Simultaneous Smoothing and Estimation of the Tensor Field from Diffusion Tensor MRI

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Abstract

Diffusion tensor magnetic resonance imaging (DT-MRI) is a relatively new imaging modality in the field of medical imaging. This modality of imaging allows one to capture the structural connectivity if any between functionally meaningful regions for example, in the brain. The data however can be noisy and requires restoration. In this paper, we present a novel unified model for simultaneous smoothing and estimation of diffusion tensor field from DT-MRI. The diffusion tensor field is estimated directly from the raw data with $L^p$ smoothness and positive definiteness constraints. The data term we employ is from the original Stejskal-Tanner equation instead of the linearized version as usually done in literature. In addition, we use Cholesky decomposition to ensure positive definiteness of the diffusion tensor. The unified model is discretized and solved numerically using limited memory Quasi-Newton method. Both synthetic and real data experiments are shown to depict the algorithm performance.

1 Introduction

Diffusion is a process of movement of molecules as a result of random thermal agitation and in our context, refers specifically to the random translational motion of water molecules in the part of the anatomy being imaged with MR. In three dimension, water diffusivity can be described by a 3x3 matrix $D$ called diffusion tensor which is highly related to the geometry and organization of the microscopic environment. The general principle is that, water diffuses preferably along ordered tissues like the brain white matter.

Diffusion tensor MRI is a relatively new MR imaging modality from which anisotropy of water diffusion can be inferred quantitatively [1], thus it provides a method to study the tissue microstructure e.g., white matter connectivity in the brain in vivo. Diffusion weighted echo intensity image $S_l$ and the diffusion tensor $D$ are related through the Stejskal-Tanner equation [1] given by:

$$S_l = S_0 e^{-b_l : D} = S_0 e^{-\sum_{i=1}^{2} \sum_{j=1}^{a_i} b_{i,j} a_{i,j}}$$  (1)

where $b_l$ is the diffusion weighting of the $l-th$ magnetic gradient, "$:$" denotes the generalized dot product for matrices. Taking log on both sides of equation (1) yields the following transformed linear Stejskal-Tanner equation:

$$log(S_l) = log(S_0) - b_l : D$$  (2)

Given several (at least seven) non-collinear diffusion weighted intensity measurements, $D$ can be estimated via multivariate regression models from either of the two equations (1) and (2). Diffusion anisotropy can then be computed to show microstructural and physiological features of tissues [2]. Especially in highly organized nerve tissue, like white matter, diffusion tensor provides a very good characterization of the restricted motion of water through the tissue that can be used to infer fiber tracts. The development of diffusion tensor acquisition, processing, and analysis methods provides the framework for creating fiber tract maps based on this complete diffusion tensor analysis.

For automatic fiber tract mapping, the diffusion tensor field must be smoothed without losing relevant features. Currently there are two popular approaches, one is to smooth the raw data $S_l$ while preserving relevant detail and then estimate diffusion tensor $D$ from the smoothed raw data [10, 13]. The raw data in this context consists of several diffusion weighted images acquired for varying magnetic field strengths and directions. Note that at least seven values at each 3D grid point in the data domain are required to estimate the six unknowns in the symmetric 2-tensor $D$ and one scale parameter $S_0$. The raw data smoothing or de-noising can be formulated using variational principles which in turn requires solution to PDEs or at times directly

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using PDEs which are not necessarily arrived at from variational principles (see [4, 13]).

Another approach to restore the diffusion tensor field is to smooth the principal diffusion direction after the diffusion tensor has been estimated using linear least-squares from the logarithmically transformed raw noisy measurements $S_I$. In Poupon et. al., [11], an energy function based on a Markovian model was used to regularize the noisy dominant eigenvector field computed directly from the noisy estimates of $\mathbf{D}$ obtained from the measurements $S_I$. Coulon et.al., [5] proposed an iterative restoration scheme for principal diffusion direction based on direction map restoration. Other sophisticated vector field restoration methods [12] can potentially be applied to the problem of restoring the dominant eigenvector fields computed from the noisy estimates of $\mathbf{D}$. Recently, Chef'd'Hotel et.al., [3] presented an elegant geometric solution to the problem of smoothing a noisy $\mathbf{D}$ that was computed from $S_I$ using the log-linearized model described above. They assume that the given (computed) tensor field $\mathbf{D}$ from $S_I$ is positive definite and develop a clever approach based on differential geometry of manifolds to achieve a smoothed tensor field which is constrained to be positive semi-definite.

We propose a novel unified variational principle to simultaneously estimate and smooth the diffusion tensor $\mathbf{D}$ with positiveness constraint on the diffusion tensor. The novelty lies in being able to directly (single step) estimate a smooth $\mathbf{D}$ from the noisy measurements $S_I$ with the preservation of the positiveness constraint on $\mathbf{D}$. In contrast, most earlier approaches used a two step method involving, (i) computation of a $\mathbf{D}$ from $S_I$ using a linear least-squares approach and then (ii) computing a smoothed $\mathbf{D}$ via either smoothing of the eigen-values and eigen-vectors of $\mathbf{D}$ or using the matrix flows approach in [3]. The problem with the two step approach to computing $\mathbf{D}$ is that the estimated $\mathbf{D}$ in the first step using the log-linearized model need not be positive definite or even semi-definite. Moreover, it is hard to trust the fidelity of the eigen values and vectors computed from such matrices even if they are to be smoothed subsequently prior to mapping out the nerve fiber tracts. Also, the noise model used in the log-linearized scheme is not consistent with the physics. Briefly, our model involves a nonlinear data term based on the original (not linearized) Stejskal-Tanner equation (1), an $L^p$ norm based matrix-valued image smoothing scheme with the positiveness of the diffusion tensor being ensured via a Cholesky factorization of $\mathbf{D}$ in the minimizer.

The rest of the paper is organized as follows: in Section 2, we present the detailed variational principle and comment on the existence of a solution to the same. Section 3 contains the details of the numerical solution to the variational principle. In Section 4, we present the experimental results and conclude in Section 5.

2 The Unified Variational Principle

Our solution to the recovery of a smooth diffusion tensor field from the measurements $S_I$ is posed as a variational principle involving a nonlinear data fidelity term with a Cholesky factorization of the diffusion tensor and an $L^p$ Norm smoothness constraint on the diffusion tensor to be estimated. The novelty of our formulation lies in the development of a unified framework for recovering and smoothing of the tensor field from the data $S_I$.

Let $S_0(x)$ be the response intensity at voxel $x = (x,y,z)^T$ when no diffusion-encoding gradient is present, $\mathbf{D}(x)$ the unknown symmetric positive definite tensor, $\mathbf{L}^T(x)$ the Cholesky factorization of the diffusion tensor $\mathbf{L}$ with $\mathbf{L}$ being a lower triangular matrix, $S_I, I = 1, \ldots N$ is the response intensity image measured after application of a magnetic gradient of known strength and direction and $N$ is the total number of intensity images each corresponding to a direction of the applied magnetic gradient. The variational principle for estimating and smoothing $\mathbf{D}(x)$ is given by

$$\min \mathcal{E}(S_0, \mathbf{L}) = \int_\Omega \sum_{I=1}^N (S_I - S_0 e^{-\beta_{I+1} L^T} )^2 dx + \int_\Omega (\alpha \nabla S_0(x)|^p + \beta |\nabla \mathbf{L}(x)|^p) dx$$  (3)

where $\Omega$ is the image domain, $\alpha$ and $\beta$ are regularization parameters, $\beta$ and $\gamma$ are the same as in equation (1). The first term is the data fidelity term obtained directly from equation (1), the second and the third term are $L^p$ smoothness constraint on $S_0$ and $\mathbf{L}$ respectively, where $p > 2/5$ for $S_0$ and $p \geq 1$ for $\mathbf{L}$. $|\nabla \mathbf{L}|^p = \sum_d |\nabla L_d|^p$, where $d = xx, yy, zz, xy, yz, zx$ are indices to the six nonzero components of $\mathbf{L}$. The lower bounds on the value of $p$ are chosen so as to make the proof of existence of a solution for this minimization (see section 2.3) mathematically tractable.

2.1 The Nonlinear Data Model

The Stejskal-Tanner equation (1) shows the relation between diffusion weighted echo intensity image $S_I$ and the diffusion tensor $\mathbf{D}$. However, multivariate linear regression based on equation (2) has been used to estimate the diffusion tensor $\mathbf{D}$ [1]. It was pointed out in [1] that these results agree with nonlinear regression based on the original Stejskal-Tanner equation (1). However, if the signal to noise ratio (SNR) is low and the number of intensity images $S_I$ is not very large (unlike in [1] where $N = 315$ or $N = 294$), the result from multivariate linear regression will differ from the nonlinear regression significantly. A robust estimator belonging to the M-estimator family was used by Poupon et.al., [11], however, its performance is not discussed in detail. In Westin et. al., [14]), an analytical
solution is derived from equation (2) by using a dual tensor basis, however, it should be noted that this can only be used for computing the tensor D when there is no noise in the measurements S_i or the SNR is extremely high.

Our aim is to provide an accurate estimation of diffusion tensor D for practical clinical use, where the SNR may not be high and the total number of intensity images N is restricted to a moderate number. The nonlinear data fidelity term based on original Stejskal-Tanner equation (1) is fully justified for use in such situations.

2.2 The Lp smoothness constraint

In Blomgren [4], it is shown that Lp smoothness constraint does not admit discontinuous solutions as the TV-norm when p > 1. However, when p is chosen close to 1, its behavior is close to the TV-norm when restoring edges. In our unified model, we need p > 6/5 for regularizing S_0 and p ≫ 1 for L to ensure existence of a solution. We can still choose a proper p which is as close to 1 as possible to have a good edge preserving smoothing scheme. In our experiment, we choose p = 1.205 for S_0 and p = 1.00 for L.

2.3 Comments on Existence of the Solution

Consider the problem:

$$\min_{(S_0, L) \in A} \mathcal{E}(S_0, L) = \int_{\Omega} \sum_{i=1}^{N} (S_i - S_0 e^{-b_i \cdot L L^T})^p dx + \int_{\Omega} (|\alpha \nabla S_0(x)|^p + \beta |\nabla L(x)|^p) dx \quad (4)$$

Where $A = \{(S_0, L) | L \in BV(\Omega), L_{ij} \in L^2(\Omega), d = xx, yy, zz, xy, yz, zx \ \text{and} \ S_0 \in W^{1,p}(\Omega), p > 6/5\}$. $BV(\Omega)$ denotes the space of bounded variation functions on the domain $\Omega$, $L^2(\Omega)$ is the space of square integrable functions on $\Omega$ and $W^{1,p}(\Omega)$ denotes the Sobolev space of order $p$ [6].

**Theorem 1** Suppose $S_0 \in L^2(\Omega)$, then the minimization problem (4) has a solution $(S_0, L) \in A$.

**Proof Outline:** Let $(S_0^{(n)}, L^{(n)})$ be a minimizing sequence, using the compact embedding results [6], we can find a convergent subsequence. We also can prove the following results:

1) Lower semi-continuity of the $L^p$ smoothness constraint terms.

2) Continuity of the data fidelity term for $S_0 \in W^{1,p}(\Omega)$ when $p > 6/5$ and $\Omega \subset \mathbb{R}^n$.

Thus, we have the convergence of the minimizing subsequence which is the solution of the minimization problem (4)(see [6]).

Finding a solution using the unified model is much more difficult than when dealing with the problems of recovering and smoothing separately. However, there are benefits of posing the problem in a unified framework, namely, one does not accumulate the errors from a two stage process. Moreover, the unified framework incorporates the nonlinear data term which is more appropriate for low SNR values prevalent when $b_i$ is high. Also, the noise model is based on the original nonlinear data model unlike the log-linearized case. Lastly, in the unified formulation, it is now possible to pose mathematical questions of existence and uniqueness of a solution – which was not possible previously.

2.4 The Positive Definite Constraint

The diffusion tensor $D$ is supposed to be a positive definite matrix however, due to the noise in the measurements, $S_i$, it is hard to recover a $D$ that retains this property unless explicitly included as a constraint. In general, a matrix $A \in \mathbb{R}^{n \times n}$ is said to be positive definite if $x^T A x > 0$, for all $x \neq 0$ and $\epsilon \in \mathbb{R}^n$. By applying the Cholesky factorization theorem, which states that: If $A$ is a symmetric positive definite matrix then, there exists a unique factorization $A = LL^T$ where, $L$ is a lower triangular matrix with positive diagonal elements. After doing the Cholesky factorization, we have transferred the inequality constraint on the matrix $D$ to an inequality constraint on the diagonal elements of $L$. This is still hard to satisfy theoretically because, the set on which the minimization takes place is an open set. However, in practise, with finite precision arithmetic, testing for a positive definiteness constraint is equivalent to testing for positive semi-definiteness. To answer the question of positive semi-definiteness, a stable method would yield a positive response even for nearby symmetric matrices. This is because, $\tilde{D} = D + E$ with $||E|| \leq \epsilon \ ||D||$, where $\epsilon$ is a small multiple of the machine precision. Because, with an arbitrarily small perturbation, a semi-definite matrix can become definite, it follows that in finite precision arithmetic, testing for definiteness is equivalent to testing for semi-definiteness. Thus, we repose the positive definiteness constraint on the diffusion tensor matrix as, $x^T D x \geq 0$ which is satisfied when $D = LL^T$.

3 Numerical Methods

We discretized the variation principle (3) directly and applied the limited memory Quasi-Newton method to find a numerical solution. The discretized variational principle is given by,

$$\min_{S_0, L} \mathcal{E}(S_0, L) = \sum_{i,j,k} \sum_{l=1}^{N} R_{ij,k}^2 + \sum_{i,j,k} (\alpha |\nabla S_0|_{ij,k}^p + \beta |\nabla L|_{ij,k}^p) \quad (5)$$

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Where,
\begin{align*}
R_{t,ij,k} &= S_{t,ij,k} - S_{0,ij,k}e^{-b_tL_{ij,k}^2} \\
|\nabla S_0|_{ij,k} &= \left( \Delta_{x}^2 S_0 \right)^2 + \left( \Delta_{y}^2 S_0 \right)^2 + \left( \Delta_{z}^2 S_0 \right)^2 + \epsilon_{ij,k} \\
|\nabla L_{ij,k} &= \left( \Delta_{x}^2 L_{ij,k} \right)^2 + \left( \Delta_{y}^2 L_{ij,k} \right)^2 + \left( \Delta_{z}^2 L_{ij,k} \right)^2 + \epsilon_{ij,k} \\
|\nabla L_{ij,k}|^p &= |\nabla L_{xx,ij,k}| + |\nabla L_{yy,ij,k}| + |\nabla L_{zz,ij,k}| + |\nabla L_{xz,ij,k}| + |\nabla L_{yz,ij,k}| + |\nabla L_{zx,ij,k}| \\
\Delta_{x}^2, \Delta_{y}^2, \text{ and } \Delta_{z}^2 \text{ are forward difference operators, } \epsilon \text{ is a small number to avoid singularities of } L^p \text{ norm when } p < 2.
\end{align*}

Numerical optimization techniques can be applied to solve the above discrete minimization problem. Due to the large number of unknown variables in the minimization, we apply the limited memory Quasi-Newton technique [8]. Quasi-Newton like methods compute the approximate Hessian matrix at each iteration of the optimization by using only the first derivative information. In Limited-Memory Broyden-Fletcher-Goldfarb-Shanno (BFGS), search direction is computed without storing the approximated Hessian matrix.

Let \( \mathbf{x} = (S_0, \mathbf{L}) \) be the vector of variables, \( f(\mathbf{x}) = E(S_0, \mathbf{L}) \) the energy function to be minimized. At \( k \)th iteration, let \( \mathbf{s}_k = \mathbf{L}_{k+1} - \mathbf{x}_k \) be the update of the variable vector \( \mathbf{x}_k = \nabla f_{k+1} - \nabla f_k \) the update of the gradient and \( \mathbf{H}_k^{-1} \) the approximation of inverse of the Hessian. Using the Sherman-Morrison-Woodbury formula [7], we have the following expression for the inverse of the approximate Hessian \( \mathbf{H}_k^{-1} \)
\begin{equation}
\mathbf{H}_{k+1}^{-1} = \mathbf{H}_k^{-1} - \frac{\mathbf{H}_k^{-1}\mathbf{y}_k\mathbf{y}_k^T\mathbf{H}_k^{-1}}{\mathbf{y}_k^T\mathbf{H}_k^{-1}\mathbf{y}_k} + \frac{\mathbf{s}_k\mathbf{s}_k^T}{\mathbf{y}_k^T\mathbf{s}_k}. \tag{7}
\end{equation}

We now use the L-BFGS (limited memory BFGS) two-loop recursion which computes the search direction \( \mathbf{H}_k^{-1}\nabla f_k \) efficiently by using last \( m \) pair of \((\mathbf{s}_k, \mathbf{y}_k)\) (see [8] for details).

The gradient of our energy function can be computed analytically as,
\begin{equation}
\nabla f(\mathbf{x}) = \frac{\partial E(S_0, \mathbf{L})}{\partial S_0} \frac{\partial E(S_0, \mathbf{L})}{\partial L_{xx}} \frac{\partial E(S_0, \mathbf{L})}{\partial L_{yy}} \frac{\partial E(S_0, \mathbf{L})}{\partial L_{zz}} - 2 \sum_{i=1}^{N} R_{t,ij,k} \frac{\partial R_{t,ij,k}}{\partial S_{0,ij,k}} + \sum_{i' \neq i, j', \neq j} \frac{\partial |\nabla S_0|_{ij,k}^p}{\partial S_{0,ij,k}} \tag{8}
\end{equation}

Where
\begin{align*}
\frac{\partial E(S_0, \mathbf{L})}{\partial S_{0,ij,k}} &= 2 \sum_{i=1}^{N} R_{t,ij,k} \frac{\partial R_{t,ij,k}}{\partial S_{0,ij,k}} + \sum_{i' \neq i, j', \neq j} \frac{\partial |\nabla S_0|_{ij,k}^p}{\partial S_{0,ij,k}} \\
\frac{\partial E(S_0, \mathbf{L})}{\partial L_{ij,k}} &= 2 \sum_{i=1}^{N} R_{t,ij,k} \frac{\partial R_{t,ij,k}}{\partial L_{ij,k}} + \sum_{i' \neq i, j', \neq j} \frac{\partial |\nabla L_{ij,k}|^p}{\partial L_{ij,k}} \\
d &= xx, yy, zz, xy, yz, xz \tag{9}
\end{align*}

Here \( \sum_{i',j',k'} \) is over a neighborhood of the voxel \( (i, j, k) \) where the forward differences involves the variables \( S_{0,ij,k} \) or \( L_{ij,k} \). Each term in equation (9) can be computed analytically, for example
\begin{equation}
\frac{\partial R_{t,ij,k}}{\partial L_{xx,ij,k}} = S_{0,ij,k}e^{-b_tL_{ij,k}^2} \ast (2b_{t,xx}L_{xx,ij,k} + 2b_{t,xy}L_{xy,ij,k} + 2b_{t,xz}L_{xz,ij,k}) \tag{10}
\end{equation}

4 Experimental Results

Two sets of experiments were performed, one using synthetic data and one using a normal rat brain DT-MRI.

4.1 Synthetic Data

We synthesized an anisotropic tensor field on a 3D lattice of size 32x32x8. The diffusion weighted images \( S \) are generated using the Stejskal-Tanner equation at each voxel \( \mathbf{x} \) by:
\begin{equation}
S_t(\mathbf{x}) = S_0(\mathbf{x})e^{-b_tD_t(\mathbf{x}) + n(\mathbf{x})}, \quad n(\mathbf{x}) \sim N(0, \sigma) \tag{10}
\end{equation}
where \( N(0, \sigma) \) is a zero mean Gaussian noise with standard deviation \( \sigma \).

We choose the 7 commonly used gradient configurations (see [14]) and 3 different field strengths in each direction for \( b_0 \) values. The Gaussian noise we added has a \( \sigma = S_0 \ast 0.1 \). In figure 1, we use the ellipsoid visualization technique to show the original diffusion tensor field and restored diffusion tensor field using the following methods: linear regression from equation (2), nonlinear regression from equation (1), weighted TV-norm (WTV-norm) method [13], modified WTV-Norm method and our current proposed method respectively. The modified WTV-Norm (MWTV-Norm) method is used to smooth the raw data first and then \( \mathbf{D} \) is estimated using a nonlinear regression method. It is evident from this figure that the MWTV-Norm method and the new unified model both yield very good estimates of the original tensor field. However, further experimentation using quantitative measures (described below) reveals the superiority of the proposed unified model.

To quantitatively assess the proposed unified model, we compute certain scalar measures such as fractional anisotropy (FA) [2], which is defined as \( FA = \sqrt{\lambda_1 - \lambda_2^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2} \), where, \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are the largest, smallest and average eigen values of the diffusion tensor \( \mathbf{D} \) respectively. FA values range from 0 to 1, where, larger values indicate more anisotropy. Let \( \Delta_{FA} \) be the difference of FA values between the estimated diffusion tensor field and the original known tensor field, and \( \theta_{DEV} \) be the angle(in degrees) between the dominant eigen vector of the estimated diffusion tensor field and the original tensor field.
Figure 1. Results from synthetic data: First image is the original tensor field, and the other images arranged from left to right, top to bottom are estimated tensor field using the following methods: linear regression, nonlinear regression, WTV-Norm, MWTV-Norm and the proposed model.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\mu(\Delta F_A)$</th>
<th>$\sigma(\Delta F_A)$</th>
<th>$\mu(D_{FV})$</th>
<th>$\sigma(D_{FV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.0870</td>
<td>0.0725</td>
<td>17.730</td>
<td>12.755</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>0.0654</td>
<td>0.0570</td>
<td>9.908</td>
<td>6.467</td>
</tr>
<tr>
<td>WTV-Norm</td>
<td>0.0274</td>
<td>0.0367</td>
<td>3.270</td>
<td>4.758</td>
</tr>
<tr>
<td>MWTV-Norm</td>
<td>0.0114</td>
<td>0.0166</td>
<td>1.853</td>
<td>1.805</td>
</tr>
<tr>
<td>Unified Model</td>
<td>0.0111</td>
<td>0.0107</td>
<td>1.333</td>
<td>1.084</td>
</tr>
</tbody>
</table>

Table 1. Comparison of different methods.

Table 1 shows the mean and standard deviation for these two measures when computed using various methods discussed thus far. The parameters for each method were empirically chosen to yield best results in each case. A better estimator is one that yields values of these scalar measures closer to zero. From table 1, we can see the unified model yields lower values than all the methods including the modified WTV-Norm method, although the difference between them in figure (1) is not apparent.

4.2 DT-MRI from a Normal Rat Brain

The normal rat brain data we used here has 21 diffusion weighted images measured using the same configuration of $b_1$ as in the previous example, each image is a 128x128x78 volume data. We extract 20 slices in the region of interest, namely the corpus callosum, for our experiment. Figure 3 depicts images of the six independent components of the estimated diffusion tensor, the computed FA, the trace(D) and $S_0$ (echo intensity without applied gradients) obtained using our proposed unified model. As a comparison, figure 2 shows the same images computed using linear least squares fitting based on the linearized Stejskal-Tanner equation from the raw data. For display purposes, we use the same brightness and contrast enhancement for displaying the corresponding images in the two figures. The effectiveness of edge preserving smoothing in our method is clearly evident in the off-diagonal components of $D$. In addition, fiber tracts were estimated as integral curves of the dominant eigen vector field of the estimated $D$ and is visualized using particle systems [9]. The mapped fiber tracts are found to follow the expected tracts quite well from a Neuroanatomical perspective as shown in figure 4. The quality of results obtained is reasonably satisfactory for visual inspection purposes, however intensive quantitative validation of the mapped fibers needs to be performed and will be the focus of our future efforts.

5 Conclusions

We introduced a novel and unified approach for smoothing and estimating the diffusion tensor field with positive definiteness constraint from DT-MRI and mapping the nerve fibers from the same. We used the Cholesky decomposition to incorporate the positive definiteness constraint on the diffusion tensor being estimated. Existence of a solution for the unified model was briefly outlined and a nu-
merical method based on limited memory quasi-Newton for the discretized variation principle was presented. Finally, results from both synthetic and real experiments consisting of the estimated diffusion tensor, computed fractional anisotropy were demonstrated and in addition, mapped fiber tracts of the normal rat brain were depicted using a particle-based visualization scheme. The estimated diffusion tensors are smooth and retain essential detail from a visual perspective. The mapped fiber tracts further show the effectiveness of the new model. However, quantitative comparison of the restored diffusion tensor is essential and will be the focus of our future efforts.

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