete probability space \( \Omega, F, \mathbb{P} \) with \( \Omega \) being a sample space, \( F \) being a \( \sigma \)-field, and \( \mathbb{P} \) being the probability measure. \( f_i(y) \) and \( g_i(y), i = 1, 2, \ldots, n \) are unknown smooth nonlinear functions. The uncertain functions \( h_1, h_2, \text{and} \Delta_i \) are locally Lipschitz.

In control engineering, radial basis function (RBF) NN is usually used as a tool for modeling nonlinear systems because of their good capabilities in function approximation. The RBF NN can be considered as a two-layer network in which the hidden layer performs a fixed nonlinear transformation with no adjustable parameters, i.e., the input space is mapped into a new space. The output layer then combines the outputs in the latter space linearly. Therefore, they belong to a class of linearly parameterized networks. In this paper, the following RBF NN is used to approximate the continuous function \([5, 11]\),
\[
h_{\text{nn}}(y) = W^T S(y)
\]
where the input vector \( y \in \Omega_0 \subset \mathbb{R}^l \) is the input vector with \( q \) being the neural networks input dimension, weight vector \( W = (w_1, w_2, \ldots, w_l)^T \in \mathbb{R}^l \), \( l > 1 \) is the neural networks node number, and \( S(y) = [s_1(y), s_2(y), \ldots, s_l(y)]^T \) means the basis function vector with \( s_i(y) \) being chosen as the commonly used Gaussian function of the form
\[
s_i(y) = \exp \left[ \frac{(y - \mu_i)^T(y - \mu_i)}{\zeta_i^2} \right], \quad i = 1, 2, \ldots, l
\]
where \( \mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{i2l}]^T \) are the center of the receptive field and \( \zeta_i > 0 \) are the width of the basis function \( s_i(y) \). It has been proven that neural network can approximate any continuous function over a compact set \( \Omega_0 \subset \mathbb{R}^l \) to arbitrary any accuracy as
\[
h(y) = W^T S(y) + \delta(y)
\]
where \( W^* \) is the ideal constant weight vector and is defined as \( W^* = \arg \min \{\sup(h(z) - W^T S(y))\} \), and \( \delta(y) \) denotes the approximation error.

The main results of this paper are based on the following assumptions:

**Assumption 1.** [36] There exist a known positive function \( \varphi(y) \) and an unknown positive constant \( \theta \) such that
\[
\delta(y) \leq \theta \varphi(y)
\]

**Assumption 2.** [10] For each \( 1 < i < n \) there exist unknown constants \( \theta_i \geq 0 \) such that
\[
|\Delta_i(x_1, \ldots, x_i)| \leq \theta_i(\varphi_1(y) + \varphi_2(z_i) + \delta_i(y)) \leq \theta_i \varphi_3(y) \]
where \( \varphi_1(y) \) and \( \varphi_2(z_i) \) are known nonnegative smooth functions with \( \varphi_3(y) = \varphi_4(y) + \delta_i(y) \), and such as
\[
\varphi_1(y) = \varphi_{i-1, 1}(y) + k_i \varphi_{p1}(y), \quad \varphi_3(z_i) = \varphi_{p1, 1}(z_i) + k_i \varphi_{p2}(z_i).
\]

but \( \varphi_3(z_i) \) is unknown nonnegative smooth functions.

**Assumption 3.** [11] For the \( \gamma \)-system, there exists a Lyapunov function \( V_0(z) \) such that
\[
L_0(z) \leq V_0(z) \leq \bar{L}_0(z)
\]
\[
LV_0 + a_0(z) + a_0(y) + a_0(y) = 0
\]
where \( V_0(z) \in C^2, \bar{a}_0, a_0, \bar{a}_0, y_0, y_0 \in \Omega_{\infty}, \text{constant} \bar{a}_0 > 0 \).

**Remark 2.** From Assumption 2 there exist smooth functions \( \varphi_1(y), \varphi_2(z_i), \varphi_3(y), \varphi_4(z_i) \) such that
\[
\varphi_1(y) = \tilde{\varphi}_1(y), \quad \varphi_2(z_i) = \tilde{\varphi}_2(z_i), \quad \varphi_3(y) = \tilde{\varphi}_3(y), \quad \varphi_4(z_i) = \tilde{\varphi}_4(z_i)
\]

3.1. **Control objective**

The control objective of this paper is to design an adaptive NN output feedback controller using the output \( y \) and state estimations \( \hat{x}_i \) so that the closed-loop system is ISS in probability and the outputs of the system can be regulated to a small neighborhood of the origin in probability.

4. **Reduced-order observer design**

Since the states \( x_i (i = 2, \ldots, n) \) are unavailable, the following reduced-order observer are introduced
\[
d\hat{x}_i = (\hat{x}_{i-1} + k_i(x_i + k_i y)dt \quad 1 \leq i \leq n - 2
\]
\[
d\hat{x}_{n-1} = (u - k_{n-2}(\hat{x}_i + k_i y)dt
\]
where \( k = (k_1, \ldots, k_{n-1})^T \) is chosen such that

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control law is designed at each step, \( A_0 = \begin{bmatrix}
- k_1 & 1 & 0 & \cdots & 0 \\
- k_2 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
- k_{n-2} & 0 & \cdots & 1 \\
- k_{n-1} & 0 & \cdots & 0 \\
\end{bmatrix} \)

is asymptotically stable. Define the observer error \( \hat{x} \) as

\[ \hat{x} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_{n-1}), \quad \hat{x}_i = \frac{1}{\theta_i} (x_{i+1} - \bar{x}_i - k_i x_i). \]  

(18)

where \( \theta_i = [\theta_1, \theta_2, \ldots, \theta_n] \), which satisfies \( d\tau = [A_0 x + F(y) + \Delta(\hat{x}, \zeta)] d\tau + G(y) d\tau \), where \( G = (1/\theta_i) \zeta_i - k_i x_i, \ldots, \theta_n - k_n x_n \), \( \Delta = (\Delta_1, \Delta_2, \ldots, \Delta_{n-1}) \). \( w = (\nu_1, \nu_2, \ldots, \nu_0) \). \( F(y) = (1/\theta_i) k_1 f_1 + \cdots + f_{n-1} - k_{n-1} f_{n-1} \). From systems (10) and (18), we get

\[ dy = [x_1 + \sigma x_1 + k_1 y + f_j(y) + \Delta_j] dt + g_j(y) dw_j, \]  

(19)

Hence, the complete system can be expressed as

\[ d\tau = [A_0 x + F(y) + \Delta(\hat{x}, \zeta)] d\tau + G(y) d\tau \]

\[ dy = \hat{x}_1 + \sigma \hat{x}_1 + k_1 y + f_j(y) + \Delta_j] dt + g_j(y) dw_j, \]  

(20)

Consider the quartic function \( V(\hat{x}) = r_0 \hat{x}^T P \hat{x} \), where \( P \) is a positive-definite constant matrix and \( r_0 \) is design parameter. The infinitesimal generator of \( V(\hat{x}) \) is

\[ LV(\hat{x}) = r_0 \hat{x}^T P \hat{x} + d \hat{x}^T P \hat{x} \]  

(21)

According to the well-known Mean Value Theorem, the following equalities hold

\[ f_i(y) = y \hat{f}_i(y) F(y) = y \hat{F}(y) \]

where \( \hat{f}_i(y) \) are completely unknown nonlinear functions that will be compensated only by an NN in this paper. So the function \( \hat{F}(y) \) is also unknown, then

\[ 2r_0 \hat{x}^T P \hat{x} = 2r_0 \hat{x}^T P \hat{x} = 2r_0 \hat{x}^T P \hat{x} = 2r_0 \hat{x}^T P \hat{x} = 2r_0 \hat{x}^T P \hat{x} = 2r_0 \hat{x}^T P \hat{x} \]

(22)

From Assumption 2 and Young’s inequality, one has

\[ 2r_0 \hat{x}^T P \hat{x} = \frac{1}{4} r_0 \hat{x}^T P \hat{x} + 4r_0 P \hat{x}^T P \hat{x} \leq \frac{1}{4} r_0 \hat{x}^T P \hat{x} \]

\[ + 8r_0 P \hat{x}^T \sum_{i=1}^{n-1} \hat{\phi}_i(y) + 8r_0 P \hat{x}^T \sum_{i=1}^{n-1} \hat{\phi}_i^2(y) \]  

(23)

\[ r_0 Tr(P^T P \hat{G}) \leq r_{max}(P)||\hat{G}||^2 \]  

(24)

where \( r_{max}(P) \) denotes the maximum eigenvalue of symmetric real matrix \( P \).

From (22)-(24), it follows that

\[ LV(\hat{x}) \leq \frac{1}{4} r_0 \hat{x}^T P \hat{x} + 4r_0 P \hat{x}^T P \hat{x} + r_{max}(P)||\hat{G}||^2 \]

\[ + 8r_0 P \hat{x}^T \sum_{i=1}^{n-1} \hat{\phi}_i(y) + 8r_0 P \hat{x}^T \sum_{i=1}^{n-1} \hat{\phi}_i^2(y) \]  

(25)

5. The DSC controller design and stability analysis

In this section, to overcome the problem of “explosion of complexity”, the DSC technique proposed in [25] will be introduced to a class of stochastic nonlinear systems described by (20). Similar to the traditional backstepping design method, the recursive design procedure contains \( n \) steps. From Step 1 to Step \( n-1 \), the virtual control law is designed at each step, finally an overall control law \( u \) is constructed at step \( n \). For the purpose of simplicity the time variable and the state vector will be omitted from the corresponding functions in the following part.

**Step 1:** Introduce a new transformation: \( z_1 = \rho'(y) \hat{y} \), where \( \rho'(y) \) is the value of the derivative of \( \rho(y) \) at \( y \). \( \hat{y} \) is a sufficiently smooth function and \( \rho'(y) > 0, \rho(0) = 0 \). This will be chosen appropriately such that \( \rho'(y) \) is strictly positive over \( R^+ \).

Set \( z_2 = \bar{x}_1 - \bar{a}_1, \quad z_3 = \bar{a}_1, \) where \( a_1 \) is the virtual controller \( \bar{a}_1 \) is a new state variable, \( \zeta_1 \) is the time constant, \( h_1 \) is the first filter error.

According to the differentiation operator mentioned above, the differential of \( h_1 \) is

\[ dh_1 = \left[ -\frac{h_1}{s_1} + B_1 (\hat{x}_1, y, \hat{W}, \hat{\vartheta}) \right] dt + C_1 (\hat{x}_1, y, \hat{W}, \hat{\vartheta}) dw_1 \]  

(26)

where \( B_1 \) and \( C_1 \) are smooth function, which have their maximum denoted by \( M_1 \) and \( N_1 \), respectively. \( \hat{W}, \hat{\vartheta} \) are the estimated value of \( W \) and \( \vartheta \), respectively.

Consider the Lyapunov function candidate

\[ V_1 = \frac{1}{2} \hat{y}^2 \]  

(27)

According to the Itô’s differential rules, the differential of the above function can be found as follows

\[ LV_1 = LV_1 + \rho'_y \hat{y} z_1 + \rho''_y \hat{y}^2 z_1 + \rho'_y \hat{y} z_2 + \rho''_y \hat{y}^2 \]  

\[ = -\frac{1}{2} \rho''_y \hat{y}^2 \]  

\[ + \frac{1}{2} (\hat{y} z_1 + \rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2) + \frac{1}{2} (\hat{y} z_2 + \rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2) + \frac{1}{2} (\hat{y} z_3 + \rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2) \]

As for the terms \( \rho'_y \hat{y} z_1 \) and \( \rho''_y \hat{y}^2 \), we have

\[ \rho'_y \hat{y} z_1 \leq \frac{\rho'_{max}(P) z_1}{4} + \frac{\rho''_{max}(P) z_1^2}{4} \leq \frac{\rho'_{max}(P) z_1}{4} + \frac{\rho''_{max}(P) z_1^2}{4} \]

\[ \rho''_y \hat{y}^2 \]  

(28)

Simplifying the above equations into (28) yields

\[ LV_1 \leq \frac{1}{4} r_0 \hat{x}^T P \hat{x} + \frac{1}{4} \rho''_{max}(P) ||\hat{G}||^2 \]  

\[ + \frac{1}{4} (2\rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2 \hat{y} z_1) + \frac{1}{4} (2\rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2 \hat{y} z_1 + \rho''_y \hat{y}^2 \hat{y} z_1) \]

\[ \leq \frac{1}{4} r_0 \hat{x}^T P \hat{x} + \frac{1}{4} \rho''_{max}(P) ||\hat{G}||^2 \]

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\[ + \frac{1}{2} (2 \beta y^2 + \beta' g_1^2 (y) + z_f y + \frac{1}{2} \xi^2 + \frac{1}{2} \xi' \xi' + z_2 (a + k_i y + \psi_{11}(y) + \theta_1 (y) d) + d y^2 + r^{-1} \beta' \psi_{11}(y) \psi_{11}(y) d + \frac{1}{2} \frac{1}{4 \xi^2} \frac{3 \xi^2 + \xi^4}{4 + 4 \xi^2} h_1^2 + 3 \quad (29) \]

where \( \tilde{\theta} = \theta - \hat{\theta} \).

**Step 2:** Defined the 2nd error surface \( z_2 = \hat{x}_1 - \hat{a}_1 \), and the 3rd error surface \( z_3 = \hat{x}_2 - \hat{a}_2 \), the differential of \( z_2 \) is
\[ d z_2 = \left[ \hat{x}_2 + k_2 y - k_1 (\hat{x}_1 + k_i y) \right] \frac{h_1}{\xi_1} dt \]
where \( \hat{a}_2 \) is a new state variable, and \( \alpha_2 \) is a design parameter. Let \( \alpha_2 \) pass through a first-order filter with time constant \( \hat{\xi}_2 \) which will be chosen later to obtain \( \hat{a}_2 \).

\[ \beta \hat{a}_2 = \alpha_2, \quad \hat{a}_2(0) = \alpha_2(0) \]

Then, define the first filter error \( h_2 \).
\[ h_2 = \hat{a}_2 - a_2 \]

According to the differentiation operator mentioned above, from the above equations, the differential of \( h_2 \) is
\[ d h_2 = \left( - \frac{h_2}{\xi_2} + B_2 (\hat{x}_2, y, \hat{W}, \hat{\theta}, \hat{a}_2, \hat{a}_2) \right) dt + C_2 (\hat{x}_2, y, \hat{W}, \hat{\theta}, \hat{a}_2, \hat{a}_2) d w_2 \]

Similar to the equation of (26) mentioned above, \( B_2 \) and \( Tr (C_2^t C_2) \) have their maximum, denoted by \( M_2 \) and \( N_2 \) respectively. Consider the Lyapunov function candidate
\[ V_2 = V_1 + \frac{1}{2} \sum \alpha_j^2 + \frac{1}{4} h_2^2 \]

According to the Itô’s differential rules, the differential of the Lyapunov function \( V_2 \) can be found as follows
\[ L V_2 = L V_1 + \frac{1}{2} \sum \left[ \hat{x}_2 + k_2 y - k_1 (\hat{x}_1 + k_i y) - \frac{1}{\xi_1} \right] \frac{h_1}{\xi_1} \]
\[ + \frac{1}{2} \frac{1}{4 \xi^2} \frac{3 \xi^2 + \xi^4}{4 + 4 \xi^2} h_1^2 + 3 \]
\[ \leq \frac{1}{2} \left[ \frac{1}{4 \xi^2} \frac{3 \xi^2 + \xi^4}{4 + 4 \xi^2} h_1^2 + \frac{1}{2} \frac{1}{4 \xi^2} \frac{3 \xi^2 + \xi^4}{4 + 4 \xi^2} h_1^2 + \frac{1}{2} \frac{1}{4 \xi^2} \frac{3 \xi^2 + \xi^4}{4 + 4 \xi^2} h_1^2 + \frac{1}{2} \frac{1}{4 \xi^2} \frac{3 \xi^2 + \xi^4}{4 + 4 \xi^2} h_1^2 + 3 \quad (30) \]

**Step 3** (3 \( \leq i \leq 11 \)): Define the ith error surface as \( z_i = x_i - a_i \), the differential of it is
\[ d z_i = \left[ \hat{x}_i + k_i y - k_{i-1} (\hat{x}_{i-1} + k_j y) - \frac{1}{\xi_{i-1}} \right] \frac{h_{i-1}}{\xi_{i-1}} \]
Define the virtual controller \( a_i \), which is also a design parameter. Introduce a new state variable \( \hat{a}_{i-1} \) and let \( a_{i-1} \) pass through a first-order filter with time constant \( \hat{\xi}_{i-1} \) which will be chosen later to obtain \( \hat{a}_{i-1} \).
\[ \beta \hat{a}_{i-1} + \hat{a}_{i-1} - a_{i-1}, \quad \hat{a}_{i-1}(0) = a_{i-1}(0) \]

Then, define the first filter error \( h_i \)
\[ h_i = \hat{a}_{i-1} - a_{i-1} \]

According to the differentiation operator mentioned above, from the above equations, the differential of \( h_i \) is
\[ d h_i = \left( - \frac{h_i}{\xi_{i-1}} + B_i (\hat{x}_i, y, \hat{W}, \hat{\theta}, \hat{a}_i, \alpha_i) \right) dt + C_i (\hat{x}_i, y, \hat{W}, \hat{\theta}, \hat{a}_i, \alpha_i) d w_i \]

Similar to the equation of (26) mentioned above, \( B_2 \) and \( Tr (C_2^t C_2) \) have their maximum, denoted by \( M_2 \) and \( N_2 \) respectively. Consider the Lyapunov function candidate \( V_i \)
\[ V_i = V_{i-1} + \frac{1}{2} \sum \alpha_i^2 + \frac{1}{4} h_i^2 \]

According to the Itô’s differential rules, the differential of the Lyapunov function \( V_i \) can be found as follows
\[ L V_i = L V_{i-1} + \frac{1}{2} \sum \left[ \hat{x}_i + k_i y - k_{i-1} (\hat{x}_{i-1} + k_j y) - \frac{1}{\xi_{i-1}} \right] \frac{h_{i-1}}{\xi_{i-1}} \]
\[ + \frac{1}{2} \frac{1}{4 \xi_{i-1}} \frac{3 \xi_{i-1}^2 + \xi_{i-1}^4}{4 + 4 \xi_{i-1}^2} h_{i-1}^2 + 3 \]
\[ \leq \frac{1}{2} \left[ \frac{1}{4 \xi_{i-1}} \frac{3 \xi_{i-1}^2 + \xi_{i-1}^4}{4 + 4 \xi_{i-1}^2} h_{i-1}^2 + \frac{1}{2} \frac{1}{4 \xi_{i-1}} \frac{3 \xi_{i-1}^2 + \xi_{i-1}^4}{4 + 4 \xi_{i-1}^2} h_{i-1}^2 + \frac{1}{2} \frac{1}{4 \xi_{i-1}} \frac{3 \xi_{i-1}^2 + \xi_{i-1}^4}{4 + 4 \xi_{i-1}^2} h_{i-1}^2 + \frac{1}{2} \frac{1}{4 \xi_{i-1}} \frac{3 \xi_{i-1}^2 + \xi_{i-1}^4}{4 + 4 \xi_{i-1}^2} h_{i-1}^2 + 3 \quad (31) \]

where \( z_{i+1} = \hat{x}_i - \hat{a}_i \)

**Step n** This is the final step. The final control law will be derived in this step. Define the nth error surface as \( z_n = \hat{x}_n - a_{n-1} \), the virtual controller \( a_{n-1} \), which is also a design parameter. Introduce a new state variable \( \hat{a}_{n-1} \) and let \( a_{n-1} \) pass through a first-order filter with time constant \( \hat{\xi}_{n-1} \) which will be chosen later to obtain \( \hat{a}_{n-1} \).
\[ \hat{\xi}_{n-1} \hat{a}_{n-1} + \hat{a}_{n-1} = a_{n-1}, \quad \hat{a}_{n-1}(0) = a_{n-1}(0) \]

Then, define the first filter error \( h_{n-1} \)
\[ h_{n-1} = \hat{a}_{n-1} - a_{n-1} \]

The differential of \( z_n \) is
\[ d z_n = \left[ u - k_n (\hat{x}_n + k_j y) - \frac{1}{\xi_{n-1}} \right] \frac{h_{n-1}}{\xi_{n-1}} \]

Consider the Lyapunov function candidate \( V_n = V_{n-1} + \frac{1}{2} \sum \alpha_n^2 + \frac{1}{4} h_{n-1}^2 \)

where \( W = W - \hat{W} \) denotes the estimate errors of the unknown NN weight vector \( W, \hat{W} \) is the estimate of the unknown NN weight vector \( W \).

According to the Itô’s differential rules, the differential of the Lyapunov function \( V_n \) can be found as follows
\[ L V_n = L V_{n-1} + \frac{1}{2} \left[ u - k_n (\hat{x}_n + k_j y) - \frac{1}{\xi_{n-1}} \right] \frac{h_{n-1}}{\xi_{n-1}} \]
\[ + \frac{1}{2} \frac{1}{4 \xi_{n-1}} \frac{3 \xi_{n-1}^2 + \xi_{n-1}^4}{4 + 4 \xi_{n-1}^2} h_{n-1}^2 + 3 \]

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\[\begin{align*}
&+ n^{-1} \sum_{i=2}^{n} \left( \frac{\alpha_{n-1} - \alpha_{n-1}}{\xi_{i-1} - \xi_i} \right)^4 + \sum_{i=2}^{n} z_i^2 \left( \frac{5}{2} \alpha_i + k_i y - k_{i-1} (x_1 + k_1 y) \right) \\
&+ \hat{W}^T (1 - \hat{W} + \hat{W}^T y) \psi (y) \\
&\leq r_0 \beta_{\text{max}} (P) \left( \frac{n}{\rho^2} \sum_{i=2}^{n} g_i^2 + \frac{1}{\rho^2} \sum_{i=1}^{n} g_i^2 k_i^2 \right) \\
&\leq r_0 \beta_{\text{max}} (P) \left( \frac{n}{\rho^2} \sum_{i=2}^{n} \theta_i \psi (y) + \frac{1}{\rho^2} \sum_{i=1}^{n} \theta_i \psi (y) \right) \\
&\leq \frac{1}{2} (2 \rho y^2 + \beta \theta (y)) \psi (y) \\
&\leq \frac{1}{2} (2 \rho y^2 + \beta \theta (y)) \psi (y)
\end{align*}\]

As for the terms \(r_0 \beta_{\text{max}} (P) ||\phi||^2\) and \((1/2)(2 \rho y^2 + \beta \theta (y))\), we have

\[\begin{align*}
&\text{Define a smooth function } \psi_{12} (y), \text{ let} \\
&\psi_{12} (y) = \frac{y}{2} (r_0 (P) \rho ||F(y)||^2 + r_0 \beta_{\text{max}} (P) \sum_{i=2}^{n} \theta_i \psi (y)) \\
&+ \frac{1}{2} (2 \rho y^2 + \beta \theta (y)) \psi (y) \sum_{i=2}^{n} \theta_i \psi (y) \\
&\leq \frac{1}{2} \psi (y) \\
&\leq W \psi (y) + \phi (y) \\
&\leq W \psi (y) + \phi (y)
\end{align*}\]

Substituting the above equations into (32), we have

\[\begin{align*}
&\text{LV}_n \leq - \frac{1}{2} r_0 \psi (y) + r_0 \phi (\psi) + \psi \psi \psi + \psi (y) \psi \psi \\
&\leq \frac{1}{2} \psi (y) + \phi (y) + \psi (y)
\end{align*}\]

where \(\psi (y)\) is a smooth nondecreasing function to be chosen later. By choosing the virtue control \(\alpha_i\) and \(\alpha_i\), the adaptive law \(u\) and the adaptive laws \(\hat{W}\) and \(\hat{W}\) can be selected as

\[\begin{align*}
&\alpha_i = - \frac{1}{2} z_i^2 - k_i y_1 - y_1 (y_1 \hat{y} - W \hat{S} (y)) \\
&- \psi (y) \psi (y) \\
&- \frac{1}{2} \psi (y) \\
&- \frac{1}{2} \psi (y) \\
&\leq \frac{1}{2} \psi (y) \\
&\leq \frac{1}{2} \psi (y) \\
&\leq \frac{1}{2} \psi (y)
\end{align*}\]

\[\begin{align*}
&\alpha_i = \frac{1}{2} z_i^2 - k_i y_1 - y_1 (y_1 \hat{y} - W \hat{S} (y)) \\
&+ \frac{1}{2} \psi (y) + \psi (y)
\end{align*}\]

\[\begin{align*}
u_1 (y) = \beta (y) \theta (y) + \beta (y) \psi (y)
\end{align*}\]

\[\begin{align*}
&\hat{W} = - \alpha_2 T \hat{W} + T \psi (y) \\
&\text{If we choose } \alpha_2 \text{ and } \psi \text{ to satisfy } \beta (y)^2 \psi (y) \geq \beta (y)^2 \text{, and substitute the above equations into (33), we have}
\end{align*}\]

\[\begin{align*}
&\text{LV}_n \leq \frac{1}{2} \psi (y^2) - \frac{1}{2} \psi (y^2) \psi (y) \\
&+ \psi (y) \psi (y)
\end{align*}\]

\[\begin{align*}
&\text{Remark 3. The control law and adaptive law have been chosen, but whether the positive functions } \beta (y^2) \text{ and } \psi (y^2) \text{ can be found to meet the conditions mentioned above, it need to be settled.}
\end{align*}\]

If there exists a class \(\infty\) function \(\pi\) such that

\[\begin{align*}
&\pi (\psi) \leq \psi (V (\psi)) + D \\
&\text{where}
\end{align*}\]

\[\begin{align*}
&c = \min \left\{ c_1, c_2, \ldots, c_n, \frac{1}{2} \rho \right\} \psi (y) + \frac{1}{2} \psi (y) \psi (y) \\
&\beta (y^2) \geq \frac{1}{2} \psi (y^2) + \frac{1}{2} \psi (y) \psi (y) \\
&- \psi (y) \psi (y)
\end{align*}\]

As in paper [10], there always exist a smooth function \(\beta (y^2)\) and arbitrarily small parameters \(\delta\), such that

\[\begin{align*}
&\beta (y^2) \geq 4r_0 \psi (y) + \frac{1}{2} \beta (y) \psi (y) \\
&\beta (y^2) > 0
\end{align*}\]

where \(\beta (y) = 0\), according to Assumption 3 and \(y \leq (1/2d_2) y^2 + (1/2d_2)\), we have

\[\begin{align*}
&\text{where} \\
&\text{where}
\end{align*}\]

\[\begin{align*}
&\text{Substituting (33), we have}
\end{align*}\]

\[\begin{align*}
&\text{where} \\
&\text{where}
\end{align*}\]
and substituting $r_1(s)$ and $r_2(s)$ into $(1 + \rho_1)r_1 + (1 + \rho_2)r_2(s)$, we get

$$(1 + \rho_1)r_1 + (1 + \rho_2)r_2(s) \leq s$$

By Theorem 2, for any $\varepsilon > 0$, there exist KL function $\beta_c$ and nonnegative constant $d_n$, such that

$$P\{X(t) \leq \beta_c(X(0), t) + d_n \geq 1 - \varepsilon, t \in [0, \infty)$$

where $X(t) = (\xi(t), y(t), z_1(t), \ldots, z_m(t), \tilde{b}(t))$.

**Remark 4.** If the value of $\rho_1$ and $\rho_2$ are chosen large enough, the constant can become arbitrarily small. So we can conclude the following results, if Assumptions 2 and 3 hold for system (10), the output-feedback system consisting of (10), (20) and (34) has a unique practically stable solution and uniformly bounded in probability. Furthermore, the output can be regulated to an arbitrarily small neighborhood of zero.

6. A simulation example

Consider the following nonlinear system

$$\dot{z} = \left( -\frac{5}{2}z + 1 \right) dt + \frac{\sqrt{3}}{2} x_1^2 dw_0$$

$$dx_1 = [x_2 + 0.8 \sin x_1 + 0.9(x_1^2 + \cos x_1)] dt + 0.9x_1^2 dw_1$$

$$dx_2 = \left[ u + \frac{x_1}{x_1^2 + 1} + 0.9z \cos x_1 \right] dt + 0.9x_1^2 dw_2$$

$$y = x_1$$

Choosing the observer

$$\dot{x} = u - k(x - ky)$$

According to the results in Section 5, the adaptive output feedback virtue control law, control law and adaptive law are given by

$$\alpha_1 = -\frac{1}{2}\frac{1}{x_1} - k_1y - \psi_1(y) - \psi_1(y)\frac{\dot{W}}{y} - \tilde{b}(y) - \frac{1}{2}\frac{1}{x_1}$$

$$u = k_1(x_1 + k_1y) - c_1z_2 - z_2$$

$$\dot{\tilde{b}} = -r_1\tilde{b} - r_2\psi_1(y) - r_3\psi_1(y)$$

$$\dot{W} = -\sigma_2 T\dot{W} + T\psi_1(y)$$

For the $z_i$-subsystem in (38), the Lyapunov function can be chosen as $V_0(z_i) = \frac{\sigma_1}{2}\|z_i\|^2 = \frac{\sigma_1}{2}\|z_i\|^2$, we can prove that $LV_0(z_i) \leq -4V_0(1/2)y^2 + d_0$. It is easy to obtain that $\psi_1 = y^2$, $\phi_1 = 0$. Then we can get $s = d_2, s = 12r_0p^2(1 + k^2), p = 1/(2k)$. From (36), we can get

$$r(s) = \frac{\frac{1}{2}(1 + \rho_1)}{d_r} \left( \frac{2d_1(1 + \rho_2)d_1}{c} - \left( \frac{8d_22\psi_1}{c_{10}} \right) \right) \psi_1$$

where $c = \min(1.4c_2, 1.4p, r_1, r_2)$.

Choosing the function $\beta_c(y^2)$ as $\beta_c(y^2) = \psi_1(y^2) + 4d_2y^2$, where $c_2 = 4(1 + \rho_1)2d_1(1 + \rho_2)d_1^{10}\psi_1^2/(d_2c_{10}^2\psi_1^2)$. Hence, the function $\psi_1(y^2) = ((c_22\psi_1/y^2)^{10} - 4d_2)\left(\psi_1^{2}\psi_1^{10} - 1.5d_2\right)$. In the simulation the initial values and the suitable parameters can be chosen as $x_1(0) = 0.5, x_2(0) = -1.2, z(0) = -2.0, \xi(0) = -1.6, \theta(0) = 0.1, c_1 = 1, d_1 = d_2 = 0.2, d_3 = 10, d_4 = 1; c_0 = 4, c_2 = 5, r_0 = 0.2, r = 0.5, v_1 = 5, v_2 = 4, \rho_1 = 1, \rho_2 = 0.1, \sigma_1 = \sigma_2 = 2, d_1 = 0.2, d_2 = 1$. The simulation results are shown in Figs. 1–3.

7. Conclusions

In this paper, the DSC based adaptive NN output feedback control has been proposed for a class of stochastic nonlinear systems with unmodeled dynamics and unmeasured states. Using the proposed technique we can eliminate the problem of ‘explosion of complexity’. To simplify the design procedure and reduce the computation loads, only one neural network is constructed to compensate for all unknown functions depending on the system output. Furthermore, the reduced-order observer is designed to estimate the unmeasured states. It is shown that all the solutions of the closed-loop system are bounded in probability, and the output of the system converges to a small neighborhood of the origin by choosing design parameters appropriately. The
effectiveness of the proposed approach has been verified by a simulation example.

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References


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