Radio Resource Management for the Multi-user Uplink Using DFT-precoded OFDM

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Abstract—In this paper we investigate resource allocation for an uplink DFT-precoded OFDMA system, a subset of which is agreed to be used in the Third Generation Partnership Project-Long Term Evolution (3GPP-LTE) uplink. Different from the well studied OFDMA resource allocation problem, each user equipment (UE) is assigned a set of sub-carriers, within which individual power allocation per sub-carrier basis is unfavorable due to the additional added complexity and possibly high peak to average power ratio of the resulting waveform. We assume a widely used linear equalizer at the receiver with multiple antennas and formulate the problem as the sub-carrier allocation problem for OFDMA with equal gain power allocation. Furthermore, we present an optimum resource allocation algorithm for the single user case and a heuristic sub-optimum algorithm for the multi-user case that can efficiently allocate resources for DFT-precoded OFDMA system.

I. INTRODUCTION

OFDM system has many favorable properties such as high spectral efficiency, robustness to multipath fading and easy equalization. Recently OFDM has been extended to a multiple access scheme (OFDMA) to accommodate multiple users in communication systems, where each UE has a set of selected sub-carriers. However, a major disadvantage is that OFDMA as well as OFDM waveform suffers from the high peak to average power ratio (PAPR) problem [1]. In a mobile cellular system, this is very power inefficient, especially in the uplink and problematic for cell edge users to overcome the large path loss. One way to mitigate the high PAPR problem is to use DFT-precoding techniques in an OFDMA system as depicted in Fig. 1, where the user’s signal will be first transformed to frequency domain using Discrete Fourier Transform (DFT). The DFT output is then mapped to the desired sub-carriers in an OFDMA manner depending on the mapping strategy. The overall transmitted signal can be produced essentially as a single carrier signal for two types of sub-carrier mappings: localized mapping (LFDMA) and equidistant distributed mapping (IFDMA), which both have inherently lower PAPR than OFDMA [2], [9]. Recently, this multiple access scheme, also known as single carrier (SC) FDMA, has been agreed on to be used in the 3GPP LTE uplink to increase the terminal power efficiency. Other mapping strategies than the two described above may result in higher PAPR than that of the SC-FDMA signal which is out of the scope of our discussion. However, regarding the system performance other mapping strategies may be preferred due to their higher degrees of freedom to perform frequency depending scheduling if the channel knowledge of each UE is available at the base station (BS), where resource allocation takes place. The resource allocation information is transmitted over a signaling channel in the downlink. It includes resource block assignment, power allocation, and selection of modulation and coding schemes.

One important issue is how to efficiently use the frequency resources in the DFT-precoded OFDMA uplink in terms of maximization of the sum data rate. Similar problems and their optimum solutions as well as sub-optimum approximations with applications for OFDMA system without DFT precoding have been addressed in [3], [4], [5], [6] and the references therein. Those solutions all suggest a joint sub-carrier and power allocation for each UE. However, different from the well studied OFDMA resource allocation problem, in this paper we consider on the one hand each UE is assigned a set of sub-carriers, within which individual power allocation per sub-carrier basis is unfavorable due to the additional added complexity and possibly high PAPR of the resulting waveform. In other words, we assume that data symbols with equal mean energy will be first transformed by DFT and then fed to the OFDMA transmitter without any intermediate power allocation stage. On the other hand a widely used zero forcing (ZF) equalizer is considered at the receiver for the sake of simple linear processing in practice. Under these practical constraints we are interested in the optimum sub-carrier mapping strategy of DFT outputs to the sub-carriers in terms of maximization of the aggregated sum data rate at the BS.

Another important issue is how to strike a balance between the system sum rate and fairness among UEs. In this paper we further apply a score-based scheduler [7] in the DFT-precoded OFDMA system and show the performance.

This paper is organized as follows: Section II derives the system model, states the problem formulation and provides the optimum solution for single user sub-carrier allocation. Section III proposes a sub-optimum multi-user sub-carrier allocation algorithm. Simulation results are given in Section IV. Finally, conclusions are drawn in Section V.

II. RESOURCE ALLOCATION

A. System model

In this section, the system model for the uplink sub-carrier allocation will be discussed where DFT-precoded OFDMA trans-
mission scheme is always assumed. We assume an FDD system where channel knowledge in the uplink of all the UEs is perfectly measured by the BS. Depending on the actual channel quality of each uplink transmission, the BS will assign sub-carriers, bits and power properly to each UE with individual power constraint so that the maximum possible sum rate of the whole system in terms of bits/symbol is achieved.

For simplicity, we start with a single user system where the UE has a single antenna and the BS has \( K \) multiple antennas. The block diagram of the transmitter and receiver under consideration is shown in Fig. 2. A data stream is converted from serial to parallel (S/P) into data blocks of size \( N \). Then each block is transformed to frequency domain via an \( N \)-point DFT. Next, the output of the DFT representing \( N \) frequency components of the signal is mapped to the \( N \times N \) transformed to frequency domain via an \( N \)-point DFT. Next, the output of the DFT representing \( N \) frequency components of the signal is mapped to the \( N \times N \) matrix

\[
\mathbf{F}_N = \frac{1}{\sqrt{N}} e^{-j 2\pi n k / N},
\]

where

\[
\mathbf{F}^H_N \text{ denotes the inverse Fourier matrix. Further on, the assignment of data symbols } \mathbf{d} \text{ to specific sub-carriers is described by a } Q \times N \text{ sub-carrier mapping matrix } \mathbf{M} \text{ with the entry}
\]

\[
m_{q,n} = \begin{cases} 1 & \text{n\text{th} data symbol assigned to } q\text{th sub-carrier} \\ 0 & \text{n\text{th} data symbol not assigned to } q\text{th sub-carrier} \end{cases}
\]

\[
1 \leq q \leq Q, 1 \leq n \leq N.
\]

After CP removal at the receiver, the received block at the \( k \)th branch (\( 1 \leq k \leq K \)) can be written as

\[
y_k = \mathbf{H}_k \mathbf{F}_Q^H \mathbf{M} \mathbf{F}_N \mathbf{d} + \mathbf{v}_k
\]

where \( \mathbf{d} \) is the input data vector of size \( N \) with \( \mathbf{R}_d = E\{\mathbf{d}\mathbf{d}^H\} = E_d \mathbf{I} \) and \( \mathbf{v}_k \) is the \( Q \times 1 \) AWGN vector with \( E\{\mathbf{v}_k\mathbf{v}_k^H\} = N_0 \mathbf{I} \) at the \( k \)th branch, and \( \mathbf{H}_k \) is a \( Q \times Q \) circular channel matrix of the \( k \)th branch with its first column \( \mathbf{h}_k = [h_{k,0}, h_{k,1}, \cdots, h_{k,L}, 0, \cdots, 0]^T \) consisting of a zero padded channel impulse response and the transfer factors of the channel is defined as \( \mathbf{h}_k = \sqrt{Q} \mathbf{Q} \mathbf{h}_k \). By collecting \( K \) received blocks in a vector, the transmitted and received signal is related by

\[
y = \mathbf{H} \mathbf{d} + \mathbf{v}
\]

where

\[
y = [y_1, y_2, \cdots, y_K]^T, \quad \mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_K]^T \quad \text{and} \quad \mathbf{v} = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_K]^T.
\]

The ZF equalizer is then given by the Moore-Penrose pseudoinverse of \( \mathbf{H} \), i.e.,

\[
\mathbf{W}_{ZF} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{W}_{ZF}.
\]

Next, the discrete symbols are inserted between each block to prevent inter-block interference and the guard interval longer than the maximum delay of the channel is considered. Then the accumulated symbols are mapped to the output of the DFT representing \( N \) sub-carriers is described by a

\[
\mathbf{F}_N
\]

specific sub-carriers is described by a

\[
\mathbf{F}_N
\]

\[
Q
\]

where

\[
E\{\mathbf{y} \mathbf{H}^H\} = E_d \mathbf{I}
\]

\[
E\{\mathbf{v}_k \mathbf{v}_k^H\} = N_0 \mathbf{I}
\]

and the remaining signal is fed to the time domain ZF equalizer applied in the time domain

\[
\mathbf{d} = \mathbf{W}_{ZF} \mathbf{y} = \mathbf{d} + \mathbf{W}_{ZF} \mathbf{v}.
\]

By denoting the noise term \( \eta = \mathbf{W}_{ZF} \mathbf{v} \), the post-detection SNR of the \( n \)th component of the transmitted data block can be calculated as

\[
\gamma_{ZF,n} = \frac{E_{\mathbf{d}}}{E\{\eta \mathbf{H}^H\}_{nn}} = \frac{N E_d}{N_0 (\sum_{n=1}^{N} \sum_{k=1}^{K} |h_{f,k,n}|^2)} = \gamma_{ZF}. \quad (3)
\]

By defining the equivalent sub-carrier channel gain to noise ratio (CNR), i.e.,

\[
g_n = \frac{\sum_{k=1}^{K} |h_{f,k,n}|^2}{N_0},
\]

(3) can be written as

\[
\gamma_{ZF} = \frac{P_{\text{total}}}{\sum_{n=1}^{N} (1/g_n)} \quad (5)
\]

where \( P_{\text{total}} = N E_d \) which stands for the total power constraint of the UE. From (3) and (5) we can see that the post-detection SNR is the same for all the components which is a function of the maximum ratio combining of the selected sub-carrier channel gains of all the branches. Therefore, according to (2) the \( N \) transmitted data symbols can be viewed as passing \( N \) Gaussian sub-channels with the same noise power. Therefore, using Shannon’s capacity formula, the total achievable rate can be calculated as a sum of rates on all used sub-carriers, namely

\[
C_{ZF} = N \log_2 (1 + \gamma_{ZF}) = N \log_2 (1 + \frac{P_{\text{total}}}{\sum_{n=1}^{N} (1/g_n)}). \quad (6)
\]

In the case of multiple users where each UE has a set of selected sub-carriers, the post-detection SNR of the \( n \)th component of the transmitted block for the \( p \)th UE can be similarly obtained as

\[
\gamma_{ZF,p} = \frac{N_p E_{d,p}}{N_0(\sum_{n=1}^{N_p} \sum_{k=1}^{K} |h_{f,k,n,p}|^2)} = \frac{P_{\text{total,p}}}{\sum_{n=1}^{N_p} (1/g_{n,p})},
\]

where \( h_{f,k,n,p} \) is the frequency response of the \( n \)th selected sub-carrier channel gain seen by the \( k \)th antenna for the \( p \)th UE with its corresponding equivalent CNR denoted as \( g_{n,p} \) and \( N_p \) is the total number of selected sub-carriers of the \( p \)th UE, which corresponds to the size of the DFT precoder of that UE. Thus the total achievable sum rate for the \( p \)th UE reads

\[
C_{ZF,p} = N_p \log_2 (1 + \gamma_{ZF,p})
\]

and the system sum rate with \( U \) UEs is then given by

\[
C_{ZF,\text{multi}} = \sum_{p=1}^{U} C_{ZF,p} = \sum_{p=1}^{U} N_p \log_2 (1 + \frac{P_{\text{total,p}}}{\sum_{n=1}^{N_p} (1/g_{n,p})}). \quad (7)
\]

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B. Problem formulation

We start with the single user case and consider maximizing the system sum rate. By assumption the UE has a total power constraint \( P_{\text{total}} \) which is equally divided in the transmitted signal. According to (6) the degrees of freedom is to choose proper sub-carriers over all available sub-carriers in the system such that the system sum rate is maximized. This is done by choosing a proper size of DFT in the precoder and mapping the output of DFT to the desired sub-carriers, i.e.,

\[
(g_n^*, N^*) = \arg \max_{1 \leq n \leq Q} C_{ZF}(N, g_n)
\]

\[
= \arg \max_{1 \leq n \leq Q} N \log_2(1 + \beta P_{\text{total}})
\]

where the last step follows from (6) by denoting \( \beta = \frac{1}{\sum_{n=1}^{N} (1/g_n)} \). The \( C_{ZF}(N, g_n) \) is a discrete function with the set of possible allocated sub-carriers as domain and the set of achievable sum rate as range. At a first glance, this is a combinatorial optimization problem which is difficult to solve. However we observe that \( \beta \) resembles the total resistance of a parallel resistor circuit if \( g_n \) is interpreted as the resistance of the \( n \)th component of the circuit. It can be proved that \( \beta \) is a monotonically increasing function w.r.t. \( g_n \), therefore for a given number of allocated sub-carriers \( N \), with \( N \in \{1, \cdots, Q\} \), sum rate is maximized if the \( N \) strongest sub-carriers are chosen. This provides us a basis to sort the sub-carriers in descending order according to their equivalent CNR and map the precoded symbols to the strongest sub-carriers. An addition of one more sub-carrier, resulting in a decreasing \( \beta \), will decrease the rate of the allocated sub-carriers. (Similar with adding a resistor in a parallel circuit reduces the total resistance.) As long as the loss of the rate of the allocated sub-carriers is smaller than the rate of added sub-carriers, sum rate over all the new allocated sub-carriers will increase. It can be expected that the sum rate will reach a maximum for a specific number of used sub-carriers. After the maximum an addition of one weaker sub-carrier will not bring any rate gain, since the rate gain obtained by adding a weaker sub-carrier is smaller than the rate loss of allocated sub-carriers. The aim is then to find the optimum number of sub-carriers \( N^* \) to be assigned to the UE in order to maximize \( C_{ZF} \), i.e.,

\[
N^* = \arg \max_{1 \leq n \leq Q} C_{ZF}.
\]

C. Optimum sub-carrier allocation (mapping)

The above problem may be solved by using its continuous extension like the technique used in [8]. However, this technique requires a proof of necessary and sufficient conditions irrespec- tive of whether the problem is concave or not. Moreover, even if the above conditions are found, a brute force search starting from \( N=1 \) and increasing the number of allocated sub-carriers until maximum in (9) is achieved can have lower complexity. In summary, the optimum sub-carrier allocation which achieves maximum sum rate in the single user case consists of two steps according to our previous discussion:

- Sort the equivalent CNRs of all the sub-carriers in descending order \( g_{n, \text{sorted}} > g_{n+1, \text{sorted}}, \forall n \in \{1, \cdots, Q - 1\} \)

- Start with one allocated sub-carrier, i.e., \( N=1 \) and increase the number sub-carriers until (9) is found.

Then \( N^* \) is the optimal number of sub-carriers that should be allocated and \( C_{ZF}(N^*) \) is the maximum achievable sum rate. Note that PAPR is not considered here for optimality.

D. Equivalence with OFDM equal gain power allocation

In this section we will show that the maximum sum rate achieved by ZF-equalized DFT-precoded OFDM using equal power allocation is equivalent to that achieved by non-precoded OFDM with equal gain power (EGP) allocation among the allocated sub-carriers. For notational simplicity, only a single branch receiver is considered but the result can be straightforward applied to multiple branches.

An OFDMA system transforms the frequency selective channel into a set of parallel AWGN channels of different gains. The sum rate using this scheme can be calculated as the sum rate of all the AWGN channels \( N_c \), i.e.,

\[
C_{\text{OFDM}} = \sum_{n=1}^{N_c} \log_2(1 + \frac{P_n|h_{f,n}|^2}{N_0})
\]

where \( h_{f,n} \) is the channel gain of the \( n \)th selected sub-carrier and \( P_n \) represents the power assigned to that sub-carrier. The last step follows from the definition of (4). EGP allocation strategy tries to pre-equalize the transmitted signal so that all gains of the allocated sub-carriers \( P_{n,g_n} \) are equal, i.e.,

\[
\begin{cases}
P_n g_n = \text{constant}, & \forall n \in \text{subject to } \sum_n P_n = P_{\text{total}}.
\end{cases}
\]

The solution is given by

\[
P_{\text{eg},n} = \frac{P_{\text{total}}}{g_n \sum_{l=1}^{N_c} (1/g_l)}, \quad (n = 1, \cdots, N_c).
\]

Upon insertion of (12) into (10), we have

\[
C_{\text{OFDM}} = \sum_{n=1}^{N_c} \log_2(1 + \frac{P_{\text{total}}}{\sum_{l=1}^{N_c} (1/g_l)}) = N_c \log_2(1 + \frac{P_{\text{total}}}{\sum_{l=1}^{N_c} (1/g_l)}) \text{bits/OFDM symbol.}
\]

Obviously, with the same occupied sub-carriers (6) and (13) are equivalent, hence the sum rate achieved by ZF-equalized DFT-precoded OFDMA with equal power allocation subject to a certain power constraint is equivalent to the sum rate achieved by a non-precoded OFDMA system using equal gain power allocation among selected sub-carriers.

It would be interesting to point out that EGP allocation shares the similar geometrical interpretation with waterfilling (WF) as illustrated in Fig. 3. It is well known that for WF (left) we can imagine that \( \text{CNRI} \) is the bottom of a container and a fixed amount of water (power), \( P_{\text{total}} \), is poured into the container. The water (power) will distribute inside the container with a water level. Then the distance between the container bottom and water level \( P_n \) in linear scale represents the power allocated to the \( n \)th sub-carrier. EGP allocation can be interpreted in the similar way if the bottom of the container is represented by CNR instead of \( \text{CNRI} \), and both CNR and the resulting \( P_n \) are measured in logarithmic scale (dB) instead of linear scale as depicted in Fig. 3 (right). This can be easily verified by taking the logarithm operation at both sides of (11). It is also worthy to mention that the product \( \beta P_{\text{total}} \) in (8) describes the final water level after power allocation.
III. MULTI-USER SUB-CARRIER ALLOCATION

In this section, we will investigate the multi-user scenario where the total available sub-carriers in the system are shared in FDMA manner by all the UEs at any time instance. Our objective is still to maximize the system sum rate. Such a problem can be straightforward extended from (8) as

$$
(g_{n,p}^*, N_p^*) = \arg \max_{1 \leq N_p \leq Q} \sum_{p=1}^U C_{gF}(N_p, g_{n,p})
$$

where $U$ is the total number of UEs and the last step follows from (7) with $\beta_p = \frac{1}{\sum_{n,p} (1/g_{n,p})}$ for the $p$th UE. Seeking an optimum sub-carrier allocation algorithm for the above problem is a challenging task since the objective function depends on both the allocated sub-carriers and individual power constraint of each UE. Instead of solving the difficult discrete optimization problem, we propose a sub-optimum solution motivated by the optimum single user sub-carrier allocation algorithm. The algorithm mainly consists of an initialization step performing the single user optimum sub-carrier allocation and a multi-user “compare and compete” step. By associating the preferred sub-carrier set $J_p$ and the allocated sub-carrier set $A_p$ with each UE, the algorithm tries to allocate proper sub-carriers from the preferred sub-carrier sets to the UEs. In the first step, each UE’s preferred sub-carrier set is initialized to the set of optimum sub-carriers determined by the single user optimum sub-carrier allocation algorithm (refer to Section II-C) assuming all the sub-carriers in the system are available for each UE. In other words, each UE is assumed not to be aware of other UEs and it performs optimum single user sub-carrier allocation individually in the system to initialize the preferred sub-carrier set. At this stage, each UE’s allocated sub-carrier set is initialized to be empty and the available sub-carriers are the total sub-carriers in the system. In the second step, the post-detection SNR $\gamma_p$ is computed individually for each UE assuming it gets its preferred sub-carriers. The UE with the highest post-detection SNR (water level) will be assigned its first preferred sub-carrier and this sub-carrier will be considered not available any more for the other UEs. Hence, for the UEs having the same preferred sub-carrier, they should base on the remaining available sub-carriers and update their preferred sub-carrier set using the single user optimum sub-carrier allocation algorithm, where the update procedure should decide whether to take more sub-carriers or reduce the occupied sub-carriers to maximize their individual sum rate. Afterwards all the UEs will compete for the remaining resources again using the same criterion until all the resources have been allocated or all the UEs have been assigned their maximum preferred sub-carriers. The whole procedure can be summarized as follows:

Initialize:
- Sort $g_{n,p}$ in descending order for each UE $p \rightarrow g_{n,p}^*$
- Apply single user optimum sub-carrier allocation algorithm to get optimum set of sub-carriers $N_p^*$ for each UE $p$
- Initialize set of preferred sub-carriers $J_p$ with $g_{n,p} \geq g_{n,p}^*$
- Initialize set of allocated sub-carriers $A_p = \emptyset$, $\forall p$, and set of available sub-carriers $A = \{1, \cdots, Q\}$

Repeat:
- Compute $\gamma_p = \frac{P_{total,p}}{\sum_{n,p} (1/g_{n,p})}$. Find $p^* = \arg \max_p \{\gamma_p\}$, $n_{p^*} = \arg \max_{n,p} \{g_{n,p}\}$ with $n_{p^*} \in J_{p^*} \setminus A_{p^*}$ and $|J_{p^*}| < N_{p^*}$.
- Set $A_{p^*} = A_{p^*} \cup \{n_{p^*}\}$, and $J_{p^*} = J_{p^*} \setminus \{n_{p^*}\}$, $\forall p \neq p^*$.
- Re-estimate $J_{p^*}$ and $N_{p^*}$ using the single user optimum sub-carrier allocation algorithm $\forall p \neq p^*$.

Until $A = \emptyset$ or $A_p = N_p$, $\forall p$.

IV. SIMULATION RESULTS

In this section, the performance of the single and multiple user sub-carrier allocation algorithms will be evaluated by simulations. First we consider the impact of different sub-carrier mapping methods on the maximum achievable sum rate for the single user case. Both SISO and SIMO configurations are taken into account and for the latter a BS with 2 receive antennas separated by 10 wavelength is assumed. The channel used consists of many single time slots generated from the LTE-like parameters. All total available 1200 sub-carriers are mapped to the middle part of the total bandwidth of 30.7 MHz consisting of 2048 sub-carriers, where the amplitude of DC sub-carrier and the rest un-mapped sub-carriers are set to zero. As a result, the 1200 sub-carriers occupy 18 MHz bandwidth with 1 MHz for analogue filtering at each side. These sub-carriers are further divided into 100 resource blocks (RBs) with each consisting of 12 sub-carriers, which is assumed to be the minimum addressable resource unit for one UE in frequency domain. For simplicity, we assume that sub-carriers inside a RB experience the same channel and this sub-carrier will be considered not available any more for the other UEs.

Fig. 4 shows the average sum rate achieved by different sub-carrier mapping methods subject to the transmit power constraint which would correspond to an SNR level of 0dB with 20 MHz
bandwidth, if equal power allocation among all the RBs is employed. It can be observed that the single user optimum sub-carrier allocation method (magenta) almost achieve the channel capacity computed by the well known water-filling (WF) solution (blue) for up to 30 allocated RBs in both SISO and SIMO configurations. Regarding the maximum sum rate marked with ”o” in the figure, optimum allocation reaches around 90% of the channel capacity with less allocated sub-carriers. For other widely used sub-carrier allocation methods LFDMA and B-IFDMA [10], they can only achieve around 50% of it for SISO and around 80% for SIMO. Without loss of generality, we assume in fixed LFDMA the starting position of consecutively mapped sub-carriers is kept fixed to be the first sub-carrier in the system. In dynamic LFDMA, the starting position will be searched until the highest achievable sum rate occurs subject to the number of the allocated consecutive sub-carriers. In the case of B-IFDMA, it is assumed that blocks containing 12 adjacent sub-carriers equidistantly distribute over the total available bandwidth. It should be noted that a strict sense B-IFDMA only allows the number of allocated RBs to be 1, 2, 4, 5, 10, 20, 25, 50 and 100 in order to fulfill the equidistant allocation property. Therefore, the number of assigned RBs have to be floored to the nearest number of allowed RBs although more RBs are available.

Next, we evaluate the performance of the proposed multi-user sub-carrier allocation algorithm. Fig. 5 shows the average sum rate of each UE achieved by different sub-carrier allocation strategies in the multi-user SISO system, where the distance of each UE to the BS is given below: We again use the same LTE-like parameter setting as for the single user case. It is further assumed that each UE moves at 10m/s randomly in the cell and the time sample density is chosen as 2 samples/half wavelength. Sub-carrier allocation will then be applied on the time sample (frame) basis. The transfer factors of users’ channels are generated from the SCME (Spatial Channel Model Extended) “urban-macro” scenario [11] with 5000 time samples. In order to compare the performance of the proposed resource allocation algorithm, maximum sum rate and resource fair policies are considered. For the former, multi-user WF [5] subject to individual power constraint is adopted representing the channel capacity. For the latter we consider the score-based scheduler [7] and Round Robin scheduler combined with LFDMA and B-IFDMA for sub-carrier allocation. It is assumed that each UE occupies 10 RBs for LFDMA and B-IFDMA and all UEs access the frequency resources in the Round Robin manner for different time frames. Irrespective of the used resource allocation strategies, the resulting achievable sum rate can be calculated according to (7) based on the actual allocated sub-carriers. It can be viewed as performing EGP allocation among the allocated sub-carriers for each UE depending on the resource allocation algorithm. Therefore, comparison allows us to get some insight on how the system and individual sum rate are affected using different policies in the DFT-precoded OFDMA system. It can be seen from Fig. 5 that the proposed algorithm (EGP-Proposed) achieves more than 93% of the system sum rate obtained by WF. Round Robin has inherent fairness but they neglect actual channel state information of the UEs. EGP with score-based scheduler (EGP-SB) achieves significant gains over LFDMA and B-IFDMA at least by a factor of two for cell edge users.

\[
\text{User Equipment Index, last index represents all the UEs.}
\]

Fig. 5. Comparison of different resource allocation strategies in uplink “urban-macro” scenario. System contains 10 UEs with index 1 to 3 denoting closed to BS UEs, index 4 to 7 representing UEs in the middle of the cell and index 8 to 10 standing for the cell edge UEs.

**V. CONCLUSION**

We analyzed resource allocation in zero forcing equalized DFT-precoded OFDMA uplink. An optimum sub-carrier allocation algorithm was found for the single user case and the performance was compared to other popular sub-optimum allocation strategies, i.e., LFDMA and B-IFDMA. A heuristic sub-optimum multi-user resource allocation algorithm was presented and shown to achieve more than 93% of the system sum rate obtained by multi-user waterfiling algorithm. Furthermore, we showed that the score-based scheduler achieved significant gains for cell edge UEs, at least by a factor of two over LFDMA and B-IFDMA with Round Robin.

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