Achieving Close-Capacity Performance with Simple Concatenation Scheme on Multiple-Antenna Channels

Nghi H. Tran†, Tho Le-Ngoc‡, Tad Matsumoto‡, and Ha H. Nguyen§
†Department of Electrical & Computer Engineering, McGill University, Montreal, QC, Canada
‡Information Theory and Signal Processing Laboratory, School of Information Science, Japan Advanced Institute of Science and Technology, Japan
§Department of Electrical & Computer Engineering, University of Saskatchewan, Saskatoon, SK, Canada

nghi.tran@mcgill.ca, tho.le-ngoc@mcgill.ca, matumoto@jaist.ac.jp, ha.nguyen@usask.ca

Abstract—This paper proposes a simple yet capacity-approaching concatenation of a mixture of short memory length convolutional codes and simple rate-1 block code followed by either complex 1-dimensional (1-D) anti-Gray or Gray mapping over multiple antenna channels with quadrature phase-shift keying (QPSK). By interpreting rate-1 code together with 1-D mapping as a multi-D mapping employed over multiple transmit antennas, the error performance is analyzed in two regions, the error-floor and turbo pinch-off regions. In the former one, a tight union bound and design criterion on the asymptotic performance are first derived, which provide an useful tool to predict the error performance. Based on the design criterion, an optimal rate-1 code for each 1-D mapping is then constructed to achieve the best asymptotic performance. In the turbo pinch-off area, by using extrinsic information transfer (EXIT) chart, the most suitable mixed codes are selected for both symmetric and asymmetric antenna setups. It is demonstrated that the simple concatenation scheme can achieve near-capacity. Furthermore, its error performance is comparable to that obtained by using well-designed irregular low-density parity-check (LDPC) and repeat accumulate (RA) codes, and thereby, outperforms a scheme employing a parallel concatenated turbo code.

I. INTRODUCTION

With the recent developments in iterative decoding, a number of pragmatic approaches using powerful turbo-like codes have been proposed [1]–[3] to achieve near-capacity over multiple-antenna channels. For instance, by using a turbo code followed by complex 1-D Gray mapping under a bit-interleaved coded modulation (BICM) framework [4], it was shown in [1] that a close-capacity performance can be attained in a symmetric antenna setup. However, the error performance of such turbo coded systems experiences a significant degradation when the antenna scenario is asymmetric [2], i.e., the number of receive antennas is smaller than the number of transmit antennas. As an alternative, reference [2] proposes an LDPC coded modulation scheme that performs very close to the capacity limit in any antenna configuration. Equally good performances are also obtained in [3] by using outer irregular RA codes. Similar to [1], 1-D Gray mapping is also employed independently and identically over each transmit antenna in [2], [3] for a superior performance. To our knowledge, the designs in [2], [3] are still the most effective coded modulation techniques over multiple antenna wireless fading channels.

This paper proposes an attractive alternative for close-capacity performances over multiple antenna channels applicable for both anti-Gray and Gray mappings. In particular, a simple yet capacity-approaching serial concatenation of a mixture of short memory length convolutional codes and a short and simple rate-1 block code followed by either 1-D anti-Gray or Gray mapping with QPSK is introduced. By interpreting rate-1 code together with 1-D mapping as a multi-D mapping [5], [6] employed over multiple transmit antennas, the error performance is analyzed in the error-floor and turbo pinch-off regions. In the former one, a tight union bound and design criterion on the asymptotic performance are derived, which provide an useful tool to predict the error performance. Optimal rate-1 block codes for anti-Gray and Gray mappings are then determined to achieve the best asymptotic performance. In the turbo pinch-off area, by using EXIT chart [7], the most suitable mixed code for each antenna configuration is selected. It is demonstrated that the simple concatenation scheme can achieve near-capacity over both symmetric and asymmetric multiple-antenna channels. Furthermore, its error performance is comparable to that using well-designed irregular LDPC and RA codes in [2], [3], thereby, outperforms a scheme employing a parallel concatenated turbo code, especially when there are more transmit than receive antennas.

II. SYSTEM MODEL

![Fig. 1. The proposed concatenation scheme equipped with $N_t$ transmit and $N_r$ receive antennas.](image-url)
A. Transmitter

A block diagram of the transmitter of the proposed concatenation system equipped with $N_t$ transmit and $N_r$ receive antennas is depicted in Fig. 1 (a). First, a binary information block $u$ of length $L_u$ is divided into two binary sequences $u_I$ and $u_I$ of length $L_I$ and $L_{II}$, respectively. Each sequence $u_l$, $l \in \{I, II\}$, is encoded by a suitable rate-$k_l/n_l$ binary encoder into a coded sequence $c_l$ consisting of $T_l = L_l/n_l/k_l$ coded bits. These binary encoders could be a simple convolutional code (cc) and shall be determined later. A coded sequence $c$ of length $T_c = T_I + T_{II}$ is then constructed by serially combining coded sequences $\{c_l\}$. This encoder structure inherited from code doping technique proposed in [8] is referred to as a mixed code of $K = 2$ binary codes, with code doping ratio $\alpha = L_I/L_u$.

After being interleaved, each group of $Q = 2N_l$ coded bits $v = (v_1, v_2, ..., v_Q)$ is transformed to a vector of $Q$ binary bits $b = (b_1, b_2, ..., b_Q)$ using a rate-1 block code with generator matrix $G$ of size $Q \times Q$ over Galois field 2 (GF(2)) as:

$$b = G \cdot v$$  \hspace{1cm} (1)

To guarantee that there is one-to-one correspondence between $v$ and $b$, a condition of invertibility is imposed on $G$, i.e., $G$ is a full-rank matrix. Then two consecutive bits $(b_{2i-1}, b_{2i})$, $1 \leq i \leq N_t$, are grouped together and mapped to a complex QPSK symbol $s_i$ using either complex 1-D anti-Gray or Gray mapping. A sequence of $N_t$ complex 1-D symbols $\{s_i\}$ is considered to be a super symbol $s = [s_1, s_2, ..., s_{N_t}]^\top$ in a complex $N_t$-D constellation $\Psi$ with cardinality $|\Psi| = 2^{Q_l}$. Finally, a direct transmission over multiple antennas is implemented as in [1], [2], where each signal $s_i$ is transmitted by the $i$th transmit antenna.

The combination of $G$ and 1-D mapping above can be interpreted as a special case of multi-D mapping technique in which a vector of $Q$ binary bits $v = [v_1, v_2, ..., v_Q]^\top$ are mapped directly to a super symbol $s$ according to a mapping rule $\xi$ [5], [6]. Vector $v$ is referred to as the label of $s$. As we shall see shortly, given an outer mixed code, there exists an optimal rate-1 code for each 1-D mapping that leads to the same mapping rule $\xi$ and achieves the best asymptotic performance.

B. Receiver

Consider an ergodic frequency-flat Rayleigh fading channel. The $N_r \times 1$ received vector $r$ is given as $r = H \cdot s + n$, where $H$ is an $N_r \times N_t$ matrix and its components are $CN(0, 1)^1$. $n$ is an $N_r \times 1$ vector representing additive white Gaussian noise whose entries are $CN(0, N_0)$. As similar to [1]–[3], we only consider the case that the channel is known perfectly at the receiver but not the transmitter.

At the receiver, a typical concatenation of a conventional detector, a posteriori probability (APP) bit decoder of rate-1 block code, and a soft-input soft-output (SISO) outer decoder can be applied. Similar to the design in [3], the detector and rate-1 block decoder can be combined in one block as shown in Fig. 1 (b) to reduce decoding complexity and improve robustness. More specifically, by representing rate-1 block code and 1-D mapping as a multi-D mapping $\xi$, the optimal combined detector performs APP detection to provide the extrinsic probability of the $k$ coded bit $v_k$. $1 \leq k \leq Q$, being set at $b, b \in \{0, 1\}$, as:

$$P(v_k = b; O) = \sum_{\rho \in \Psi^b} \left[ \exp \left( -\frac{|| r - h \cdot s ||^2}{N_0} \right) \right] \cdot \prod_{j \neq k} P(v_j = v_j(s); I).$$  \hspace{1cm} (2)

In (2), $\Psi^b$ denotes a subset of $\Psi$ that contains all symbols whose labels have the value $b$ at the $k$th position. Clearly, $\Psi^b$ is determined by the mapping rule $\xi$. Furthermore, $v_j(s)$ is the value of the $j$th bit in the label of $s$ and $P(v_j = v_j(s); I)$ is the a priori probability of the other bits, $j \neq k$, on the same channel symbol. Observe that the computation of the extrinsic information of the coded bit in (2) involves the set of $2^{Q-1}$ super symbols in $\Psi^b$, which has the same complexity as that of the conventional detector [1], [2].

After being deinterleaved, the extrinsic information of the corresponding $T_I$ and $T_{II}$ coded bits computed by the combined detector is forwarded to the two SISO channel decoders, respectively. For convolutional codes, the SISO channel decoder uses the forward-backward algorithm [9]. There is an iterative processing between the combined detector and the outer channel decoder to exchange the extrinsic information as shown in Fig. 1 (b).

III. Tight Union Bound and Design Criterion on The Asymptotic Performance and Optimal Rate-1 Codes

By representing rate-1 block code together with 1-D mapping as a multi-D mapping $\xi$, this section presents a tight union bound on the asymptotic performance of the proposed system. The derivation is based on the assumption of an ideal interleaver and perfect a priori information of coded bits fed back from the decoder to the combined detector as normally seen in the analysis of BICM with iterative decoding [10]. The derived bound can be used to accurately predict the asymptotic BER performance without the need of time-consuming simulations. Furthermore, a design criterion is introduced, which is helpful in developing an optimal rate-1 code together with either anti-Gray or Gray mapping.

A. Tight Union Bound and Design Criterion on The Asymptotic Performance

Due to space limitation, the tight union bound and design criterion on the asymptotic performance are briefly introduced. The detailed analysis can be found in [11].

With an assumption of ideal interleaver, i.e., infinite interleaver, one can treat coding and modulation as separate components in a BICM system [4]. As a result, the bit error
probability (BEP) for a BICM system using a mixed code with code doping ratio $\alpha$ can be expressed as:

$$P_b = \alpha P_b^{(I)} + (1 - \alpha) P_b^{(II)},$$

where $P_b^{(I)}$ and $P_b^{(II)}$ are the BEPs of BICM systems using binary codes I and II, respectively. When the $i$th component code is a rate-$k_i/n_i$ convolutional code, the union bound on $P_b^{(I)}$ is expressed as:

$$P_b^{(I)} \leq \frac{1}{k_i} \sum_{d=d_{ij}}^\infty c_{ij}^{(I)} f(d, \Psi, \xi).$$

In (4), $c_{ij}^{(I)}$ is the total information weight of all of the error events at Hamming distance $d$ and $d_{ij}$ is the free Hamming distance of the binary code $I$. The function $f(d, \Psi, \xi)$ is an average pairwise error probability (PEP), which depends on the Hamming distance $d$, the constellation $\Psi$, and the mapping rule $\xi$. For the system under consideration, the mapping $\xi$ is determined by rate-1 block code $G$ and 1-D mapping.

Owing to the success of iterative decoding steps, it can be assumed that one has perfect a priori information of the coded bits fed-back to the combined demodulator. As a consequence, the union bound on $f(d, \Psi, \xi)$ can be computed by averaging over all signal points $s$ and $p$ in the $N_r$-D constellation $\Psi$ whose labels differ in only 1 bit at position $k$, $1 \leq k \leq Q$, as [11]

$$f(d, \Psi, \xi) \leq \frac{1}{\pi} \int_{0}^{\pi/2} E \left\{ \left(1 + \frac{|s-p|^2}{4N_0 \sin^2 \theta} \right)^{-N_r} \right\} d\theta,$$

where

$$E \left\{ \left(1 + \frac{|s-p|^2}{4N_0 \sin^2 \theta} \right)^{-N_r} \right\} = \frac{1}{Q^22^q} \sum_{s,u \in \Psi} \sum_{k=1}^{Q} \left(1 + \frac{|s-p|^2}{4N_0 \sin^2 \theta} \right)^{-N_r}.$$

As shall be verified later, (5) provides an accurate approximation on the BER performance in the error-floor area. At high enough SNR, the function $f(d, \Psi, \xi)$ can be approximated as [11]:

$$f(d, \Psi, \xi) \approx \frac{1}{2} \left(4N_0 \right)^{-N_r} \hat{\delta}(\Psi, \xi) d,$$

where

$$\hat{\delta}(\Psi, \xi) = \frac{1}{Q^22^q} \sum_{s \in \Psi} \sum_{k=1}^{Q} |s-p|^{-2N_r}.$$  

The parameter $\hat{\delta}(\Psi, \xi)$ above does not depend on SNR and it can be conveniently used to characterize the effect of a mapping $\xi$ to the asymptotic performance. More specifically, $\hat{\delta}(\Psi, \xi)$ should be made as small as possible to minimize the BER. In the next subsection, in combining with either anti-Gray or Gray mapping, an optimal rate-1 code is constructed to minimize $\hat{\delta}(\Psi, \xi)$ in (7).

### B. Optimal Rate-1 Linear Block Codes

Without loss of generality, assume that the coordinates of the four QPSK symbols are $[+1,+1], [+1,-1], [-1,+1],$ and $[-1,-1]$. By representing the super constellation $\Psi$ as a hypercube in $N_r$-D signal space, it was shown in [5] that for any symbol $s$, there is only one symbol $p$ at the largest squared Euclidean distance $4Q$ to $s$. Furthermore, there are $Q$ symbols $\{p\}$ at the second largest squared Euclidean distance $4(Q-1)$ to $s$. This implies the following lower bound on $\hat{\delta}(\Psi, \xi)$:

$$\hat{\delta}(\Psi, \xi) \geq \frac{4^{-N_r}}{Q} \left[ Q^{-N_r} + (Q-1)^{-(N_r-1)} \right].$$

It is simple to see that a mapping rule $\xi$ that satisfies the following condition achieves the equality in (8):

**Condition 1:** For any symbol $s \in \Psi$, let $\Psi_s$ be a set of $Q$ symbols $\{p\}$ whose labels differ in only 1 bit to that of $s$. In $\Psi_s$, there are one symbol at squared Euclidean distance $4Q$ and $(Q-1)$ symbols at squared Euclidean distances $4(Q-1)$ to $s$.

In combining with an 1-D mapping, a rate-1 code $G$ is called optimal if it is invertible and yields to a mapping $\xi$ that satisfies **Condition 1**. This optimal code is determined below for both anti-Gray and Gray mappings. For convenience, the notations $W$ and $F$ are used to indicate rate-1 code for anti-Gray and Gray mappings, respectively.

### 1) Optimal Codes for Anti-Gray mapping:

When anti-Gray mapping is used, it is straightforward to verify that a group of 2 binary bits $(b_{2i-1}, b_{2i})$, $1 \leq i \leq N_r$, shall be mapped to a QPSK symbol $s = [2(b_{2i-1} \oplus b_{2i}) - 1, 2b_{2i} - 1]$, where $\oplus$ denotes the plus operation in $GF(2)$. As a result, a symbol $s \in \Psi$ with label $v$ carrying $Q$ bits $b$, $b = G \cdot v$, can be represented as:

$$s = [2(b_1 \oplus b_2) - 1, 2b_1 - 1, \ldots, 2(b_{N_r-1} \oplus b_{N_r}) - 1, 2b_{N_r} - 1]^T. \quad (9)$$

One then has the following theorem for having an optimal $W$:

**Theorem 1:** Let $w_k = [w_{1,k}, \ldots, w_{Q,k}]^T$ be the $k$th column of $W$. If $W$ is optimal then $w_{2i-1,k}, w_{2i,k} = [1,0]$ for at least $(N_r - 1)$ values of $i$, $1 \leq i \leq N_r$.

**Proof:** Consider two symbols $s = [s_1, \ldots, s_N]$ and $p = [p_1, \ldots, p_N]$ whose labels $v$ and $y$ differ in only 1 bit at position $k$. Also, let $b = W \cdot v$ and $a = W \cdot y$. It then follows that:

$$b \oplus a = W \cdot (v \oplus y) = w_k. \quad (10)$$

Since $W$ is optimal, $||s-p||^2 \geq 4(Q - 1)$. Equivalently, $||s_i - p_i||^2 \geq 8$ for at least $(N_r - 1)$ values of $i$, $1 \leq i \leq N_r$. Furthermore, from (9), one has:

$$||s_i - p_i||^2 = 4 \left( (b_{2i-1} - a_{2i-1})^2 + (b_{2i} - a_{2i})^2 \right). \quad (11)$$

It can be observed from (11) that $||s_i - p_i||^2 = 8$ if and only if $[b_{2i-1} \oplus a_{2i-1}, b_{2i} \oplus a_{2i}] = [1,0]$. Combining this result with (10), **Theorem 1** is proved.

Based on **Theorem 1**, the below theorem provides an optimal $W$ for anti-Gray mapping.

**Theorem 2:** For anti-Gray mapping, a rate-1 block code $W$ whose entries are given as:

$$[w_{2i-1,k}, w_{2i,k}] = \begin{cases} [1,0], & k = 1, 1 \leq i \leq N_r, \\
[1,0], & k > 1, i \neq (k + 1) \text{ div } 2, 1 \leq i \leq N_r, \\
[0,1], & k > 1, k \text{ mod } 2 = 0, i = (k + 1) \text{ div } 2 \\
[1,1], & k > 1, k \text{ mod } 2 = 1, i = (k + 1) \text{ div } 2 \end{cases} \quad (12)$$

is optimal.

**Proof:** The invertible property of $W$ can be proved as follows. Let $m = [m_1, \ldots, m_Q]^T$ be a vector of $Q$ binary
bits and \( m = m_1 \oplus m_2 \oplus \ldots \oplus m_Q \). Consider the following linear combination:
\[
x = m_1 w_1 + m_2 w_2 + \ldots + m_Q w_Q
\]
(13)
It then follows from (12) that:
\[
x = [m \oplus m_2, m_2, \ldots, m \oplus m_{2i-1}, m_{2i-1} \oplus m_2, \ldots, m_Q, m_{Q-1} \oplus m_Q]^T
\]
(14)
Therefore, \( x = 0 \) if and only if \( m = 0 \). As a result, \( W \) is invertible. The optimality of \( W \) then follows closely Theorem 1. In particular, let \( s \) and \( p \) be two symbols whose label differ in only 1 bit at position \( k \). Consider two separate cases of \( k \) as follows:
- If \( k = 1 \), it can be verified that \( \|s_i - p_i\|^2 = 8 \) for all \( 1 \leq i \leq N_t \). This makes \( \|s - p\|^2 = 4Q \).
- If \( k > 1 \), \( \|s_i - p_i\|^2 = 8 \) for all \( 1 \leq i \leq N_t \) but \( i = (k + 1) \) div 2. When \( i = (k + 1) \) div 2, it follows from (12) and (11) that \( \|s_i - p_i\|^2 = 4 \). Therefore, \( \|s - p\|^2 = 4(Q - 1) \).

Theorem 2 is thus proved.

Besides the optimal \( W \) in (12), it is worth noting that by permuting any two columns of \( W \), another optimal code can also be obtained. The proof is straightforward and omitted here for brevity of the presentation.

2) Optimal Codes for Gray mapping: With the optimal \( W \)
in (12), define \( F \) be a \( Q \times Q \) matrix over GF(2) whose elements are given as:
\[
\begin{align*}
f_{2i-1,k} &= w_{2i-1,k} + w_{2i,k} \\
f_{2i,k} &= w_{2i-1,k}
\end{align*}
\]
(15)
The following theorem shows the optimality of \( F \).

Theorem 3: The use of rate-1 code \( F \) in (15) together with Gray mapping results in the same mapping rule \( \xi \) attained by combining rate-1 code \( W \) in (12) and anti-Gray mapping. Consequently, \( F \) in (15) is optimal for Gray mapping.

Proof: Let \( v \) be a vector of binary inputs. When \( W \) in (12) is used together with anti-Gray mapping, a symbol \( s \in \Psi \) with label \( v \) carrying \( Q \) bits \( b, b = G \cdot v \), is given in (9). On the other hand, with rate-1 code \( F \) followed by Gray mapping, a symbol \( p \in \Psi \) carrying \( Q \) bits \( a, a = F \cdot v \), can be expressed as:
\[
p = [2a_1 - 1, 2a_2 - 1, \ldots, 2a_{Q-1} - 1, 2a_Q - 1]^T
\]
(16)
From (15), one has:
\[
\begin{align*}
a_{2i-1} &= f^{2i-1} \cdot v = (w^{2i-1} \oplus w^{2i}) \cdot v = b_{2i-1} \oplus b_{2i} \\
a_{2i} &= f^{2i} \cdot v = w^{2i-1} \cdot v = b_{2i-1}
\end{align*}
\]
(17)
where \( f^{(k)} \) and \( w^{(k)} \) are the \( k \)th rows of \( F \) and \( W \), respectively. It then follows from (9), (16), and (17) that \( s = p \). It means that a combination of either \( F \) and Gray mapping or \( W \) and anti-Gray mapping leads to the same mapping rule \( \xi \). Theorem 3 is proved.

Combining the above results, it can be concluded that the combination of rate-1 code \( W \) in (12) followed by anti-Gray mapping is equivalent to that of rate-1 code \( F \) in (15) together with Gray mapping. Furthermore, they minimize \( \delta(\Psi, \xi) \) in (7). It means that for a given outer mixed code, the proposed concatenation is optimal as far as the asymptotic performance is concerned.

IV. DESIGN USING EXIT CHARTS

The analysis presented in Section III is only useful to predict the error performance at the BER floor region. This section analyzes the convergence property of the proposed scheme for a close-capacity performance at the turbo pinch-off region by means of EXIT chart [7]. Using the same notations as in [7], let \( I_{A1} \) and \( I_{E1} \) denote the mutual information between the \( a \ priori \) LLR and the transmitted coded bit, and between extrinsic LLR and the transmitted coded bit at the input and output of the detector, respectively. Similarly, \( I_{E2} \) and \( I_{A2} \) are the mutual information representing the \( a \ priori \) knowledge and the extrinsic information of the coded bits at the input and output of the SISO decoder.

In the following, a difference between the conventional MIMO detector using Gray mapping and the combined detector considered in this paper is first demonstrated with the aid of EXIT curves. Note that the combined detector is the same for both two cases of rate-1 code \( W \) in (12) with anti-Gray mapping and rate-1 code \( F \) in (15) with Gray mapping. Then a combination of the combined detector and a mixture of simple convolutional decoders, with which close-capacity performance can be achieved, is proposed by having the combined detector EXIT curve matched to the decoder EXIT curve. For illustrative purposes, the number of transmit antennas is fixed at \( N_t = 4 \). Also, we only consider the same rate-1/2 component codes, which results in an overall rate \( r_e = 1/2 \) outer mixed code. The ratio of energy per information bit at the receiver over noise \( E_b/N_0 \) is defined as \( E_b/N_0(\text{db}) = E_s/N_0(\text{db}) + 10 \log_{10} \frac{N_t}{r_e N_c} \) [1], [2], where \( E_s \) is total energy used over \( N_t \) transmit antennas.

A. EXIT curves of the MIMO detector and combined detector

Figure 2 shows EXIT curves of the MIMO detector and combined detector at \( E_b/N_0 = 5 \text{dB} \).

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.
channels [2]. However, for an asymmetric channel, EXIT curve of the MIMO detector exhibits a steeper slope. This phenomenon causes a performance degradation when Gray mapping is used together with those above-mentioned turbo-like codes [2]. This adverse behavior can be overcome by using well-designed irregular LDPC or RA codes as in [2], [3].

In the case of the combined detector, we have shown in [11] that the bitwise mutual information with perfect a priori information, i.e., \( I_E(1,1) \), can be significantly improved over that of the MIMO detector. However, since the proposed block codes are of rate 1, the areas under the combined detector and MIMO detector must be equal [11]. Consequently, it can be observed from Fig. 2 that its EXIT curve exhibits much higher slope over that of the MIMO detector, with larger mutual information at the right end of the curve. As shown in the next subsection, this makes the combination of either rate-1 code \( W \) in (12) with anti-Gray mapping or rate-1 code \( F \) in (15) with Gray mapping a perfect match to proposed mixed codes having also decayed EXIT curve. More interestingly, close-capacity performance can be achieved over both symmetric and asymmetric multiple-antenna channels.

### B. EXIT curve matching

This subsection applies EXIT chart technique [7] to select a suitable mixed code for the combined detector.

![Fig. 3. EXIT charts of the combined detector with \( N_t = N_r = 4 \) at \( E_b/N_0=1.82dB \), rate-1/2 4-state cc with \( g_1 = [1,1,1] \) and \( g_2 = [1,0,1] \), rate-1/2, 2-state cc with \( g_1 = [1,1] \) and \( g_2 = [1,0] \), and a mixed code of 4-state and 2-state codes with code doping ratio \( \alpha = 0.35 \).](image1)

We first examine the symmetric case with \( N_t = N_r = 4 \). Figure 3 plots the EXIT curve of the combined detector at \( E_b/N_0=1.82dB \) and the EXIT curves of two standards rate-1/2, 2-state convolutional code (cc) with generator polynomials \( g_1 = [1,1,1] \) and \( g_2 = [1,0,1] \), rate-1/2, 4-state cc with generator polynomials \( g_1 = [1,1,1] \) and \( g_2 = [1,0,1] \). The above SNR is chosen to make sure that the middle point of the detector EXIT curve \( I_E(0.5) \) is larger than 0.5. Also note that the axes of the EXIT curve of the decoder are swapped so that the convergence behavior can be visualized [7]. It is clear from Fig. 3 that the EXIT curve of the standard rate-1/2, 4-state cc with generator polynomials \( g_1 = [1,1,1] \) and \( g_2 = [1,0,1] \) does not fit well to the detector EXIT curve, since the two EXIT curves quickly intersect and the intersection point falls in the lower left quadrant of the EXIT plane. Because a more powerful rate 1/2 cc exhibits a sharper slope at the beginning, it is not suitable either. The EXIT curve of rate-1/2, 2-state cc intersects the combined detector EXIT curve in the upper right quadrant of the EXIT plane, but at a low mutual information, which does not guarantee low BER.

To overcome the above disadvantages, a mixed code of the two standard convolutional codes can be used to achieve better curve matching. In particular, Fig. 3 also shows the EXIT curve of a mixture of 4-state and 2-state codes with code doping ratio \( \alpha = 0.35 \). It is interesting to see that the EXIT curve of this mixed code matches very well to the detector EXIT curve. The two EXIT curves do not intersect until reaching the ending point \( I_A(1) \) with very high mutual information, leading to a low BER. This match is very similar to that obtained in [2], [3] using an irregular LDPC or RA code and Gray mapping alone, where the curves fit at the pinch-off limit \( E_b/N_0 = 1.8dB \). Furthermore, this curve fit happens close to the capacity limit, which is at \( E_b/N_0 = 1.47dB \).

![Fig. 4. EXIT charts of the combined detector in various asymmetric scenarios using \( N_t = 4 \) and \( N_r = 3 \) at \( E_b/N_0=2.285dB \), \( N_t = 4 \) and \( N_r = 2 \) at \( E_b/N_0=3.285dB \), and rate-1/2, mixed codes with \( \alpha = 0.2 \) and \( \alpha = 0.07 \), respectively.](image2)

In the case of asymmetric configurations, mixed codes can also be applied to match with the combined detector EXIT curves. Figure 4 provides the combined detector EXIT curves in two asymmetric setups i) \( N_t = 4 \) and \( N_r = 3 \) at \( E_b/N_0=2.285dB \), \( N_t = 4 \) and \( N_r = 2 \) at \( E_b/N_0=3.285dB \), and rate-1/2, mixed codes with \( \alpha = 0.2 \) and \( \alpha = 0.07 \). Observe that the EXIT curves are matched very well in all cases. Similar to the symmetric scenario, these results are very comparable to those achieved in [2], [3].

Table I summarizes the results obtained above and those in [2], [3] for comparison. The respective capacity limits are also provided. Clearly, the pinch-off \( E_b/N_0 \) in Table I shows that the concatenation scheme employing only simple outer mixed codes and inner rate-1 block code can approach near-capacity for both symmetric and asymmetric antenna setups.

Before closing this section, it is worth mentioning that the
TABLE I
CAPACITY AND PINCH-OFF POINTS OF THE PROPOSED SYSTEM AND THOSE ACHIEVED IN [2], [3].

<table>
<thead>
<tr>
<th>( N_t \times N_r ) channel</th>
<th>Proposed system Curves fit at ( E_b/N_0 )</th>
<th>Schemes in [2], [3] Curves fit at ( E_b/N_0 )</th>
<th>Capacity ( E_b/N_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 4</td>
<td>1.82dB</td>
<td>1.80dB</td>
<td>1.47dB</td>
</tr>
<tr>
<td>4 x 3</td>
<td>2.285dB</td>
<td>2.30dB</td>
<td>1.97dB</td>
</tr>
<tr>
<td>4 x 2</td>
<td>3.285dB</td>
<td>3.30dB</td>
<td>2.95dB</td>
</tr>
</tbody>
</table>

The proposed scheme can also perform very well over an asymmetric channel in which multiple antennas are only equipped at the base-station. Those results, however, are not presented here due to space limitation.

V. ILLUSTRATIVE RESULTS

This section provides numerical and simulation results to verify the analysis made in the previous sections. A random interleaver of length \( 1 \times 10^5 \), which is the same to those considered in [2], [3], is used. In the computation of the asymptotic bound for \( P_0 \) in (3) and (4), the first 20 Hamming distances in the distance spectrum of each component code are included.

![Fig. 5. BER performance of the proposed systems equipped with \( N_t = 4 \) transmit and \( N_r = 4 \), \( N_r = 3 \), and \( N_r = 2 \) receive antennas. The outer codes are rate-1/2 mixed codes of the standard rate-1/2 4-state cc and rate-1/2, 2-state cc with code doping ratios \( \alpha = 0.35 \), \( \alpha = 0.2 \), and \( \alpha = 0.07 \), respectively.](image)

Figure 5 shows the BER performances after 80 iterations of the concatenation scheme using \( N_t = 4 \) transmit antennas and \( N_r = 4 \), \( N_r = 3 \), and \( N_r = 2 \) receive antennas. The outer codes are rate-1/2 mixed codes of the standard rate-1/2 4-state convolutional code and rate-1/2, 2-state convolutional code with code doping ratios \( \alpha = 0.35 \), \( \alpha = 0.2 \), and \( \alpha = 0.07 \), respectively. It can be seen from Fig. 5 that the analytical results obtained by EXIT charts agree with the BER curves. In particular, the turbo pinch-off region happens around \( E_b/N_0 = 2.0 \text{dB} \), \( E_b/N_0 = 2.48 \text{dB} \), and \( E_b/N_0 = 3.50 \text{dB} \), and one achieves a BER around \( 10^{-2} \) with 80 iterations at \( E_b/N_0 = 2.05 \text{dB} \), \( E_b/N_0 = 2.52 \text{dB} \), and \( E_b/N_0 = 3.56 \text{dB} \) over 4 x 4, 4 x 3, and 4 x 2 channels, respectively. Compared to the similar BER obtained by employing the standard UMTS parallel concatenated turbo code constructed from memory-three constituent codes, with feedback and feedforward generator polynomials \([1, 0, 1, 1]\) and \([1, 1, 0, 1]\) at \( E_b/N_0 = 2.5 \text{dB} \) over 4 x 4 channel, at \( E_b/N_0 = 3.2 \text{dB} \) over 4 x 3 channel, and at \( E_b/N_0 = 5.4 \text{dB} \) over 4 x 2 as reported in [2], it can be seen that the simple concatenation system outperforms the turbo-coded scheme, especially in the asymmetric scenarios. The proposed scheme also has a lower receiver complexity, which can be roughly explained by the fact that in each iteration, SISO decoder needs to perform two times in the turbo-code system. The tightness of the asymptotic bound in the error floor area is also clearly observed for all systems under consideration. This makes it effective in predicting the error performance at reasonable low BER regions.

VI. CONCLUSIONS

This paper introduced an attractive coded modulation scheme over multiple-antenna channels with QPSK using a concatenation of a simple outer mixed code of short memory length convolutional codes and a short rate-1 linear block code followed by either 1-D anti-Gray or Gray mapping. In the error-floor area, the error bound and design criterion were first derived, which can be used to accurately predict the error performance. Optimal rate-1 block codes were then developed for both anti-Gray and Gray mappings to achieve the best asymptotic performance. In the turbo pinch-off region, it has been shown through EXIT chart analysis that the proposed system achieves comparable performances to those using well-designed LDPC and RA codes. As a result, it approaches a close-capacity performance over both symmetric and asymmetric multiple-antenna channels, and thereby, outperforms a scheme employing a standard parallel concatenated turbo code at lower decoding complexity.

REFERENCES