Modeling and Compensation of Hysteresis in Piezoceramic Transducers for Vibration Control

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ABSTRACT: Hysteretic behavior in piezoceramic transducers is investigated theoretically and experimentally. The applicability of the rate-independent generalized Maxwell resistive capacitor (MRC) hysteresis model is established. Methods for MRC and inverse MRC online model identification are developed by first establishing that the MRC and its inverse are the same particular cases of the classical Preisach hysteresis model. This enables use of the extensive mathematical framework that has been developed for Preisach models. A method of incorporating the MRC model in a feedforward control scheme for hysteresis compensation is also presented. Experimental studies on a 1-3 piezoceramic composite support the theoretical developments and their applicability to piezoceramics.

1. INTRODUCTION

A number of studies have shown that significant hysteresis is present in piezoceramic transducers, such as those based on lead zirconate (PZT). Hysteresis can have a detrimental effect on the performance of the piezoceramic in position or vibration control applications. Hysteresis can cause multiple output states for a given input state, thus frustrating open-loop control, and it can generate unwanted amplitude-dependent phase shifts and harmonic distortion which reduce the effectiveness of feedback control. On the other hand, it can be argued that hysteresis could potentially be harnessed as a means of dissipating unwanted vibration.

Before hysteresis in piezoceramics can be properly accounted for and/or utilized in control algorithms, system models and design strategies, its fundamental behavior must be better understood and properly modeled. To this end, a number of studies can be found in the literature employing a range of hysteresis operators. Two of the more promising operators, which are causal in nature and thus potentially useful in active control algorithms, are the Preisach hysteresis model (Hughes and Wen, 1997; Ge and Jouaneh, 1995, 1996, 1997) and Maxwell resistive capacitor model (Goldfarb and Celanovic, 1997; Royston and Houston, 1998; Lee and Royston, 2000). Preisach models have been applied primarily to ferromagnetic hysteresis and more recently to magnetostrictive hysteresis, although they are not domain specific (Mayergoyz, 1991; Mayergoyz and Friedman, 1988, 1990; Adly and Mayergoyz, 1991; Schafer and Janocha, 1995). Their mathematical properties have been rigorously developed and practical issues for control and simulation, such as a system identification methodology and the existence of an inverse, have been addressed (Mayergoyz, 1991; Krasnoselskii and Pokrovskii, 1983). The Maxwell resistive capacitor model has been employed by a few researchers to describe dielectric piezoceramic hysteresis (Goldfarb and Celanovic, 1997; Royston and Houston, 1998; Lee and Royston, 2000) even though it has its roots in describing mechanical hysteresis. Unlike the Preisach models, its properties with respect to piezoceramic hysteresis have not been rigorously defined and issues of identification and existence of an inverse have not been adequately investigated. Nonetheless, an advantage of the Maxwell resistive capacitor model over that of the Preisach model is its ease of implementation in numerical algorithms for simulation or control (Royston and Houston, 1998).

Primary objectives addressed in this paper include an investigation of the relationship between the classical Preisach model (CPM) and the Maxwell resistive capacitor model (MRC) and its inverse. Specifically, it is shown that the MRC and its inverse are a particular subset of the CPM model, thus endowing them with certain properties. MRC and inverse MRC identification procedures are established by utilizing their formulation as CPM models. Implementation of an MRC-based algorithm for feedforward hysteresis compensation is also investigated.

2. HYSTERESIS IN PIEZOCERAMICS

The 1-3 piezoelectric ceramic composite considered in this study is shown schematically in Figure 1. The 1-3 consists of PZT-5H rods oriented in the thickness or 3 direction, which are uniformly spaced and separated by a compliant polymer material. The volume fraction of rods considered in this study is 15%. Any change in the length of the rods (3 direction), due to the application of a voltage across the elec-
trodes, will appear as a change in the thickness of the entire layer. Ideally, the compliant material absorbs any lateral strain in the rods (1 and 2 direction). In terms of volume displacement, this results in an actuator with greater authority than that of an equivalent volume of PZT.

See Royston and Houston (1998) for a detailed derivation of the constitutive equations of the 1-3 including nonlinearity. It is sufficient to state here that, under a specified range of operating conditions, the 1-3 may be treated like a one-dimensional homogeneous piezoelectric component with effective dielectric, mechanical and piezoelectric constants that are based on weighted averages over the domain of the 1-3. Consequently, the linear constitutive equations may be expressed in the following form.

\[
S = s^D T + g D \tag{1a}
\]
\[
E = -g T + \beta^T D \tag{1b}
\]

Here, nomenclature generally follows that of ANSI/IEEE standard 176-1987 on piezoelectricity. In this formulation independent variables are mechanical stress \( T \) and electric displacement \( D \). Dependent variables are mechanical strain \( S \) and electric field \( E \). Superscripts \( D \) and \( T \) refer to “at constant” electrical displacement or mechanical stress, respectively. Coefficients \( s \), \( g \), and \( \beta \) refer to the elastic compliance coefficient, piezoelectric constant, and the dielectric impermeability, respectively.

A number of studies have shown that, even at relatively low electrical and/or mechanical stress levels, piezoelectric ceramics exhibit substantial rate-independent hysteretic behavior that is not accounted for in the linear Equations (1a,b). For example, Goldfarb and Celanovic (1997) and Main and Garcia (1995, 1997a, 1997b) have measured a strong hysteretic relation between electrical displacement \( D \) and field \( E \). Take hysteretic behavior in the 1-3 to be in the dielectric \( \beta^T \) relation. It will be denoted as a bracket \{ \} in the following equations.

\[
S = s^D T + g D \tag{2a}
\]
\[
E = -g T + \{\beta^T D\} \tag{2b}
\]

These equations do agree with experimental observations of PZT reported in the literature. For example, Goldfarb and Celanovic (1997) observed that the applied electrical displacement \( D \) vs. strain \( S \) relation under zero stress \( T \) was reversible, but that applied electric field \( E \) vs. \( S \) under zero \( T \) was not. They also observed that the mechanical stress-strain relation under constant electric displacement was reversible whereas the relation under constant electric field was hysteretic. Damjanovic (1997) observed that the applied stress vs. electrical displacement relation was hysteretic.

3. THE MAXWELL RESISTIVE CAPACITOR HYSTERESIS MODEL AND ITS INVERSE

In a previous article (Royston et al., 1999) it has been proven that the Maxwell resistive capacitor (MRC) hysteresis model and its inverse are in fact a form of the classical Preisach model (CPM). In that article the particular Preisach model for the MRC was derived and an online identification routine based on Preisach principles was formulated. In this article some of these results are briefly summarized and an identification technique for the inverse MRC model is developed.

3.1 MRC Model Development

Though the MRC model has its roots in describing me-
Mechanical hysteresis between stress and strain, it is not domain specific. It is schematically represented in Figure 2 where the formulation is in terms of electrical field and displacement, $E$ and $D$. Referring to Figure 2, the model may be implemented into the otherwise linear constitutive Equations (2a) and (2b) as follows:

\[ S = \beta^T D + g D \]  
\[ E = -g T + MRC(\beta^T D) \]

with $MRC(\beta^T D) = \sum_{i=1}^{n} E_{rc}^{(i)}$ where

\[ \text{if } |\beta^{(i)}(D - D_{b}^{(i)})| < e_{rc}^{(i)} \text{ then } E_{rc}^{(i)} = \beta^{(i)}(D - D_{b}^{(i)}) \]
\[ \text{otherwise } E_{rc}^{(i)} = e_{rc}^{(i)} \text{ sign}[\dot{D}] \]  

with $D_{b}^{(i)}$ is set such that $|\beta^{(i)}(D - D_{b}^{(i)})| = e_{rc}^{(i)}$

Here, the terms $\beta^T$, $e_{N}$, $\mu$, $e_{rc}$, and $D_{b}$ may be viewed as electrical analogies to a mechanical spring stiffness, normal force, Coulomb friction coefficient, the force due to Coulomb friction and the displacement from an equilibrium position of the massless box.

The authors have established that the MRC hysteresis model and its inverse are particular cases of the classical Preisach hysteresis model (Royston et al., 1999). Indeed, the basic MRC unit or operator is an elementary stop hysteron, as defined in the magnetics literature (Visintin, 1994; Miano et al., 1996; Bobbio et al. 1997); hysterons are also the basic building blocks of the Prandtl-Ishlinskii hysteresis models. Additionally, the MRC operator is related to the Krasnosel’ski and Pokrovskii (KP) operator mentioned in recent studies that use it for actuator hysteresis compensation (Webb et al., 2000). The KP operator is, in the terminology of Krasnosel’ski and Pokrovskii (1989), a play hysteron, for which the stop hysteron is, in some cases, its inverse. Put simply, the classical Preisach model (CPM) combines the outputs of independent bi-stable relays to form its output according to the formula (Mayergoyz, 1991)

\[ f(t) = \int_{x,y} \mu(x,y) \gamma_{xy}[u(t)] dx dy \]  

Here, $f(t)$ is the output, $u(t)$ is the input, $\mu(x,y)$ is the weight function and $\gamma_{xy}$ is the simple hysteresis relay operator whose value is determined by the input operation as depicted in Figure 3. And, $x$ and $y$ correspond to up and down switching values of the input, respectively.

Consider an MRC model with $n = 1$ elasto-slide elements. It is easily shown that the following Preisach function represents this behavior

\[ \mu(x,y) = \frac{1}{2} \beta^{(1)} \left[ \delta(x - y) - \delta(x - y - w_{1}) \right] \]  

where $\delta$ denotes the Dirac delta function and $w_{1} = 2e_{rc}^{(1)}/\beta^{(1)}$. For the single elasto-slide element an inverse is not well-defined. To address this issue, now consider a very simple MRC model depicted in Figure 4 with one completely reversible linear “spring” element $\beta^{(2)}$ and one linear spring element $\beta^{(1)}$ that is a part of a slip element with a “slip force” of $e_{rc}^{(i)}$. Generalizing this, one may consider that there are slip elements associated with both springs but $e_{rc}^{(2)}$ is suffi-
ciently large that it is never reached and $D_{b}^{(2)} = 0$ for all time. The fact that a classical Preisach model can also describe this then directly follows.

It is easily determined (Mayergoyz, 1991) for the relationship depicted in Figure 4 that the Preisach weighting function can be expressed as follows:

$$\mu(x, y) = \frac{1}{2} \left\{ \left( \beta^{T(1)} + \beta^{T(2)} \right) \delta[x - y] - \beta^{T(1)} \delta[x - y - w_1] \right\}$$

This leads to the following expression to determine $E$ for a given input history of $D$, with stress $T = 0$:

$$E(t) = \int_{t_1}^{t_2} \frac{1}{2} \left\{ \left( \beta^{T(1)} + \beta^{T(2)} \right) y_{x,x} - \beta^{T(1)} y_{x,x,w_1} \right\} D(t) dx$$

(7)

The above formulation can be generalized to the case of Figure 2. Suppose that $E_{b}^{(n)}$ is sufficiently large that it is never reached and $D_{b}^{(n)} = 0$ for all time. Then we have the following:

$$\mu(x, y) = \frac{1}{2} \left\{ \sum_{i=1}^{n} \left( \beta^{T(i)} \delta[x - y] - \sum_{i=1}^{n-1} \beta^{T(i)} \delta[x - y - w_i] \right) \right\}$$

(8)

$$E(t) = \int_{t_1}^{t_2} \frac{1}{2} \left\{ \sum_{i=1}^{n} \left( \beta^{T(i)} y_{x,x} - \sum_{i=1}^{n-1} \beta^{T(i)} y_{x,x,w_i} \right) \right\} D(t) dx$$

(9)

This Preisach function has some unique features as can be observed in Figure 5(a) where the function $\mu(x, y)$ is graphed. Essentially, the MRC model represents a subset of classical Preisach functions with the following properties: (1) the $\mu(x, y)$ function consists of a countable number of lines parallel to the $x = y$ line; (2) the number of lines corresponds to the number of MRC hysteretic elasto-slide elements; (3) each line has a constant value along its length and (4) this constant value and the line’s distance from the $x = y$ line are directly related to MRC elasto-slide element properties.

In terms of MRC model identification, one could first establish whether or not a particular hysteretic relationship meets the criterion of congruency and wiping out to be described by the CPM (Mayergoyz, 1991). Having established this, one then could check for the validity of an MRC representation by checking for a constant value along lines parallel to the $x = y$ line.

For CPM identification and numerical simulation, one rarely directly uses the functional relationships of Equations (7) and (8). Instead, identification is accomplished by determining the first order transition (reversal) curves. These can be graphically represented in terms of the following function:

$$F(x', y') = \frac{1}{2} (f_{x'} - f_{y'})$$

(10)
in the \( T(x,y) \) triangle, similar to \( \mu(x,y) \) as shown in Figure 5(b). These are obtained by first monotonically increasing the input value from negative saturation to \( x' \) obtaining the output value \( f_{x'} \). The input is monotonically decreased to \( y' \) obtaining the output value \( f_{x' y'} \). This results in the following:

\[
F(x', y') = \int_{T(x', y')} \mu(x, y) dx dy
\]  

(11)

where \( T(x', y') \) denotes the triangular region bordered by the maximum value of \( x' \), the minimum value of \( y' \) and the line \( x = y \).

Based on Equations (8)–(11) we have for the MRC model in Figure 2 that

\[
F(x', y') = \left\{ \frac{1}{2} \sum_{i=1}^{n} \beta^{T(i)} \right\} (x' - y') \\
- \frac{1}{2} \sum_{i=1}^{n} [\beta^{T(i)}(x' - y' - y_i) H[x' - y' - y_i]]
\]  

(12)

where \( H[\alpha] \) denotes the Heaviside function as

\[
H[\alpha] = \begin{cases} 
1 & \alpha > 0 \\
0 & \alpha < 0 
\end{cases}
\]  

(13)

Like \( \mu(x,y) \), the function \( F(x,y) \) of the MRC hysteresis model has a constant value along lines parallel to the \( x = y \) line. Unlike the representation of \( \mu(x,y) \), for finite MRC elements (finite \( n \)) the resulting graph of \( F(x,y) \) is continuous over the region \( T(x,y) \) but its derivatives are discontinuous. Consequently, having established congruency and wiping out properties per requirement of any CPM representation, one then could determine \( F(x,y) \) based on the first order transition curves. If the resulting graph of \( F(x,y) \) has the property of a constant value along lines parallel to the \( x = y \) line, then it may be described by an MRC model. Note that the level of accuracy or resolution that is needed in approximating the experimentally measured relationship will determine the number of MRC elasto-slide elements.

3.2 Model Identification

A method for identification of the MRC model is now presented based upon recognition that it is a classical Preisach model. The developed methodology assumes a symmetric hysteresis region with \( |x_{\text{max}}| = |y_{\text{max}}| \) but can be generalized to the nonsymmetric case. The \( T(x,y) \) triangle is divided by \( n \) lines parallel to the \( x = y \) line that are equally separated in distance with the \( n \)th line intersecting the point \( (x,y) = (x_{\text{max}}, y_{\text{max}}) \). Hence, we have \( w_i \) values of equal increments in length denoted by the following:

\[
w_i = \frac{i2}{n} x_{\text{max}} \quad i = 1, \ldots, n
\]  

(14)

Given the experimentally determined function \( F(x,y) \), it is possible to average its values along lines parallel to the \( x = y \) line at distances \( w_i \) from the \( x = y \) line. These averaged values will be denoted:

\[
F_{c_i} = \frac{1}{\sqrt{2(2x_{\text{max}} - w_i)}} \int_{y_{\text{min}}}^{x_{\text{max}}} F(x', x' - w_i) dx
\]  

(15)

and we define

\[
F_x' = \begin{bmatrix} F_{c_{11}} \\ \vdots \\ F_{c_{n1}} \end{bmatrix}
\]  

(16)

Given this, Equations (12)–(16) may be written in the following form, solving for the vector \( \beta^T \) of the unknown MRC elastic coefficients:

\[
\beta^T = \begin{bmatrix} \beta^{T(1)} \\ \vdots \\ \beta^{T(n)} \end{bmatrix} = \frac{n}{x_{\text{max}}} \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & \cdots & 2 & 2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 2 & \cdots & 3 & \cdots & n \end{bmatrix} F_x'
\]  

(17)

Upon determining the elastic coefficients, the sliding constants can be determined using the following:

\[
e_{r_{c}}^{(i)} = \frac{i x_{\text{max}}}{n} \beta^{T(i)} \quad i = 1, \ldots, n - 1
\]  

(18)

Note that a value for \( e_{r_{c}}^{(n)} \) is not needed as this is not used in Equation (12) since it is assumed that the input values do not exceed the region of \( T(x,y) \).

3.3 The Inverse Maxwell Resistive Capacitor (IMRC) Hysteresis Model

The inverse of the Maxwell slip model of Figure 4 is shown in Figure 6 and corresponds to the following Preisach function where an overhead bar is used to denote that it is for the inverse.

\[
\bar{\mu}(x,y) = \frac{1}{2} \left( \frac{1}{\beta^{T(1)} + \beta^{T(2)}} \right) \delta[x - y] + \left( \frac{1}{\beta^{T(2)}} - \frac{1}{\beta^{T(1)} + \beta^{T(2)}} \right) \delta[x - y - w_i (\beta^{T(1)} + \beta^{T(2)})] \right)
\]  

(19)

While superposition of the simple elements of Figure 4 could be used to construct the general form of Figure 2, su-
perposition of the inverse functions has a different meaning. It would be akin to combining the slip elements in series instead of in parallel. The inverse function for Figure 2 can be constructed essentially by taking the reciprocal of the sum of reciprocals of the terms depicted in Equation (19) which leads to the following:

\[ \frac{1}{\beta^{T(1)} + \beta^{T(2)}} \]

(20a)

where,

\[ \sum_{i=1}^{n} \beta^{T(i)} = 1 \]

(20b)

\[ \bar{\beta}^{T(i)} = \left( \sum_{j=1}^{n} \beta^{T(j)} \right) - \left( \sum_{j=1}^{n} \beta^{T(j)} \right) \]

(20c)

for \( i = 1, \ldots, n - 1 \), which implies that

\[ \bar{\beta}^{T(n)} = \left( \sum_{j=1}^{n} \beta^{T(j)} \right) - \left( \sum_{j=1}^{n} \beta^{T(n)} \right) \]

(20d)

and

\[ w_i = \sum_{j=1}^{n} \beta^{T(j)} \]

\[ \bar{w}_i = \begin{cases} w_i & i = 1 \\ \bar{w}_{i-1} + (w_i - \bar{w}_{i-1}) \sum_{j=1}^{n} \beta^{T(j)} & i > 1 \end{cases} \]

(20e)

Note that the IMRC model contains the same unique features observed in the direct MRC model that were listed in Section 3.1. Since the IMRC with certain constraints (\( \beta^{T(n)} \neq 0 \)) is also a classical Preisach model, a similar identification methodology should follow. From previous discussions it also logically follows that the Everett function for the IMRC may be represented as follows.

\[ F(x', y') = \left\{ \frac{1}{2} \sum_{i=1}^{n} \bar{\beta}^{T(i)} \right\} (x' - y') + \frac{1}{2} \sum_{i=1}^{n} \left[ \tilde{\beta}^{T(i)}(x' - y' - \bar{w}_i) \right] H[x' - y' - \bar{w}_i] \]

(21)

The practical difference between the MRC and IMRC is subtle. Refer to Figure 7. The slope of the Everett function \( F(x, y) \) decreases in value along lines parallel to \( x = y \) for the MRC model. It \( (\tilde{F}(x, y)) \) increases in value along lines parallel to \( x = -y \) for the IMRC model.

Figure 6. Dielectric relationship for simple MRC model of Figure 4 with \( E \) as input and \( D \) as output under \( T = 0 \) condition.

Figure 7. Everett function \( F(x', y') \) based on experimental results: (a) \( E \) vs. \( D \) for CPM simulation and inverse MRC simulation and (b) \( D \) vs. \( E \) for inverse Preisach model and MRC simulation.
3.4 Inverse Model Identification

The developed identification methodology here assumes a symmetric hysteresis region with \([\bar{x}_{\text{max}}] = [\bar{y}_{\text{max}}]\) but can be generalized to the nonsymmetric case. The \(T(x, y)\) triangle is divided by \(n\) lines parallel to the \(\bar{x} - \bar{y}\) line that are equally separated in distance with the \(n\)th line intersecting the point \((\bar{x}, \bar{y}) = (\bar{x}_{\text{max}}, \bar{y}_{\text{max}})\). Hence, we have \(\bar{w}_i\) values of equal increments in length denoted by the following.

\[
\bar{w}_i = \frac{i^2}{n} \bar{x}_{\text{max}} \quad i = 1, \ldots, n
\]  

Then, given the experimentally determined function \(F(x, y)\), it is possible to average its values along lines parallel to the \(\bar{x} = \bar{y}\) line at distances \(\bar{w}_i\) from the \(\bar{x} = \bar{y}\) line. These averaged values will be denoted:

\[
F_{x'i} = \frac{1}{2(\bar{x}_{\text{max}} - \bar{w}_i)} \int_{\bar{y}_{\text{min}}}^{\bar{y}_{\text{max}}} F(\bar{x}', \bar{x}' - \bar{w}_i) d\bar{x}' \quad i = 1, \ldots, n
\]

and we define

\[
\bar{F}_{x'} = \begin{bmatrix} F_{x'1} \\ \vdots \\ F_{x'n} \end{bmatrix}
\]

Given this, Equations (20)–(24) may be written in the following form, solving for the vector of \(\bar{\beta}^T\) the unknown MRC elastic coefficients.

\[
\bar{\beta}^T = \begin{bmatrix} \bar{\beta}^{T(1)} \\ \vdots \\ \bar{\beta}^{T(n)} \end{bmatrix}
\]

\[
\bar{F}_{x'} = \frac{n}{\bar{x}_{\text{max}}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 3 & 2 & 2 & \cdots & 2 & 2 \\ 5 & 4 & 3 & \cdots & 3 & 3 \\ \vdots & 6 & \ddots & \ddots & \vdots & \vdots \\ 2n - 3 & \vdots & 2n - 5 & \cdots & n - 1 & n - 1 \\ 2n - 1 & 2n - 2 & 2n - 3 & \cdots & n & n \end{bmatrix}^{-1}
\]

Using Equation (19) we can extract the dielectric impermeability coefficients \(\beta^{T(i)}\) as follows:

\[
\beta^{T(n)} = \left[ 1 + \sum_{j=1}^{n} \bar{\beta}^{T(j)} - \bar{\beta}^{T(n)} \right]^{-1}
\]

\[
\beta^{T(i)} = -\sum_{j=i+1}^{n} \bar{\beta}^{T(j)} + \frac{1}{1 - \bar{\beta}^{T(i)} \sum_{j=i+1}^{n} \bar{\beta}^{T(j)}} \quad i = 1, \ldots, n - 1
\]

The sliding constants \(e^{T(i)}\) can be determined after we first determine the values \(\bar{w}_i\). These are obtained from Equation (19) where it is noted that they have dependence on \(\bar{w}_i\) and \(\bar{\beta}^{T(i)}\) for \(i = 1, \ldots, n\).

\[
\bar{w}_i = \begin{bmatrix} \bar{w}_i \\ \sum_{j=i}^{n} \bar{\beta}^{T(j)} \\ \bar{w}_{i-1} + (\bar{w}_i - \bar{w}_{i-1}) \sum_{j=i}^{n} \bar{\beta}^{T(j)} \end{bmatrix} \quad i > 1
\]

And we have from Equation (18) that \(e^{T(i)} = w_i \bar{\beta}^{T(i)} / 2\) for \(i = 1, \ldots, n - 1\). As before, a value for \(e^{T(n)}\) is not needed as this is not used in Equation (20) since it is assumed that the input values do not exceed the region of \(T(x, y)\) or \(\bar{T}(x, y)\).

3.5 Implementing MRC Feedforward Compensation for Electrical and Mechanical Displacement Control

When piezoceramics are used for actuation, a common situation is for the voltage or electric field \(E\) to be the input and the displacement or strain \(S\) to be the output. Unfortunately, hysteresis will be involved in this relationship. For the case of \(T = 0\), in theory the hysteresis can be compensated using a feedforward scheme. Referring to Equations (2a) and (2b), when \(T = 0\) a desired strain \(S\) is linearly proportional to a desired electric displacement \(D\). If hysteresis is not present, this desired \(D\) is linearly proportional to a desired electric field \(E\). Of course, with hysteresis present applying a \(E\) based on proportional assumptions will not result in the desired \(S\) trajectory. However, if the desired \(E\) is determined using the direct MRC model of hysteresis with the desired \(D\) as input to this model, the output \(S\) will match the desired trajectory of \(S\). If \(T \neq 0\) and is dynamically varying, feedback control would also need to be employed. A simple force feedback scheme would involve sensing \(T\) and subtracting \(gT\) from the computed desired \(E\) value. These simple control schemes are schematically represented in Figure 8.

For the case that \(T = 0\), MRC and IMRC identification of experimental data with \(D\) or \(E\) as input and \(E\) or \(D\) as output were conducted. Given \(D\), values for \(S\) were also predicted based on Equation (2a). In Figure 9, for an exponentially de-
Figure 8. Proposed scheme for implementing hysteresis compensation based on the MRC model.

Figure 9. Measured and simulated relationships with T = 0: (a) D vs. E, (b) S vs. D and (c) S vs. E. Note that offsets have been applied on graph for ease of comparison. Key: (I) experimental, (II) Preisach simulation, (III) MRC (or inverse MRC) simulation, n = 20, (IV) MRC (or inverse MRC) simulation, n = 5 and (V) MRC (or inverse MRC) simulation, n = 2.
caying oscillatory input $E$ the measured and predicted $D$ vs. $E$ vs. $S$ relationships are shown for MRC and IMRC models and Equations (2a) and (2b) with $n = 2, 5$ and $n = 20$. For the cases of $n = 5$ and $20$, very good agreement with experiment is achieved. For the case of $n = 2$, decreased accuracy is apparent. Identified MRC and IMRC models of the same order $n$ had identical relationships between $E$, $D$ and $S$.

Feedforward control to obtain a desired trajectory in $S$ due to an input $E$ for the case that $T = 0$ is considered. Referring to Figure 8, cases were considered in which no compensation for hysteresis was employed and for which an MRC model with $n = 2, 5$ or $20$ was employed for compensation. Results in Figure 10 suggest that the compensation scheme is very effective as MRC model accuracy is increased.

4. CONCLUSIONS

In this paper the applicability of the rate-independent generalized Maxwell resistive capacitor (MRC) hysteresis model to piezoceramic transducers has been established via experimental studies on a 1-3 piezoceramic composite. Methods for MRC and inverse MRC online model identification have been developed and verified experimentally. A method for incorporating the MRC model in a feedforward control scheme for hysteresis compensation was also presented and validated by simulation. It is noted that the identified relationship between Preisach and Maxwell hysteresis models and the associated formulations for implementing feedforward compensation may prove useful in a wider range of applications with hysteresis than just those involving piezoceramics.

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