User Fairness Analysis of a Game Theory Based Power Allocation Scheme in OFDMA Relay Systems

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Abstract — In this paper, the performance of user fairness of resource allocation is considered in the downlink of relay-based OFDMA cellular systems. In the downlink transparent frame structure, the base stations (BSs) transmit in the first time subslot and the relay stations (RSs) transmit in the second time subslot. A non-cooperative power allocation game is defined for maximizing throughput and reducing co-frequency interference. The distributed process of this game based power allocation scheme is given. Of particular interest in the results is the fairness seen by users and this is compared with equal power allocation, the simulation results showing that the reduced user fairness is worthwhile due to a great power saving.

Keywords-OFDMA; relay; power allocation; game theory; fairness

I. INTRODUCTION

Relay networks have been widely considered to be a suitable architecture for future mobile networks, as adding relay stations (RS) can increase the capacity and reduce power consumption. Work on relay selection [1], subchannel scheduling [2] and power allocation [3-5] has been carried out in the context of a single cell, but these algorithms omitted the co-channel interference (CCI) in multiple cells, which is a major factor that affects system performance.

Fairness of resource allocation to users is an important issue that is often neglected although the work in [3-5] treated it as an explicit goal, albeit in a single cell. In this paper, the fairness resulting from a power allocation strategy derived from Game Theory is studied and compared with that using the explicit strategy in multi-cell scenario.

In OFDMA systems, there is no co-frequency interference between users located in the same cell in the downlink. However, co-frequency interference from other cells exists caused by transmitters in other cells using the same subchannel. So the power allocation problem of multi-cell OFDMA relay systems can be viewed as a non-cooperative game, in which the BSs (RSs) in multi-cell will compete with each other in co-frequency subchannel for channel capacity by increasing their transmission power selfishly.

A non-cooperative power allocation game is introduced into OFDMA systems in [6] to maximize users’ utilities in a distributed way in multi-cell OFDMA systems. [7] considers the non-cooperative power allocation game in multi-cell OFDMA relay networks, but it considers power allocation only in the second time subslot and only in a slow-fading channel. In [8], we defined power allocation game both in first and second time subslots in a fast fading channel scenario. In this paper, we look at the effect on user fairness and power efficiency with the power allocation game in multiple cell OFDMA relay systems.

The structure of this paper is: the system model is given in section II; the non-cooperative power allocation game (NPAG) algorithm in section III and simulation results in IV, and the conclusion is given at the end of the paper.

II. SYSTEM MODEL

In the multi-cell OFDMA relay system, at least one fixed decode and forward (DF) relay station (RS) is deployed in each cell. Fig. 1 shows the normal definition of links for a relay network.

Figure 1. Definition of terms in relay network

A time slot is divided into the first time subslot and the second time subslot. The transparent frame structure is used in this paper (Fig. 270a in [9]). The subchannels in the first time subslot are allocated to the direct link users and the first hop link of relay link users; the subchannels in the second time subslot are allocated to the second hop link of relay link users.

We use $n \in \{1,2,\cdots,N\}$ as the index of a cell and $s$ as the index of a transmitter in each cell; in the first time subslot the transmitter is BS ($s=0$), and in the second time subslot the transmitter is RS ($s=r$, $r$ is the index of RSs); $k$ is index of a receiver in each cell, when the RS is a receiver, $k=r$, the direct link user is $k_d$ and the relay link user is $k_r$; the time subslot is $t \in \{1,2\}$; the index of subchannel is $m \in \{1,2,\cdots,M\}$. 
Denote the interference received by transmitter \( s \) (BS or RS) and receiver \( k \) (RS or MS) in the \( n \)th cell on the subchannel \( m \) at the \( t \)th time subslot \( f_{n,m,t}^{(s,k)} \) by:

\[
I_{n,m,t}^{(s,k)} = \sum_{k' \neq k} g_{n,m,t}^{(s,k')} f_{n,m,t}^{(s,k')} + \sigma^2
\]

(1)

In (1), \( g_{n,m,t}^{(s,k)} \) and \( f_{n,m,t}^{(s,k)} \) are the link gain and transmission power on the subchannel \( m \) in the cell \( n \) between transmitter \( s \) and receiver \( k \) at the \( t \)th time subslot; \( \sigma^2 \) is the power of additive white Gaussian noise (AWGN).

The signal to interference plus noise ratio (SINR) on the \( m \)th subchannel between transmitter \( s \) and receiver \( k \) at the \( t \)th time subslot in the \( n \)th cell is

\[
\gamma_{n,m,t}^{(s,k)} = g_{n,m,t}^{(s,k)} P_{n,m,t}^{(s,k)} / I_{n,m,t}^{(s,k)}
\]

(2)

The capacity of the \( m \)th subchannel between \( s \) and \( k \) of the cell \( n \) in the time subslot \( t \) is:

\[
c_{n,m,t}^{(s,k)} = B_m \log_2 \left(1 + \gamma_{n,m,t}^{(s,k)}\right)
\]

(3)

The subchannels can be allocated with any scheduling algorithms. A fixed subchannel allocation is used in this paper to remove the effect of scheduling on system performance. For a direct link user, the achieved data rate is \( R_{1,k} \) and the achieved data rates of the relay link user in the first hop link and second hop link are \( R_{2,k} \) and \( R_{2,k}^t \) respectively. The available data rate of a relay link user is determined by the minimum capacity of the first hop link and the second hop link in the DF relay [7-8].

III. NON-COOPERATIVE POWER ALLOCATION GAME

In OFDMA systems, the subchannels are independent, so maximizing the utility of one cell is the same as maximizing the utility on each subchannel in this cell. Thus, the multi-cell non-cooperative power allocation game (NPAG) can be formulated on each co-frequency subchannel. Furthermore, the BSs transmit data in the first time subslot and the RSs transmit data in the second time subslot, so the NPAG can be formulated as \( G^{BS} = \{n\} \{P_{n,m}^{BS}, u_{n,m}^{BS}\} \) in the first time subslot, and \( G^{RS} = \{n\} \{P_{n,m}^{RS}, u_{n,m}^{RS}\} \) in the second time subslot. In \( G^{BS} \), \{n\} is the set of co-channel interference cells, i.e. a set of players competing with each other. \( P_{n,m}^{BS} = [p_{n,m,1}^{BS}, p_{n,m}^{BSmax}] \) is the strategy set of transmission power of the BS on the \( m \)th subchannel in \( n \)th cell; \( P_{n,m}^{BS} = [p_{n,m,1}^{BS}, p_{n,m,2}^{BS}, \cdots, p_{n,m}^{BS}] \) is the BS transmission power vector of all the cells on the \( m \)th subchannel; \( P_{n,m}^{BS} \) is the BS transmission power vector on the \( m \)th subchannel of all the cells excluding the power \( p_{n,m}^{BS} \) in the \( n \)th cell. So, the BS transmission power vector on the \( m \)th subchannel of all the cells can be expressed as \( P_{m}^{RS} = (P_{n,m}^{BS}, \cdots, p_{n,m}^{BS}) \). \( u_{n,m}^{BS} \) is the utility of BS on the \( m \)th subchannel in the \( n \)th cell, and each cell will maximize its utility rationally. Similarly, \( n \), \( P_{n,m}^{RS} \) and \( u_{n,m}^{RS} \) are defined in \( G^{RS} \).

Each transmitter prefers to increase its transmission power to increase its capacity and this would increase the interference and degrade the capacity of adjacent cells in which users are using the co-frequency subchannel. Then other cells whose capacities are affected would also increase their transmission power to increase their capacities. So a pricing factor is used in order to let a transmitter in each cell not only consider its capacity but also control its interference to others [6]. The utility of \( n \)th BS on the subchannel \( m \) can be defined as:

\[
u_{n,m}^{BS} = B_m \log_2 \left(1 + \sum_{k=1}^{K} \gamma_{n,m,t}^{(0,k)}\right) - \alpha^{BS} P_{n,m,t}^{0,0,k}
\]

(5)

in which, \( k_m \) represents the subchannel \( m \) is allocated to receiver \( k \) and \( P_{n,m}^{RS} = p_{n,m}^{0,0,k} \) for simplicity. \( \alpha^{BS} \) is the pricing factor of a BS which is positive scalar and can be considered to have unit bps/W. With the introduction of \( \alpha^{BS} \), a transmitter will not just increase its power, but also will consider the interference to other cells taking a view of the whole system. The bigger \( \alpha^{BS} \) is, the higher the price will be for increasing power, so each cell will use less power, which causes less interference to other cells.

In order to use the channel efficiently, the match between the second hop link and the first hop link of a RS should be considered. A match factor is introduced, which is defined as the ratio of the first hop link capacity of RS \( r \) and second hop link capacity of RS \( r \) in the cell \( n \):

\[
\beta^r_n = \frac{\sum_{k_r} R_{1,k_r}}{\sum_{k_r} R_{2,k_r}}
\]

(6)

As the BS transmits data for both direct link users and RSs in the first time subslot, the \( \alpha^{RS} \) for direct link users and RSs is different in the first time subslot, which is:

\[
\alpha^{RS} = \begin{cases} 
\alpha^{k_r} & \text{for direct link user} \\
\alpha' \beta^r_n & \text{for RS } r \text{ in cell } n 
\end{cases}
\]

(7)

\( \alpha^{k_r} \) is the pricing factor for a direct link user, and \( \alpha' \) is the base price for an RS; direct link users and the RSs use a different pricing factor to compete for transmission power. When \( \beta^r_n > 1 \), i.e., the channel capacity of the first hop link is bigger than that of the second hop link, the \( \alpha^{BS} \) increases
compared with the base pricing factor \( \alpha' \). Therefore, the BS decreases its transmission power to the RS \( r \) to match the capacity of the first hop link and the second hop link. The larger the difference between the data rate of two hop links is, the bigger \( \beta_r' \) is; as a result, \( \alpha'_{BS} \) becomes bigger, and the transmission power is more strictly controlled. When \( \beta_r' < 1 \), the BS tends to increase its transmission power to increase the first hop link capacity of RS to match that of the second hop link.

Similarly with (5), the utility of the RS in the first hop link of the center of cell and 6 RSs are uniformly placed in the cell at 0.5 of the cell radius away from the BS. The channel model [11] includes shadow fading, large scale pathloss, and multi-path fading. Detailed simulation parameters are shown in Table I.

### IV. Simulation Analysis

A 19 cells downlink OFDMA relay system is simulated with wrap-around technology being used for multi-cell interference [10]. In each single cell, one BS is positioned in the centre of cell and 6 RSs are uniformly placed in the cell at 0.5 of the cell radius away from the BS. The channel model [11] includes shadow fading, large scale pathloss, and multi-path fading. Detailed simulation parameters are shown in Table I.

Round Robin scheduling is used for subchannel allocation. The system throughput is the sum of the average throughput.
achieved by each user in one cell. The Raj Jain fairness index [12] is adopted in here to measure the fairness among users. In this paper, we examine the performance, particularly fairness, under different pricing factor and different user numbers.

### TABLE I. SIMULATION PARAMETERS FOR SYSTEM MODEL

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>10MHz</td>
</tr>
<tr>
<td>Subchannel number</td>
<td>15</td>
</tr>
<tr>
<td>Cell radius</td>
<td>1000 m</td>
</tr>
<tr>
<td>Noise PSD</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>Transmission timing interval (TTI)</td>
<td>1 ms</td>
</tr>
<tr>
<td>BS Transmission power limitation</td>
<td>20W</td>
</tr>
<tr>
<td>RS Transmission power limitation</td>
<td>10W</td>
</tr>
<tr>
<td>Target BER</td>
<td>$10^{-7}$</td>
</tr>
</tbody>
</table>

Since the transmission power from RS to relay link users is allocated first, the optimal RS pricing factor will be obtained first [9]. Fig.1 - Fig. 3 show the effect of the BS pricing factor on system performance with the optimal RS pricing factor.

It can be seen from Fig. 1 that system throughput increases first and then decreases with increasing BS pricing factor and when the BS pricing factor is zero, the system throughput is the same as that of equal power allocation. When the BS pricing factor equals $3 \times 10^5$, the largest system throughput gain is obtained and increases by 3% compared with that of BS equal power allocation.

Fig. 2 is the average BS transmission power under different RS pricing factor, which shows that the BS average transmission power decreases as pricing factor rises. At the optimum (for throughput) BS pricing factor of $3 \times 10^5$, the average power of a BS will reduce 25% compared with that of a BS using equal power allocation.

Fig. 3 is the average BS transmission power under different RS pricing factor, which shows that the BS average transmission power decreases as pricing factor rises. At the optimum (for throughput) BS pricing factor of $3 \times 10^5$, the average power of a BS will reduce 25% compared with that of a BS using equal power allocation.

The user fairness under different BS pricing factor is shown Fig. 3, from which, we can see that user fairness index declines with the BS pricing factor rises. This conclusion also can be deduced from (11), in which, the larger SINR is, the higher transmission power will achieve. However, the reduction of fairness index is relatively small compared with the improvement in power consumption.

In Fig. 4 – Fig. 6, the system performance of NPAG is compared with equal power allocation on each subchannel (named EQPA) with different user numbers. In the simulation, the BS pricing factor is $3 \times 10^5$ and RS pricing factor is $2.5 \times 10^5$, the values at which maximum throughput will be obtained.

Fig. 4 is the system throughput of each cell with different user numbers. At first, the system throughput increases directly with the number of uses, but after a certain number is reached the throughput saturates and then decreases. This is because the larger number of users will get multi-user diversity gain, but, as the Round Robin scheduling tries to keep user fairness, the system throughput will decrease as the number gets too large. It can be seen that the throughput of NPAG is nearly 4% larger than that of EQPA.
Fig. 4 shows the average transmission power of BS and RS under EQPA, NPAG. Compared with EQPA, and it is clear that the transmission power of BS and RS are reduced greatly by the use of NPAG, in which, the overall power (BS power plus and RSs power) will be reduced by about 25%.

The user fairness index with different user numbers is shown in Fig. 6. It can be seen that the NPAG will decrease the user fairness compared with EQPA, which is consistent with the result of Fig. 3. The user fairness of NPAG is nearly 4% less than that of EQPA, but this reduction is much less than the improvement in power saving.

V. CONCLUSION

In this paper, we study the power allocation for multi-cell with co-channel interference in OFDMA cellular relay systems. A distributed power allocation algorithm (NPAG) based on non-cooperative game via pricing is proposed.

The simulation results show that system throughput is improved and transmission power is reduced compared with equal power allocation without having too much effect on fairness. This is important as while power saving is important the performance seen by the users needs to be maintained for the system to be acceptable.

REFERENCES


