ML-Based Joint Estimation of Carrier Frequency Offset and Doubly Selective Channels for OFDM Transmissions

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Abstract—This paper proposes a pilot-aided joint channel estimation and synchronization scheme for burst-mode orthogonal frequency division multiplexing (OFDM) systems over time- and frequency-selective (doubly selective) channels. By exploiting the basis expansion model (BEM) for representing doubly-selective channels, a maximum likelihood (ML) cost function of carrier frequency offset (CFO) and BEM coefficients is formulated to develop a ML framework for the joint synchronization and channel estimation problem. Inheriting the properties of the ML estimation, the proposed estimator is unbiased and its mean-squared-error (MSE) performance achieves Cramèr-Rao lower bounds (CRLB) asymptotically (for a large data record). Simulation results demonstrate that, over a wide range of Doppler spreads, the proposed estimation scheme offers a high robustness against the time variation of fast fading channels and outperforms the linear minimum mean square (LMMSE) algorithm. In addition, the ML-based algorithm achieves CRLB at very low signal-to-noise ratio (SNR) regimes.

Index terms—Orthogonal frequency division multiplexing (OFDM), joint channel estimation and synchronization, time- and frequency-selective channel, basis expansion model (BEM), carrier frequency offset (CFO).

I. INTRODUCTION

The last decade has witnessed numerous intensive studies in employing orthogonal frequency division multiplexing (OFDM) for broadband communication systems to exploit its high spectral efficiency and robustness against multipath (frequency-selective) fading channels [1]. However, most of these studies have focused on time-invariant (quasi-static) multipath channels. In wireless OFDM transmissions over (fast) time-varying multipath channels, e.g., 4-G mobile communications and digital audio/video broadcasting (DAB/DVB) systems with (high-speed) mobile users, the time-variation (time-selectivity) of channel impulse responses (CIR) induce a loss of subcarrier orthogonality and in turn inter-carrier interference (ICI) [3]. Furthermore, multicarrier-based transmissions are highly vulnerable to carrier frequency offset (caused by oscillator mismatches and/or Doppler shift) that also introduces ICI [2]. Consequently, the presence of both time-selective (time-variant) multipath channels and carrier frequency offset (CFO) will incur a significant ICI power, giving rise to an irreducible error floor in OFDM receiver performance. For CFO compensation and coherent data detection/decoding, the knowledge of CFO and CIR is therefore indispensable at OFDM receiver. Hence, accurate estimation of doubly selective (time- and frequency-selective) channels and CFO under low SNR regimes (in coded transmissions) is of crucial importance to (mobile) OFDM-based systems. Due to the fact that the time-variant CIR is different at each (time-domain) sample within an OFDM symbol, doubly selective channel estimation becomes a challenging problem in mobile OFDM-based systems. Recently, a few algorithms [3]-[7] have been proposed for doubly selective channel estimation in OFDM transmissions. However, these algorithms have been developed and investigated under the assumption of perfect synchronization. In practice, such an assumption is rarely attained while imperfect synchronization conditions most likely exist and would significantly degrade the performance of these channel estimation algorithms and in turn the overall receiver performance.

This paper proposes a joint synchronization and doubly selective channel estimation scheme for single-input single-output (SISO)-OFDM transmissions. By exploiting the basis expansion model (BEM) [8]-[10] for representing doubly selective channels and the maximum likelihood (ML) principle, a ML cost function of CFO and BEM coefficients is formulated to facilitate a ML framework for the joint estimation of CFO and BEM parameters. As a ML-based estimation, the proposed estimator is unbiased and able to achieve Cramèr-Rao lower bound asymptotically (for the use of a large observation record). Unlike the LMMSE-based channel estimation algorithm [6] that requires the use of the autocorrelation channel matrix and noise power, the proposed ML-based scheme does not need this knowledge.

The rest of the paper is organized as follows. Section II describes the system model. The derivations of the proposed joint channel estimation and synchronization are delineated in Section III. Section IV presents simulation results and relevant discussions. Finally, conclusion is provided in Section V.

Notation: Upper (lower) bold-face letters will be used for matrices (column vectors). Superscripts $H$, $T$ and $\ast$ will stand for Hermitian, transpose and conjugate operators, respectively. $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ denote integer floor and ceiling operators, respectively.

II. SYSTEM MODEL

A. Transmitted signal model

Consider an uncoded single-input single-output (SISO) OFDM system using MQAM modulations. Let $X_k^{(m)}$ be a complex-valued data symbol conveyed by the sub-carrier $k$ of the $m$-th OFDM symbol. Each OFDM symbol consists of $K=N$ information bearing sub-carriers, where $N$ is FFT size. After IFFT and cyclic prefix (CP) insertion, the $n$-th sample of the $m$-th OFDM symbol is

$$x_n^{(m)} = \frac{1}{\sqrt{N}} \sum_{k=-K/2}^{K/2} X_k^{(m)} e^{j2\pi nk/N}, \quad n = 0, 1, ..., N-1$$

where $N_g$ denotes CP length.

B. Doubly-selective fading channel model

In this paper, the time-varying multipath channel response that includes the effect of transmit-receive filters and doubly

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selective propagation is denoted by \( h(t, \tau) \). Let \( H(f, \tau) \) stand for the Fourier transform of \( h(t, \tau) \). Also, \( \tau_{\text{max}} \) and \( f_{\text{max}} \) are defined as the delay spread and the Doppler spread, respectively, where \( |H(f, \tau)| = 0 \) for \( |\tau| > \tau_{\text{max}} \) or \( |f| > f_{\text{max}} \). Various existing algorithms based on short training symbols in the preamble of a burst can be used to establish symbol timing synchronization and the sampling period \( T \) at receiver. In burst-mode OFDM transmissions, the BEM [8]-[10] can be applied for an interval of \( MN_T \) seconds corresponding to one burst duration of \( M \) OFDM symbols with a length of \( N_e = N_L + N \) samples. More specifically, \( h(t, \tau) \) over one burst duration can be represented by: i) \( Q+1 \) coefficients that remain unchanged per burst duration but are allowed to vary independently in the next burst, and ii) \( Q+1 \) Fourier bases that capture the time variation of the channel are common for all burst durations. As a result, the discrete-time baseband equivalent channel response at the \( n \)-th received sample of the \( m \)-th OFDM symbol can be represented as

\[
h_{q,l}^{(m)} = \sum_{q=0}^{Q} h_{q,l}e^{j\omega_q((m+N_e)+(m-1)N)}, \quad l \in [0, L-1] \tag{2}
\]

where \( \omega_q \approx 2\pi(q - Q/2) / (MN_e) \), \( L \approx \tau_{\text{max}} / T \), \( Q \approx 2f_{\text{max}}MN_T \) and \( n = -N_e, \ldots, N-1 \).

In doubly selective channel estimation, the BEM coefficients \( h_{q,l} \) can be treated either as deterministic unknowns (in Fisher estimation) or as zero-mean complex Gaussian random variables with variance \( \sigma^2_{h,q,l} \) [6], [10] (in Bayesian estimation). The variance \( \sigma^2_{h,q,l} \) can be determined based either on the multipath intensity profile and Doppler power spectrum [10] or on the autocorrelation function of the time-varying channel [6].

This paper considers the BEM coefficients \( h_{q,l} \) as unknowns to be estimated by the proposed ML-based algorithm. As a result, the ML-based joint synchronization and channel estimation scheme does not need the knowledge of autocorrelation channel matrix and noise power as in the case of the LMMSE algorithm presented in [6].

C. Received signal model

Over the time- and frequency-selective channel, after CP removal, the \( n \)-th received sample in the \( m \)-th OFDM symbol \( y_n^{(m)} \) can be represented by

\[
y_n^{(m)} = e^{j2\pi f_n} N \sum_{l=0}^{L-1} h_{q,l}^{(m)} x_{n-l}^{(m)} + w_n^{(m)} \tag{3}
\]

where \( f \) denotes the normalized carrier frequency offset (CFO), \( L \) is the channel order, \( w_n^{(m)} \) stands for receiver noise samples with variance of \( N_0 \) and \( n = 0, \ldots, N-1 \).

III. ML JOINT ESTIMATION OF CFO AND BEM COEFFICIENTS

A. Derivation of the ML-based joint estimation algorithm

Based on the received samples at (3) in the time domain, the proposed pilot-aided algorithm attempts to jointly estimate BEM coefficients and CFO by using the maximum likelihood (ML) principle. Based on the joint probability density function of the received samples at (3), one can deduce a ML cost function of CFO and BEM coefficients as follows

\[
f = \| y - \Phi(\hat{\epsilon})S\|^2
\]

where \( y = \begin{bmatrix} y(m_0)^T, y(m_1)^T, \ldots, y(m_P)^T \end{bmatrix}^T \),

\[
y(m_p) = \begin{bmatrix} y_0, y_1, \ldots, y_{N-1}^p \end{bmatrix}^T, \quad p = 1, \ldots, P,
\]

\[
\Phi(\epsilon) = \text{diag}\begin{bmatrix} \Phi^{(m_0)}(\epsilon), \Phi^{(m_1)}(\epsilon), \ldots, \Phi^{(m_P)}(\epsilon) \end{bmatrix}^T,
\]

\[
\theta^{(m_p)}(\epsilon) = \frac{1}{N} e^{j2\pi f N [0 + \tau_{m_p} + (m_p-1)N]}, \quad \epsilon = e^{j2\pi \tau_{m_p} N},
\]

\[
S = \begin{bmatrix} s(m_0)^T, s(m_1)^T, \ldots, s(m_P)^T \end{bmatrix}^T
\]

\[
S^{(m_p)} = \left[ \begin{array}{c} \Omega_0^{(m_p)}(\epsilon) \Omega_{L}^{(m_p)}(\epsilon) \left( s(m_p) \right)^T \end{array} \right],
\]

\[
\Omega_{q}^{(m_p)} = \text{diag}\left[ e^{j2\pi \nu_{q+1} N [0 + \tau_{m_p} + (m_p-1)N]}, e^{j2\pi \nu_{q} N [1 + \tau_{m_p} + (m_p-1)N]} \right]^T
\]

\[
\Lambda^{(i)}(\epsilon) = \begin{bmatrix} \Lambda_0^{(i)}(\epsilon), \Lambda_{L}^{(i)}(\epsilon) \end{bmatrix}^T
\]

\[
h = \begin{bmatrix} h_0^T, \ldots, h_{L-1}^T \end{bmatrix}^T, \quad \hat{\epsilon} = \arg \max_{-\epsilon_{\text{max}} \leq \epsilon \leq \epsilon_{\text{max}}} \{ L(\epsilon) \}
\]

where \( L(\epsilon) = y^H \Phi(\epsilon)S^H(\epsilon)y \) with \( P = S^H S \) and \( \epsilon \) denotes the number of pilot OFDM symbols used for the ML-based estimation problem.

Setting the gradient vector of the ML cost function \( \frac{\partial f}{\partial h} \) to zeros yields the ML estimates of BEM coefficients as follows

\[
\hat{h} = \left( S^H S \right)^{-1} S^H \Phi^H(\epsilon) y \tag{5}
\]

Then, substituting \( \hat{h} = \left( S^H S \right)^{-1} S^H \Phi^H(\epsilon) y \) into the ML cost function (4) yields the ML estimate of CFO as follows

\[
\hat{\epsilon} = \arg \max_{-\epsilon_{\text{max}} \leq \epsilon \leq \epsilon_{\text{max}}} \{ L(\epsilon) \}
\]

As described in (6), the ML estimate of CFO can be obtained by using the fine one-dimensional grid search over the range of possible CFO values. To alleviate a high complexity of the exhaustive search, the Newton-Raphson [14] method can be used after a coarse CFO estimation by using the grid search at (6) with a large step size (as compared with the high-complexity fine grid search using a small step size).
B. Newton-Raphson method [14]

The Newton-Raphson method for the CFO estimation problem can be mathematically described as follows:

$$\hat{\epsilon}_{k+1} = \hat{\epsilon}_k - \left[ \frac{\partial^2 L(\epsilon)}{\partial \epsilon^2} \right]^{-1} \left[ \frac{\partial L(\epsilon)}{\partial \epsilon} \right]_{\epsilon = \hat{\epsilon}_k}$$

(7)

where $\hat{\epsilon}_k$ denotes the CFO estimate at the $k$-th iteration.

With the aid of the Newton-Raphson method [14], the proposed ML-based estimation algorithm can be summarized in the following steps:

1) Coarse CFO estimation by the grid search at (6).
2) Fine CFO estimation by the Newton-Raphson method using the coarse CFO estimate (from Step 1) as an initial guess.
3) ML estimation of BEM coefficients by using (5) and the fine CFO estimate obtained from Step 2.

C. Pilot symbol placement

For burst-mode OFDM transmissions, the burst structure used in this paper is shown in Fig. 1. As illustrated, data segment includes pilot OFDM symbols where all sub-carriers of these symbols are dedicated to pilots. The pilot pattern is similar to the one used in [6].

![Pilot pattern for joint CFO and doubly selective channel estimation](image)

Fig. 1. Pilot pattern for joint CFO and doubly selective channel estimation.

V. SIMULATION RESULTS AND DISCUSSIONS

Computer simulation was conducted to evaluate the performance of the proposed joint synchronization and doubly selective channel estimation scheme. As an illustrative example, we use the OFDM system parameters similar to those given in the IEEE 802.11a standard [13]. QPSK signal constellation is employed for OFDM symbols of 52 data sub-carriers. The data segment contains 7 pilot symbols and 7 data OFDM symbols (similar to the pilot pattern used in [6]). As used in [10], the doubly selective channel model is generated by using the multipath intensity profile $\phi_c(r) = \exp(-0.1r/T)$ and the Doppler power spectrum is

$$S_c(f) = \left( \pi f^{2}_{max} - f^2 \right)^{-1}$$

where sampling period $T=50$ns and Doppler spread $f_{max} \in [0, 900]$ (Hz).

Fig. 2 shows the MSE performance and Cramér-Rao lower bound (CRLB) of CFO estimation versus SNRs. In Fig. 2, the Doppler spread is $f_{max} = 300$Hz. The normalized CFO in each burst (in the simulation) is generated as a random variable uniformly distributed in the range $[-0.3, 0.3]$. In the first step of the ML-based estimation scheme, the coarse CFO grid search (6) uses step size of 0.01 then the Newton-Raphson method with 10 iterations is employed for the fine CFO estimation. As observed, the proposed algorithm provides highly accurate CFO estimates and achieves the Cramér-Rao lower bound in a very low SNR regime, even at SNR=0dB. In practice, the accurate CFO estimation at low SNRs is highly desired in coded OFDM transmissions.

Fig. 3 depicts the MSE performance and CRLB of channel (BEM) estimation versus SNRs. The settings of CFO, $f_{max}$, coarse and fine CFO estimation are similar to those in Fig. 2. As can be seen, in the presence of CFO (imperfect synchronization scenario), the LMMSE-based channel estimation algorithm [6] without CFO compensation has worse performance than the sequential estimation where the LMMSE scheme [6] uses CFO estimates (for CFO compensation before the LMMSE channel estimation) by an existing CFO estimation algorithm [11]. By performing joint estimation of CFO and channel responses to avoid mutual effect between channel estimation and synchronization in sequential estimation, the proposed ML-based scheme provides more accurate channel estimates than the LMMSE algorithm [6] under the above imperfect synchronization scenarios. More importantly, the ML-based estimation scheme is able to achieve CRLB in a very low SNR regime (even at SNR=-2.5dB). As SNR>-2.5dB, the performance of the ML-based scheme (without using the knowledge of autocorrelation channel matrix and noise power) is comparable with that of the LMMSE algorithm [6] (Bayesian estimation) that uses the priori knowledge under a perfect synchronization scenario (CFO=0). It is noted that CRLB is herein used as a theoretical performance limit for only the ML-based estimation scheme (Fisher estimation) and not for the LMMSE algorithm [6] (Bayesian estimation).

Figs. 4 and 5 show the MSE performance of the ML-based joint CFO and BEM estimation under various Doppler spreads, $f_{max}$. As can be seen, the proposed ML-based scheme is robust against the time variation of the multipath fading channels. As observed, the LMMSE [6] without CFO compensation provides inaccurate channel estimates under any Doppler spread. With CFO compensation using CFO estimates from [11], the performance of the LMMSE algorithm [6] is enhanced but still significantly worse than that of the proposed scheme which jointly estimates CFO and BEM coefficients by a ML framework.
In the paper, a pilot-aided joint synchronization and doubly selective channel estimation algorithm was proposed for SISO-OFDM transmissions. Using the basis expansion model (BEM) [8]-[10] to represent doubly-selective channels, a ML cost function of CFO and BEM coefficients is formulated to develop a ML framework for the joint CFO and BEM estimation. Over a wide range of Doppler spreads, the proposed ML-based scheme provides a high robustness against the time variation of fast fading channels and outperforms the linear minimum mean square (LMMSE) algorithm [6] in different imperfect synchronization scenarios. More importantly, the ML-based estimation algorithm is able to achieve CRLB at very low SNRs (practical operating SNR conditions for coded OFDM transmissions). Unlike the LMMSE-based channel estimation algorithm [6], without the knowledge of autocorrelation channel matrix and noise power, the ML-based joint synchronization and channel estimation scheme still provides the MSE performance comparable to that of the LMMSE-based algorithm [6] in perfect synchronization condition.

APPENDIX

Cramér-Rao Lower Bound Derivations

The received samples $y_n$ corresponding to $P$ pilot OFDM symbols can be represented in vector form as follows:

$$y = \Phi(\varepsilon)Sh + w.$$  \hspace{1cm} \text{(A.1)}

Then, the Fisher Information Matrix [15] can be determined by

$$F = \frac{2}{N_0} \text{Re} \left[ \frac{\partial}{\partial \omega} \Phi(\varepsilon)SH^H \right] \left[ \frac{\partial}{\partial \omega} \Phi(\varepsilon)SH \right]^T$$  \hspace{1cm} \text{(A.2)}

where $\omega = \begin{bmatrix} \text{Re}(\tau^T) & \text{Im}(\tau^T) \end{bmatrix}^T$.

The CRLB matrix can be obtained by inverting the Fisher information matrix in (A.2)

$$CRLB(h, \varepsilon) = F^{-1}.$$  \hspace{1cm} \text{(A.3)}

REFERENCES


