Abstract—This paper is dedicated to better understanding of the achievable sum rate in the single carrier FDMA (SC-FDMA) system. We investigate the achievable sum rate in a two-user SC-FDMA system, where a mix of SIMO (Single Input Multiple Output) and SDMA (Space Division Multiple Access) transmission mode is assumed for the wideband communication. The problem is formulated which appears to be non-convex. Relation to the conventional water®lling solution is pointed out for single mode (SIMO or SDMA) transmission. We show that a mix of SIMO and SDMA transmission mode in the wideband is not favorable for any pair of the user equipments. A better strategy would be to switch to SIMO mode or SDMA mode adaptively. We also propose a simple algorithm to ®nd the group of users for SDMA mode based on the wideband average spatial correlation. Performance improvement is shown by combining adaptive SIMO mode and SDMA mode selection with our proposed user grouping algorithm.

I. INTRODUCTION

Recently the single carrier FDMA (SC-FDMA) transmission scheme was agreed to be chosen for the 3GPP-Long Term Evolution (LTE) uplink [1]. It has many similarities to OFDMA and the main difference is the addition of a Discrete Fourier Transform (DFT) before the conventional OFDMA as shown in Fig. 1, which can be viewed as a DFT-precoded OFDMA system with speci®c sub-carrier mapping constraints. One prominent advantage over OFDMA is that SC-FDMA signal has low peak to average power ratio (PAPR) due to its inherent single carrier structure, which bene®ts the user equipment (UE) a lot in terms of transmit power ef®ciency, especially for low order modulation like BPSK or QPSK [2]. To obtain the SC-FDMA signal with low PAPR, the output of DFT should be mapped to the sub-carriers in one of the following two ways [3]: One way is to map them to consecutive sub-carriers, also known as the localized FDMA (LFDMA). The other way is to map them equidistantly over the entire system bandwidth, also known as the interleaved FDMA (IFDMA). Irrespective of the used mapping methods, a demapping operation masking out of the allocated sub-carriers followed by a simply frequency domain equalization and IDFT can be used to recover the transmitted signal.

However, regarding the achievable system sum rate, SC-FDMA is inferior to OFDMA due to the limited degrees of freedom to perform power and sub-carrier allocation. OFDMA allows individual power allocation per sub-carrier basis over the entire bandwidth. However, it would be only possible for SC-FDMA to adjust the power within the consecutively or equidistantly allocated sub-carriers after DFT. Nevertheless, this is unfavorable due to the additional added complexity and possibly high PAPR of the resulting waveform. Consequently, it would be realistic to use equal power allocation among the transmitted signal. On the other hand, it’s shown that the optimum sub-carrier mapping for general DFT-precoded OFDMA requires to choose the most strongest sub-carriers over the entire bandwidth for the single user case [4]. For the multi-user case, it was indicated that good sub-carriers which are generally not adjacent should be used by the UEs in order to exploit the multi-user diversity.

Assume each UE has a single antenna and the base station (BS) is equipped with multiple antennas. If the channel state information (CSI) of all the UEs is available at the BS, it’s known that SDMA can be used to serve multiple UEs simultaneously using the same frequency resources. By properly selecting UEs to operate on SDMA mode, spectral ef®ciency can be obtained hence system sum rate can be improved. However, if the UEs are highly spatially correlated, SDMA can even lead to sum rate degradation. In this case, it may be favorable that each UE operates on SIMO mode using different frequency resources. In an OFDMA system, whether to decide for SDMA or SIMO mode can be done on sub-carrier/resource block (RB) basis by comparing the rates of them. However, this is not possible for SC-FDMA system due to the sub-carrier mapping constraints. It is an open question how and when to assign frequency resource to SDMA transmission or SIMO transmission, or a combination of them in order to achieve higher system sum rate.

In order to ensure SDMA to offer high spectral ef®ciency, signi®cant work ([5] and the references therein) has been done to ®nd suitable UEs and decide whether and when to grant a resource to them. However, most of the algorithms work...
on sub-carrier basis hence the performance evaluation is not suitable for SC-FDMA system.

In this paper, we restrict ourself to LFDMA mapping and start with investigation of the achievable sum rate in a two-UE system, where a mix of SIMO and SDMA transmission mode is considered. The problem is formulated which appears to be non-convex, hence difficult to solve. Interestingly, from all our observations based on numerical evaluations, maximum sum rate occurs only with SIMO FDMA or full bandwidth SDMA and nothing in between. Therefore, we propose an adaptive SIMO and SDMA transmission mode for SC-FDMA system. Furthermore, we propose a simple algorithm to find the group of users for SDMA mode based on the wideband average correlation, which together with adaptive mode transmission offers significant performance improvement in a multi-user frequency selective environment.

This paper is organized as follows: Section II describes the system model and the problem formulation. Section III presents the proposed SDMA grouping algorithm. Simulation results are provided in Section IV. Conclusions are drawn in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

<table>
<thead>
<tr>
<th>SIMO (UE 1)</th>
<th>SDMA (UE 1 &amp; UE 2)</th>
<th>SIMO (UE 2)</th>
</tr>
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<tbody>
<tr>
<td>$N_{1}^{SIMO}$</td>
<td>$N_{SDMA}$</td>
<td>$N_{2}^{SIMO}$</td>
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Fig. 2. Frequency allocation for SIMO and SDMA transmission in an SC-FDMA system with 2 UEs

Consider the wideband LFDMA uplink transmission with 2 UEs, where the BS is equipped with $N_{r}$ antennas and each UE is equipped with a single antenna. In principle, both UEs can operate on the SIMO mode or the SDMA mode. In the SIMO mode, signals from different UEs are transmitted using different frequency resource to avoid interference and each UE’s signal is detected by maximum ratio combining at the BS. This is referred to as the SIMO FDMA. In the SDMA mode, both UEs use the same frequency resource to transmit data simultaneously and a linear zero forcing MIMO receiver is employed at the BS to recover the data streams from different UEs. An important issue is how to select the transmission mode or a combination of the transmission modes adaptively for different channel conditions so that the aggregate sum rate in the system is maximized. For simplicity, we assume a frequency flat wideband channel for each UE which corresponds to the situation where $N_{t}$ is large enough so that the diversity gain effectively mitigate the channel deep fades. As a result, regarding the achievable sum rate, it makes no difference to allocate which specific frequency resources in terms of sub-carriers to which UE and which mode. It only depends on how many frequency resources should be allocated to which UE and which mode. Without loss of generality, the frequency assignment for SIMO an SDMA transmission can then be described as in Fig. 2, where $N_{1}^{SIMO}$ sub-carriers and $N_{2}^{SIMO}$ sub-carriers are used for the SIMO mode by the first and the second UE, respectively, and $N_{SDMA}^{SIMO}$ sub-carriers are used for the SDMA mode by both UEs. They are the design parameters to be determined in the system.

Let $\hat{\gamma}_{i,j}$ denote the post-detection SNR in frequency domain before IDFT, with its first subscript $i$ representing the UE index, the second subscript $j$ the sub-carrier index and the superscript $m$ the transmission mode. The sum rate of the $i$th UE $R_{i}$ is equivalent to that achieved by equal gain power allocation among allocated sub-carriers, whose equivalent channel gain to noise ratio (CNR) equals the post-detection SNR [4], i.e.,

$$R_{i} = (N_{1}^{SIMO} + N_{SDMA})\log_{2}(1 + \beta_{i}P_{total,i})$$

with

$$\beta_{i} = \frac{1}{\sum_{j=1}^{N_{1}^{SIMO}} (1/\hat{\gamma}_{i,j}) + \sum_{j=1}^{N_{SDMA}} (1/\hat{\gamma}_{i,j})}, \quad i = 1, 2$$

where $P_{total,i}$ is the total power constraint associated with the $i$th UE. In frequency flat fading channel, the post-detection SNR is the same for the sub-carriers allocated for the same transmission mode, hence

$$\hat{\gamma}_{i,j}^{SIMO} = \hat{\gamma}_{i,j}^{SIMO}, \quad \hat{\gamma}_{i,j}^{SDMA} = \hat{\gamma}_{i,j}^{SDMA}, \quad \forall i$$

Plugging (3) into (2) we have

$$\beta_{i} = \frac{1}{(N_{1}^{SIMO}/\hat{\gamma}_{i,j}^{SIMO}) + (N_{SDMA}/\hat{\gamma}_{i,j}^{SDMA})}.$$  

Denote $N_{total}$ as the number of total available sub-carriers in the system, our objective is to maximize the sum rate of both UEs which can be formulated as the following optimization problem:

$$\text{maximize} \quad R_{1} + R_{2}$$
$$\text{subject to} \quad N_{1}^{SIMO} + N_{SDMA} + N_{2}^{SIMO} \leq N_{total}$$
$$0 \leq N_{1}^{SIMO} \leq N_{total}$$
$$0 \leq N_{SDMA} \leq N_{total}$$
$$0 \leq N_{2}^{SIMO} \leq N_{total}$$

The above problem is a discrete optimization problem which is difficult to solve. One possible approach may be to use the continuous extension technique, as used in [6]. So the objective function can be written in continuous form which is differentiable and the constraint functions affine. Then the Karush-Kuhn-Tucker (KKT) conditions can be used to discuss the optimum solution. However, note that the continuous form of the objective function is not concave, local optimums have to be compared with each other to find the global optimum. Unfortunately, solving the KKT condition of this problem is very difficult since the optimization variables can not be explicitly expressed as a function of other optimization variables which is omitted here for concision. Instead, we refer to the numerical analysis in the later session and find that the maximum sum rate only occurs at $N_{SDMA} = N_{total}$ or $N_{SDMA} = 0$ ($N_{1}^{SIMO} + N_{2}^{SIMO} = N_{total}$) for all our observations, meaning that the maximum sum rate is achieved either by both UEs...
operating on SDMA mode in the entire system bandwidth or by operating on SIMO mode in an FDMA manner. Any mix of the SIMO and SDMA modes like in Fig. 2 will result in lower achievable sum rate. In the following, these two special cases will be analyzed.

A. SIMO FDMA mode

In the case of SIMO FDMA mode transmission, \( N_{SDMA} = 0 \) \((N_{1}^{SIMO} + N_{2}^{SIMO} = N_{total})\) holds. By plugging this condition into (1) and (4), the achievable system sum rate in frequency flat fading channel can be expressed as

\[
R_{total} = N_{1}^{SIMO} \log_2 (1 + \frac{\gamma_{1}^{SIMO} P_{total,1}}{N_{total}^{SIMO}}) + N_{2}^{SIMO} \log_2 (1 + \frac{\gamma_{2}^{SIMO} P_{total,2}}{N_{total}^{SIMO}})
\]

where each UE’s sum rate can now be interpreted as equal power allocation among allocated sub-carriers with constant CNR = \( \gamma_{SIMO} \) since the term \( P_{total} \) indicates that the total power is equally divided among them. In order to obtain the maximum system sum rate, the optimum value of \( N_{1}^{SIMO} \) and \( N_{2}^{SIMO} \) have to be determined. Note that equal power allocation among flat channels is equivalent to waterfilling power allocation solution because the amount of water \( (P_{total}) \) will then distribute itself equally among all the flat sub-channels. Therefore, the maximum achievable sum rate can be found by the multi-user waterfilling solution [7], where the optimum number of allocated sub-carriers of each UE should be found by the multi-user waterfilling solution [7], where the optimum bandwidth allocation among allocated sub-carriers with constant power allocation among allocated sub-carriers with constant CNR = \( \gamma_{SIMO} \)

B. Full bandwidth SDMA mode

In the case of full bandwidth SDMA mode transmission, \( N_{SDMA} = N_{total} \) holds. By plugging this condition into (1) and (4), the achievable system sum rate in frequency flat fading channel can be similarly expressed as

\[
R_{total} = N_{total} \log_2 (1 + \frac{\gamma_{1}^{SIMO} P_{total,1}}{N_{total}^{SIMO}}) + N_{total} \log_2 (1 + \frac{\gamma_{2}^{SIMO} P_{total,2}}{N_{total}^{SIMO}})
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\]

III. SDMA GROUPING ALGORITHM

Our previous discussion assumes that the post-detection SNRs are known a priori. In reality, they are subject to the actual channel conditions. For SIMO mode, since maximum ratio combining is assumed at the receiver, \( \gamma_{SIMO} \) depends on the CNR and spatial channel correlation of \( N_{1} \) branches. For SDMA mode, \( \gamma_{SDMA} \) depends on the channel correlation between both UEs, as well as the CNR. In a multi-user system with more than 2 UEs, if we are allowed to form proper UE pairs and smartly assign resources to them, higher resulting \( \gamma_{SDMA} \) can be obtained and the system sum rate performance can be optimized without sacrificing fairness among users. The question is how to find the best UE pairs for SDMA transmission, this is referred to as the SDMA grouping problem.

In the SDMA/OFDMA system, it’s known that if spatial channels of the UEs in the same SDMA group are orthogonal, spectral efficiency gain can be obtained. On the other hand, if they are spatially correlated, SDMA can even lead to spectral efficiency loss. In other words, the degree of spatial correlation between the UEs will cause a higher or lower post-detection SNR of the two streams. Therefore, for each sub-carrier building an SDMA group with the UEs whose sub-channel is as uncorrelated as possible is a good solution for the SDMA grouping problem. As a result, it’s much likely that different sub-carriers/ RBs favor different user groups in the case of frequency selective channel. However, this solution is not applicable for the SDMA/SC-FDMA system, where individual treatment of sub-carriers is not allowed. To overcome this problem, we propose an SDMA grouping algorithm based on the average wideband channel correlations.

In order to describe the proposed algorithm, similar notations as in [5] are used. Let \( H_{kn} \in C^{N \times 1} \) denote the channel matrix between the \( k \)th UE and the BS on sub-carrier \( n \). The full channel matrix \( H_{kn} \) on sub-carrier \( n \) can be written by stacking the channel matrix \( H_{kn} \) as

\[
H_{kn} = [H_{1n}, H_{2n}, \ldots, H_{Kn}].
\]

The spatial correlation between two vector channels \( h_{i} \) and \( h_{j} \) is measured by the normalized scalar product \( \rho_{ij} \) which is given by [8]

\[
\rho_{ij} = \frac{\| h_{i} h_{j}^{H} \|}{\| h_{i} \|_2 \| h_{j} \|_2}
\]

where \( \| \cdot \| \) is the absolute value and \( \| \cdot \|_2 \) is the \( l_2 \) norm. Using (5) and (6), the channel spatial correlation between all the UEs on sub-carrier \( n \) is then given by

\[
R_{n} = \frac{N_{n} H_{n}^{H} H_{n} N_{n}}{\text{with}}
\]

\[
N_{n} = \text{diag}(\| h_{1n} \|_2^{-1}, \| h_{2n} \|_2^{-1}, \ldots, \| h_{Kn} \|_2^{-1})
\]

where \( \| \cdot \| \) applies to \( R_{n} \) element-wise, and \( \text{diag}(\cdot) \) denotes the diagonal matrix with the arguments on the diagonal elements. The proposed algorithm for SDMA grouping is to look for the UE pairs \( (p^{*}, q^{*}) \) such that the average channel spatial correlation over all the sub-carriers is minimized, i.e.,

\[
(p^{*}, q^{*}) = \arg \min_{p,q} \sum_{n=1}^{N_{total}} \frac{R_{n}|_{p,q}}{\| R_{n} \|_{F}}
\]

where \( \| \cdot \|_{F} \) denotes the Frobenius norm. Note that (7) explicitly builds an SDMA group with 2 UEs. However, it can be applied iteratively to build multiple SDMA groups with 2 UEs from the rest available UE candidates.

IV. SIMULATION RESULTS

In this section, the performance of the mixed SIMO and SDMA transmission mode and our proposed SDMA grouping algorithm will be evaluated by simulations.
First, the achievable sum rate of the mixed SIMO and SDMA transmission mode under different post-detection SNR conditions are investigated. The simulation scenario according to Fig. 2 is assumed. The total number of available sub-carriers is fixed to 100, i.e., N_{\text{total}} = 100. For simplicity, it’s further assumed that the post-detection SNRs are the same for the same transmission mode for both UEs, i.e., \( \gamma_1^{\text{SIMO}} = \gamma_2^{\text{SIMO}} = \gamma^{\text{SIMO}} \) and \( \gamma_1^{\text{SDMA}} = \gamma_2^{\text{SDMA}} = \gamma^{\text{SDMA}} \).

Fig. 3 shows the achievable sum rate in bits/s/Hz by assuming \( \gamma^{\text{SIMO}} = 30\text{dB} \) and different \( \gamma^{\text{SDMA}} \) for different number of \( N_1^{\text{SIMO}} \) and \( N_2^{\text{SIMO}} \). Interestingly, it can be observed that the maximum sum rate occurs either in the case of \( N_1^{\text{SIMO}} + N_2^{\text{SIMO}} = 100 \) or in the case of \( N_1^{\text{SIMO}} = N_2^{\text{SIMO}} = 0 \) for different post-detection SNR conditions, meaning that the maximum sum rate is achieved either using SIMO FDMA mode or full bandwidth SDMA mode as stated in section II. It’s worth mentioning that this statement also holds for all our observations by assuming different total number of available sub-carriers in the system.

We next investigate the property of the optimum bandwidth division between both UEs if they operate on SIMO FDMA mode. To this end, we keep \( N_{\text{total}} = 100 \) fixed and vary \( N_1^{\text{SIMO}} \) for different values in the simulation. Then by exhaustive search we can find the optimum \( N_1^{\text{SIMO}} \) and \( N_2^{\text{SIMO}} \) when the maximum system sum rate is achieved. Fig. 4 depicts the ratio between the optimum allocated number of sub-carriers of each UE \( N_1^{\text{SIMO}}/N_2^{\text{SIMO}} \) over the ratio between the product of the post-detection SNR and power constraint \( \gamma_1^{\text{SIMO}} \cdot P_{1,\text{total}}/(\gamma_2^{\text{SIMO}} \cdot P_{2,\text{total}}) \) of each UE. They are consistent with the theoretical results derived in Appendix. The only difference between them results from the rounding errors since the number of allocated sub-carriers are integer values in the simulation, yet in theory, they can be any non-negative values.

To evaluate the proposed SDMA grouping algorithm, simulation is performed using LTE-like parameters. A BS is equipped with \( N_T = 2 \) (or \( N_T = 3 \)) receive antennas separated by 10 wavelengths. It serves 10 single-antenna UEs. There are totally 300 sub-carriers in the 5MHz system and they are grouped in blocks of 12 adjacent sub-carriers, resulting in 25 resource blocks. Each block is assumed to be the minimum addressable resource unit in frequency domain for each UE. It’s further assumed that all UEs have the same average path loss. A fixed group size of 2 UEs is assumed. Channels are generated from the SCME (Spatial Channel Model Extended) urban-macro scenario [9] with 18,000 time slots consisting of 14 OFDM symbols each. Sub-carrier allocation will then be applied on the time slot basis. For each time slot, only one group is allowed to be assigned sub-carriers. They can either use SIMO FDMA mode or use full bandwidth SDMA mode. The rest of the groups later on access the channel in a Round Robin manner. For SIMO FDMA mode, it’s assumed that the ratio of allocated sub-carriers of the 2 UEs equals the ratio of their average receive SNR in that frame. Note that this is optimum in the case of flat fading channel as stated before. The achievable system sum rate can be calculated using (1) by assuming the maximum ratio combining receiver for the SIMO mode and the zero forcing receiver for the SDMA mode. Fig. 5 shows the average system sum rate over different receive SNR for different transmission schemes. It can be observed that the proposed SDMA grouping algorithm offers about 1.8 dB gain over random grouping at high SNR condition. It amounts to around 0.7 bits/s/Hz which translates to 3.5 Mbits/s improvement in the 5MHz bandwidth. On the other hand, SIMO FDMA achieves higher average sum rate over full bandwidth SDMA at low SNR condition. Motivated by these facts, if an additional comparison is made between SIMO FDMA mode and full bandwidth SDMA mode for each time slot and the one with higher sum rate is selected, the advantage of both SIMO and SDMA transmission can be combined in the whole interested SNR range, which is the upper curve depicted in Fig. 5. If the BS is equipped with 3 receive antennas and all other simulation parameters
version of the harmonic mean of increases. Furthermore, the rate loss of SC-FDMA reduces the channel is less frequency selective, remain the same, full bandwidth SDMA will outperform SIMO by using the 2×3 SDMA configuration is much larger (about two times) than that by the 2×2 SDMA configuration. The reason is as follows: In SDMA mode, \( \beta_i \) in (2) can be expressed as \( \beta_i = \frac{1}{\sum_{j=1}^{N_{r}} \left(1/\gamma_{i,j}^{\text{SDMA}}\right)} \), \( i = 1, 2 \), which is a scale version of the harmonic mean of \( \gamma_{i,j}^{\text{SDMA}} \)'s. It’s known that the harmonic mean is sensitive and restricted by small values. If the channel is less frequency selective, \( \gamma_{i,j}^{\text{SDMA}} \) will be less variant, which leads to a lower probability that smaller values of \( \gamma_{i,j}^{\text{SDMA}} \) would occur than in a more frequency selective channel. Due to the diversity gain provided by an additional receive antenna in the 2x3 SDMA configuration, its effective channel is less frequency selective than that of the 2x2 SDMA configuration. As a consequence, \( \gamma_{i,j}^{\text{SDMA}} \) is less variant which results in a higher \( \beta_i \) and hence the achievable sum rate increases. Furthermore, the rate loss of SC-FDMA reduces significantly due to hardening of the effective channel caused by additional receive diversity.

V. CONCLUSION

We investigate the achievable sum rate in a two-user SC-FDMA system, where a mix of SIMO and SDMA transmission mode is assumed for the wideband communication. The relation to the conventional waterfilling solution is pointed out for the single mode (SIMO or SDMA) transmission. We show for all our observations that the maximum sum rate only occurs either with SIMO FDMA mode or full bandwidth SDMA mode for wideband flat fading channel. Furthermore, a simple SDMA grouping algorithm based on the wideband average spatial correlation for frequency selective channel is proposed. Performance improvement is observed by combining it with adaptive SIMO mode and SDMA mode selection without sacrificing fairness among users.

APPENDIX

In the following, we show that the optimum number of allocated sub-carriers of each UE should fulfill \( N_{1}^{\text{SIMO}}/N_{2}^{\text{SIMO}} = (\gamma_{1}^{\text{SIMO}}/P_{1})/(\gamma_{2}^{\text{SIMO}}/P_{2}) \) for SIMO FDMA mode. The equivalent CNR of the ith UE, parameterized by \( b_i \), can be expressed as \( \text{CNR}_i = \gamma_{i}^{\text{SIMO}}/b_i \), where \( b_i \) is scalars. According to the multi-user waterfilling principle, for wideband flat fading channel the following relation holds

\[
\begin{align*}
    b_1P_{\text{total}} &= (1 - \frac{b_1}{\gamma_1^{\text{SIMO}}})\gamma_1^{\text{SIMO}} \\
    b_2P_{\text{total}} &= (1 - \frac{b_2}{\gamma_2^{\text{SIMO}}})\gamma_2^{\text{SIMO}}.
\end{align*}
\]

Furthermore, the equivalent CNR for both UEs should be the same to satisfy the power constraint of both UEs. It follows that

\[
\begin{align*}
    b_1/\gamma_1^{\text{SIMO}} &= b_2/\gamma_2^{\text{SIMO}}. \tag{10}
\end{align*}
\]

Plugging (10) into (8) and (9), we can easily arrive at the relation \( N_{1}^{\text{SIMO}}/N_{2}^{\text{SIMO}} = (\gamma_{1}^{\text{SIMO}}/P_{1})/(\gamma_{2}^{\text{SIMO}}/P_{2}) \).

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