Distributed Resource Allocation for Cognitive Radio Ad-hoc Networks with Spectrum-Sharing Constraints

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Abstract—In cognitive radio settings with highly dynamic primary activities and with small opportunities for secondary access, the requirement to fairly distribute the temporarily available spectral ranges among the unlicensed users turns out to be of particular relevance. The current paper addresses this issue by presenting a new design formulation that aims to optimize the performance of an orthogonal-frequency-division-multiple-access (OFDM) ad-hoc cognitive radio network, by means of joint subcarrier assignment and power allocation. Besides important constraint on the tolerable interference induced to primary network, to efficiently implement spectrum-sharing fairness, the optimization problem considered here strictly enforces upper and lower bounds on the total amount of temporarily available bandwidth to be granted to individual secondary users. Specifically, the system throughput is maximized via the application of Lagrangian duality theory. More importantly, the dual decomposition framework also gives rise to the realization of distributed solution. As the proposed distributed protocol requires very limited cooperation among the participating network elements, it is especially applicable for the ad-hoc networking environment under investigation, to which any central processing or control is certainly inaccessible. While the computational complexity of the devised algorithm is affordable, its performance in practical scenarios also attains the actual global optimum. The potential of the proposed approach is verified through asymptotic complexity analysis and via numerical examples.

I. INTRODUCTION

To better utilize the radio spectrum, cognitive radio [1], [2] has been identified as an efficient technology to exploit the existence of the spectrum portions unoccupied by the primary (or licensed) users. While the primary users (PUs) still have priority access to the spectrum, secondary (or unlicensed or cognitive) users are permitted to have restricted access, subject to a constrained degradation on the PUs’ performance [3]. In spectrum sharing environments, the key design challenges of a cognitive radio network are therefore to guarantee a protection of the PUs from excessive interference induced by the secondary users (SUs) as well as to meet some Quality-of-Service (QoS) requirements for the latter.

On the other hand, spectrum pooling is an opportunistic spectrum access approach that enables public access to the already licensed frequency bands [4]. The basic idea is to merge spectral ranges from different spectrum owners into a common pool, from which the SUs may temporarily rent spectral resources during idle periods of the PUs. In effect, the licensed system does not need to be changed while the SUs access unused resources. Orthogonal frequency division multiplexing (OFDM) has been recognized as a highly promising candidate for unlicensed users’ transmission in spectrum-pooling radio systems, due to its great flexibility in dynamically allocating the unused spectrum among the SUs as well as its ability to monitor the spectral activities of the PUs at no extra cost.

In [5], the authors present a solution to an energy-efficient resource allocation problem that maximizes the cognitive radio link capacity, taking into account the availability of the OFDM subcarriers and the limits on total interference generated to the PUs. Based on a risk-return model, a convex optimization problem is formulated, which incorporates a linear average rate loss function in the objective function to include the effect of subcarrier availability. Considering networks with the coexistence of multiple primary and secondary links through OFDMA-based air-interface, reference [6] utilizes the dual framework from [7] to provide centralized and distributed algorithms that improve the total sum rate of secondary networks subject to interference constraints specified at PUs’ receivers. The work in [8] studies the optimization of an ad hoc cognitive radio network coexisting with multicell primary radio networks. To jointly optimize the throughput of the ad hoc SU links, Lagrange optimization is utilized to design fast-convergent sum rate maximization schemes constrained on the power spectral mask, the transmit power of SUs, the maximum-subchannel-rate, and the minimum-rate per SU link.

Different from the existing approaches, we formulate in this paper a new design problem for OFDMA-based secondary ad-hoc networks to enhance the system throughput. In cognitive radio settings where the primary activities on the radio spectrum are highly dynamic and chances for secondary access are slim, the problem of fairly sharing the temporarily available frequency bands among the SUs becomes even more relevant than merely maximizing the system performance. Hence, in addition to the constraints on the tolerable interference limits induced by secondary network to the PUs and the maximum total transmit power at individual SUs, our formulation also incorporates the upper and lower bounds on the number of OFDM subchannels that unlicensed users are allowed to utilize. While the upper limits prevent cognitive users with favorable conditions from greedily filling all the spectrum holes, the lower thresholds provide certain guarantee of fairness in terms of bandwidth sharing to other SUs.

Moreover, the dual optimization approach proposed in this paper allows to provide practically global optimal solution with affordable complexity, as opposed to highly demanding computational burden typically required by direct methods. Further, the dual design framework gives rise to the realization of distributed algorithm, which is certainly desirable for ad-hoc networks without any central coordination at all. In implementing the distributed scheme, we also introduce the concept
of virtual timers at the SUs while requiring a limited level of cooperation from primary network.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a primary base station (BS) that transmits $N$ downlink traffic flows (not necessarily OFDM) to its $N$ subscribed PUs. Assume that each of those data streams is intended for only one PU and occupies a predetermined frequency band $B_P^{(n)}$ ($n = 1, \ldots, N$) in the radio spectrum. As the primary network does not utilize the entire available spectral ranges, a secondary ad-hoc network is also deployed to implement efficient opportunistic spectrum access. This secondary network, which consists of $G$ transmitter (Tx)-receiver (Rx) pairs, is assumed to be capable of accurately sensing the spectrum to locate the frequency bands temporarily unused by the primary network. Then, these spectral holes are merged into a common pool according to the spectrum pooling approach, from which the total bandwidth $B$ available for secondary access is divided into $K$ OFDM subchannels of equal bandwidth $B_s = B/K$. Denote by $N = \{1, \ldots, N\}$, $G = \{1, \ldots, G\}$ and $K = \{1, \ldots, K\}$ the sets of PUs, SU Tx-Rx pairs and OFDM subcarriers, respectively. Also, let $|K_g|$ represent the set of OFDM subchannels allocated to SU $g$ and $|K_{g,s}|$ its cardinality. The system setup is depicted in Fig. 1.

Although primary and secondary networks do not share the same frequency bands, their coexistence can actually lead to the mutual interference between the two due to the non-orthogonality of respective transmitted signals. Let $\Phi_{PU}^{(n)}(e^{jw})$ denote the power spectral density (PSD) of the signal transmitted from primary BS to its serviced user $n$ on frequency $B_P^{(n)}$.

The interference from this signal to subchannel $k$ is [9]

$$J(n, k) = \int_{d_k^{(n)} - B_s/2}^{d_k^{(n)} + B_s/2} \mathcal{E}\{I_K(w)\} \, dw,$$

where $d_k^{(n)} = |f_k - f_n|$ represents the spectral distance between subcarrier $k$ and the center frequency $f_n$, and $\mathcal{E}\{I_K(w)\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{PU}^{(n)}(e^{jw}) \left( \frac{\sin(w - w_k)/2}{\sin(w/2)} \right)^2 \, dw$ is the PSD of PU $n$’s signal after $K$-Fast-Fourier-Transform (FFT) processing. Also, let $h_{PS}^{SU}(g, k)$ indicate the channel from primary BS to SU-Rx $g$ on subcarrier $k$. Then, the total interference introduced from primary network to SU-Rx $g$ on $k$ can be expressed as

$$\tilde{J}(g, k) = |h_{PS}^{SU}(g, k)|^2 \sum_{n \in N} J(n, k).$$

On the other hand, the OFDM signals from a SU-Tx to its intended SU-Rx might interfere the reception at the PU receivers. Upon denoting $T_s$ the OFDM symbol duration and defining $P_{g,k}$ the power spent for transmission from SU-Tx $g$ to SU-Rx $g$ on subcarrier $k$, the PSD of this subcarrier-$k$ signal can be modeled as $\Phi_{PS}^{SU}(f) = P_{g,k} T_s \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2$. The interference caused by this signal onto PU $n$ is then [9]

$$I_{g,k}^{(n)} = P_{g,k} \tilde{J}_{g,k}^{(n)},$$

where $I_{g,k}^{(n)} = |h_{PS}^{SU}(g, k)| T_s \int_{d_k^{(n)} - B_s/2}^{d_k^{(n)} + B_s/2} \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \, df$ and $h_{PS}^{SU}(g, k)$ is the channel from SU-Tx $g$ to PU $n$ on $k$.

In this work, we assume a slow fading channel model such that the channel conditions remain unchanged during the resource allocation period (e.g., in high data rate systems and/or environments with reduced degrees of mobility). With this assumption, the channel-to-interference-plus-noise ratio (CINR) of SU $g$ on subcarrier $k$ can be shown to be

$$\gamma_{g,k} = \frac{|h_{SS}^{SU}(g, k)|^2 / \left[ \Gamma(N_0 B_s + \tilde{J}(g, k)) \right]}{\left[ \Gamma(N_0 B_s + \tilde{J}(g, k)) \right]}.$$

where $h_{SS}^{SU}(g, k)$ is the corresponding channel coefficient, $N_0$ the one-sided PSD of additive white Gaussian noise (AWGN), and $\Gamma$ the signal-to-noise ratio (SNR) gap to the capacity limit. Then, the maximum attainable rate of SU $g$ on subcarrier $k$ is indeed

$$R_{g,k} = \log_2 (1 + \gamma_{g,k} P_{g,k}).$$

The goal of this paper is to devise joint subcarrier assignment and power allocation scheme that maximizes the aggregate throughput of all secondary transmissions, while satisfying important constraints on the maximum tolerable interference at each PU and on the total transmit powers of individual SU-Tx’s. Moreover, it should be emphasized that in spectrum sharing environments where the PUs always have priority access to the spectrum, the chances for cognitive radio users to utilize the frequencies depend heavily on the dynamics of these licensed users. In many cases where the PUs are extremely active and occupy a wide range of frequency bands for a long period of time, the chances left for secondary access become slim. Therefore, it is imperative to share out these valuable but yet scarce opportunities among the secondary users in a fair and efficient manner. To this end, we propose to constrain the maximum and minimum numbers of OFDM subchannels that
individual SU Tx-Rx pairs are permitted to use. The upper limits prevent the more favorable SUs from greedily taking up all the temporarily available spectrum, whereas the lower limits guarantee other SUs with certain levels of fairness in terms of spectrum access. Specifically, the design problem of interest can be formulated as follows:

$$\max_{\{P_{g,k}\}} \sum_{g \in G} w_g \sum_{k \in K_g} \log_2(1 + \gamma_{g,k} P_{g,k})$$ \hspace{1cm} \text{(6a)}$$

subject to

$$\sum_{g \in G} \sum_{k \in K_g} P_{g,k} I_{g,k}^{(n)} \leq I_{th}^{(n)}, \forall n \in N$$ \hspace{1cm} \text{(6b)}$$

$$\sum_{k \in K_g} P_{g,k} \leq P_{g}^{\max}, \forall g \in G$$ \hspace{1cm} \text{(6c)}$$

$$P_{g,k} \geq 0, \forall g \in G, \forall k \in K_g$$ \hspace{1cm} \text{(6d)}$$

$$P_{g,k} \cdot P_{g',k} = 0, \forall k \in K, \forall g' \neq g \in G$$ \hspace{1cm} \text{(6e)}$$

$$K_{g}^{\min} \leq |K_g| \leq K_{g}^{\max}, \forall g \in G.$$ \hspace{1cm} \text{(6f)}$$

In this formulation, the weight $0 \leq w_g \leq 1$ reflects the priority given to SU $g$ where $\sum_{g \in G} w_g = 1$. With $I_{th}^{(n)}$ denoting the interference threshold, (6b) expresses the maximum allowable interference at PU $n$. While the regulatory limit on the total transmit power at the SU-Tx $g$ is represented in (6c), (6d)-(6e) enforce a disjoint subchannel assignment in OFDMA systems, i.e., one subcarrier is permitted to be assigned to at most one SU at a time. Finally, the fairness design is reflected in (6f) where the total number of subcarriers allotted to any SU $g$ is upper and lower bounded by $K_{g}^{\max}$ and $K_{g}^{\min}$, respectively. These values are strictly required to satisfy both $\sum_{g \in G} K_{g}^{\min} \leq K$ and $K_{g}^{\max} \leq K, \forall g \in G$.

III. JOINT SUBCARRIER AND POWER ALLOCATION FOR COGNITIVE RADIO NETWORKS VIA DUALITY

It is noteworthy that the optimization problem (6) is NP-hard since it requires the allocation of an optimal set of subcarriers to each SU. The computational complexity needed to directly resolve this combinatorial problem increases, at least, exponentially with the number of subcarriers $K$. In this section, we will develop optimal algorithm to efficiently resolve this challenging problem where, instead, the solution is derived in the dual domain. One of the main motivations behind this approach is that the particular structure of (6) satisfies the so-called “frequency-sharing condition” introduced in [7], which implies that the dual-domain optimal subcarrier-power allocation will become that of the primal problem (6) for a sufficiently large number of subcarriers.

A. Optimal Design with Spectrum-Sharing Constraints

The exclusive channel assignment constraint (6d)-(6e) can be expressed as $P_{g,k} \in S_1 = \{P_{g,k} \geq 0; \forall g \in G, \forall k \in K_g \mid P_{g,k} \cdot P_{g',k} = 0, \forall g' \neq g \}$. Let us define $\rho_{g,k}$ such that $\rho_{g,k}$ is equal to 1 if $P_{g,k} > 0$ and 0 otherwise. Then, the bounds on total number of subcarriers assigned each SU [see (6f)] can be rewritten as $P_{g,k} \in S_2 = \{P_{g,k}; \forall g \in G, \forall k \in K_g \mid \rho_{g,k} \leq K_{g}^{\max} \}$. The main steps of this algorithm are outlined in Table I. Notice that $\{P_{g,k}\}$ is also required to satisfy the spectrum-sharing constraints (6f). Hence, for a particular subcarrier $k \in K$ the task of optimally determining which SU to use $k$ cannot be done by simply selecting among the total $G$ power allocations from (12) the one that maximizes $D_k^{(s)}(\lambda, \mu)$. Instead, this involves searching through all $GK$ values of $D_k^{(s)}(\lambda, \mu)$ to decide the optimal subcarrier-SU matchings and subsequently the optimal power distributions for those assignments. To this end, we propose a 2-stage algorithm that designates the $P_{g,k}^{*}$'s to their eligible SUs in an optimal fashion while also satisfies (6f). The main steps of this algorithm are outlined in Table I. Specifically, Stage 1 attempts to provide minimum guarantee on the number of subcarriers allotted to all SUs. Then, in Stage 2 the remaining subcarriers are allocated on a competitive basis among the SUs to further enhance the system throughput. In any case, as soon as a certain SU has reached its maximum allowable shares of spectrum, it will be eliminated from the
TABLE I: Optimal joint allocation for a fixed value of \( \{ \lambda, \mu \} \)

<table>
<thead>
<tr>
<th>% INITIALIZATION</th>
<th>- Given ( \lambda, \mu ), compute all ( GK ) power allocation as in (11).</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>- Construct ( R := D_{g, k}^{(0)}(\lambda, \mu) \in \mathbb{R}^{G \times K} ) as in (12).</td>
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% PHASE 1 – Allocation to meet spectrum-sharing constraints

- If \( K_{g_{\min}} = 0 \), discard SU \( g \) from further consideration in Phase 1.
- Repeat
  - Perform a 2-D search on \( R \) to find \( \{ g^*, k^* \} = \arg \max_{g,k} R \).
  - Assign \( P_{g, k^*} := P_{g, k^*}^* \), and \( P_{g, k} := 0, \forall g \neq g^* \in \mathcal{G} \).
  - Discard subcarrier \( k^* \) from all subsequent searches.
  - If \( |K_{g^*}| = K_{g_{\max}} \), discard SU \( g^* \) from all subsequent searches.
  - If \( |K_{g^*}| = K_{g_{\min}} \), discard SU \( g^* \) in the rest of Phase 1.
- Until \( |K_g| \geq K_{g_{\min}}, \forall g \in \mathcal{G} \).

% PHASE 2 – Allocation to further enhance system throughput

- Repeat
  - Perform a 2-D search on \( R \) to find \( \{ g^*, k^* \} = \arg \max_{g,k} R \).
  - Assign \( P_{g, k^*} := P_{g, k^*}^* \), and \( P_{g, k} := 0, \forall g \neq g^* \in \mathcal{G} \).
  - Discard subcarrier \( k^* \) from all subsequent searches.
  - If \( |K_{g^*}| = K_{g_{\max}} \), discard SU \( g^* \) from all subsequent searches.
- Until all subcarriers \( k \in K \) have been assigned.

The keys to realize a distributed solution for (6) lie in the search for optimal \( P_{g, k}^* \) to solve (10) while also satisfying (6f). First, observe that the computation of \( P_{g, k}^* \) in (11) mainly requires the local information available at SU-Tx’s \( g \) itself, except for \( \lambda_n \) and \( \tilde{I}^{(n)}(n) \). On one hand, since both \( h_{g, k}^{26} \) and \( \tilde{I}(g, k) \) can be estimated/measured at the SU-Rx \( g \), the CINR \( \gamma_{g, k} \) can be computed and made available to its corresponding transmitter via dedicated feedback channels. On the other hand, in order to evaluate \( \tilde{I}^{(n)}(n, g) \), the SU-Tx \( g \) demands certain collaboration from the PU \( n \). For this, after performing the Lagrangian update on \( \lambda_n \) and estimating the channel gain \( h_{g, k}^{26} \), each PU broadcasts these values to the SUs. Upon receiving these values from all \( N \) PUs, SU-Tx \( g \) will be able to determine \( P_{g, k}^* \) and \( D_k(g)(\lambda, \mu), \forall k \in K, \forall g \in \mathcal{G} \) by (12).

The above-proposed procedure in Table I, which aims to fulfill the constraints on minimum/maximum subchannel sharing, can be implemented in a distributed fashion as follows. At the beginning of the allocation period, individual SU-Tx’s broadcast “MIN-SUB-REQ” flag packets which specify their required minimum number of subcarriers \( K_{g_{\min}} \). Upon receiving all these packets and by summing the indicated values together, SU-Tx’s are able to know the exact total minimum of \( K_{g_{\min}} \) subcarriers being requested. Then, based on the computed \( P_{g, k}^* \) and \( D_k(g)(\lambda, \mu) \), each SU-Tx constructs a length-\( K \) list of virtual timers \( T(g) \), whose \( k \)-entry is

\[
T(g)(k) = c \cdot \exp \left[ -D_k(g)(\lambda, \mu) \right], \forall g \in \mathcal{G}, \quad (16)
\]

with constant \( c > 0 \) made common to all the SUs. Notice that our definition of virtual timer in (16) is able deal with any real-valued \( D_k(g)(\lambda, \mu) \). This is rather different from the “virtual clock” values defined in [6] which are only valid for strictly positive \( D_k(g)(\lambda, \mu) \).

As there are \( K \) OFDM subcarriers available for secondary access, we divide the total allocation time into \( M \) minislots and let the SUs compete in a sequential manner. At the beginning of each minislot \( i \), all SU-Tx’s whose \( K_{g_{\min}} > 0 \) pick the largest value from their own list \( T^{(i)} \) and start the virtual timer corresponding to that value. Assume that SU-Tx \( g \) has its largest value \( D_k^{(i)}(\lambda, \mu) \) being the maximum over all SUs’ virtual timer lists. Therefore, this SU’s timer will expire first, and consequently be allowed to transmit with power \( P_{g, k}^{(i)} \) over subcarrier \( k^{(i)} \). It then broadcasts an “EXPRIE” flag packet to indicate that subcarrier \( k^{(i)} \) has already been occupied. Since
TABLE II: D-TMSC algorithm

<table>
<thead>
<tr>
<th>AT SECONDARY USERS</th>
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<tbody>
<tr>
<td><strong>% PHASE 1 – SU Initialization</strong></td>
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</tr>
<tr>
<td>- Each SU-Tx $g$ broadcasts a “MIN-SUB-REQ” flag packet which indicates its required minimum number of subcarriers $K_{g}^{\text{min}}$. Upon receiving all these “MIN-SUB-REQ” packets, SU-Tx’s are able to determine there are totally $K_{g}^{\text{tot}}$ subcarriers being requested.</td>
<td></td>
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<tr>
<td>- Each SU-Tx $g \in G$ computes all $P_{k,g}^{*}$ by (11) and constructs a list of virtual timers $T_{(g)}^{(i)}$ by (12) and (16).</td>
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</tr>
<tr>
<td>- Each SU-Tx $g$ initializes $\mu_{g}$ and sets $t := 0$ and $i := 1$.</td>
<td></td>
</tr>
<tr>
<td><strong>% PHASE 2 – Subcarrier-power allocation for fair spectrum sharing</strong></td>
<td></td>
</tr>
<tr>
<td>- SU-Tx whose $K_{g}^{\text{min}} = 0$ remains silent until it has received a total of $K_{g}^{\text{tot}}$ “EXPIRE” flag packets.</td>
<td></td>
</tr>
<tr>
<td>- Other SU-Tx’s start their respective largest virtual timer for this slot $i$.</td>
<td></td>
</tr>
<tr>
<td>- SU-Tx $g$ expires first (denoted as $g^{*}$) is eligible to use subchannel $k^{(i)}$.</td>
<td></td>
</tr>
<tr>
<td>- If SU-Tx $g^{<em>}$ recognizes that $[K_{g^{</em>}}^{\text{r}}] = K_{g^{<em>}}^{\text{max}}$, it will compete until receiving $K_{g^{</em>}}^{\text{tot}}$ “EXPIRE” flag packets (including those generated by itself).</td>
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</tr>
<tr>
<td>- Otherwise, SU-Tx $g^{*}$ deletes the entry corresponding to $k^{(i)}$ subcarrier from its virtual timer list, and moves to the next competing time slot (i.e., $i := i + 1$).</td>
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</tr>
<tr>
<td><strong>% PHASE 3 – Subcarrier-power allocation to enhance throughput</strong></td>
<td></td>
</tr>
<tr>
<td>- All SU-Tx’s start their respective largest virtual timers for this minislot $i$.</td>
<td></td>
</tr>
<tr>
<td>- SU-Tx $g$ starts transmitting with power $P_{k,g}^{*}$ on subcarrier $k^{(i)}$.</td>
<td></td>
</tr>
<tr>
<td>- Upon receiving this “EXPIRE” packet, other SU-Tx’s stop their virtual timers, back off, delete the entry corresponding to $k^{(i)}$ from their virtual timer lists, and move to the next competing time slot (i.e., $i := i + 1$).</td>
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<tr>
<td>- SU-Tx $g^{<em>}$ will be eligible to occupy this subchannel $k^{(i)}$, when it has received $K_{g^{</em>}}^{\text{tot}}$ “EXPIRE” flag packets (including those generated by itself).</td>
<td></td>
</tr>
<tr>
<td>- Otherwise, SU-Tx $g^{*}$ deletes the entry corresponding to $k^{(i)}$ subcarrier from its virtual timer list, and moves to the next competing time slot (i.e., $i := i + 1$).</td>
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<tr>
<td><strong>% PHASE 4 – SU Lagrangian updates</strong></td>
<td></td>
</tr>
<tr>
<td>- Upon receiving all $K^{\text{tot}}$ “EXPIRE” packets, each SU-Tx $g \in G$ updates its $\mu_{g}$ based on (15), resets its counter of the “EXPIRE” messages, sets $t := t + 1$, and resets $i := 1$.</td>
<td></td>
</tr>
<tr>
<td>- Return to Phase 2 and repeat until convergence.</td>
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</tr>
</tbody>
</table>

AT PRIMARY USERS

| % PHASE 1 – PU Initialization |  |
| - Each PU $n \in N$ initializes and broadcasts $\lambda_{n}$. |  |
| **% PHASE 2 – PU Lagrangian updates** |  |
| - Upon receiving all $K^{\text{tot}}$ “EXPIRE” packets, each PU $n \in N$ updates its $\lambda_{n}$ based on (14), broadcasts these values to all SUs, resets its counter of the “EXPIRE” messages, sets $t := t + 1$. |  |
| - Return to Phase 2 and repeat until convergence. |  |

Fig. 2: Distribution of spectrum in the numerical examples.

messages are to be passed among the SUs: “MIN-SUB-REQ” and “EXPIRE.”

- **Between cognitive radio and primary networks:** (i) The PUs listen to “EXPIRE” packets from all SUs to determine when to update their $\lambda$, and (ii) The PUs broadcast the computed $\lambda$ and the estimated $h_{g,k}^{\text{SP}}$ to all SUs.

IV. PERFORMANCE EVALUATION

A. Complexity Analysis

Assume that searching through an unsorted 1-D list of dimension $M$ requires a complexity of $O(M^2)$. For a fixed $\{\lambda, \mu\}$, the centralized design to maximize system throughput in (6), which involves a 2-D search over a $G \times K$ matrix, demands $O(G^2 K^2)$ operations to solve (9). Further, the subgradient method to update $\{\lambda, \mu\}$ converges in $\Delta$ iterations, which is typically small for appropriate choices of step size. As there are $K$ subcarriers to be assigned and the PUs only need to update and broadcast their Lagrangian variables, the total complexity of the centralized scheme can be shown to be $O((G^2 K^3 + N) \Delta + \chi_{c})$, where $\chi_{c}$ expresses the communicating overheads necessary to obtain all the global information about the two networks under investigation. For the purpose of efficient processing, individual SUs in the distributed scheme D-TMSC may sort their list of virtual timers in decreasing order. This calls for $O(K \log K)$ operations. During every one of the $K$ competing minislots, each SU searches through its own list to find and remove the subcarrier that has already been used, implying a $O(K)$ complexity. Totally, the D-TMSC algorithm entails an asymptotic complexity of $O((K \log K + K^2) \Delta + \chi_{SU}) = O(K^2 \Delta + \chi_{SU})$ at individual SU-Tx’s, with $\chi_{SU}$ representing the number of message passings and updates.

B. Numerical Examples

Consider a communication scenario in which a primary BS transmits downlink data to its $N = 2$ subscribed users over the predetermined frequencies in the available spectrum. All the PU signals are assumed to be elliptically filtered white noise with equal amplitude $P_{PU} = 1$. The frequency bands left unused by primary network are filled with $K = 24$ OFDM subchannels, as depicted in Fig. 2, over which $G = 3$ cognitive radio TX-Rx pairs are allowed to communicate to exploit opportunistic spectrum access.

We perform our numerical examples in MATLAB environment (version 7.8.0) on a PC equipped with 2.67-GHz Intel(R) Core(TM) i5 CPU, 64-bit operating system and 8-GB RAM. In each simulation run, 100 sets of independent channel gains $\{h_{g,k}^{\text{SS}}\}, \{h_{g,k}^{\text{PS}}(g,k)\}$ and $\{h_{g,k}^{\text{SP}}(g,k)\}$ are randomly generated according to the Rayleigh distribution. The average channel gains, $N_0$, $T_s$ and $B_s$ are all normalized to 1. We further assume perfect coding, i.e., $\Gamma = 1$. Since all the spectral
In this paper, we have proposed a distributed algorithm to optimally allocate subcarriers and power in a cognitive radio ad-hoc network employing OFDMA technology. The novel scheme also takes into account the issue of fair utilization of the spectral holes, by placing lower and upper limits on the number of subchannels that individual SUs may occupy. This design requirement is particularly relevant in network settings where primary spectral activities are highly dynamic, leaving tiny opportunities for secondary access. Specifically, the proposed solution which is derived from Lagrangian dual optimization is able to offer system throughput maximization, subject to tolerable interference introduced to the primary network. The distributed implementation devised here is especially applicable for ad-hoc networks that have no central coordination of any kind. Simulation results and asymptotic complexity analysis have confirmed the advantages of our proposed approach.

\section*{REFERENCES}


\begin{table}[h]
\centering
\caption{Average number of subchannels assigned to SUs}
\begin{tabular}{|c|c|c|c|}
\hline
SU & Actual allocation (fixed $P_{\text{max}}$) & Conservative & D-TMSC \\
\hline
SU 1 & 6.6517 & 10.7417 & 6.6067 \\
\hline
SU 2 & 10.5750 & 8.0833 & 5.3417 \\
\hline
SU 3 & 6.7267 & 10.6150 & 6.6583 \\
\hline
\end{tabular}
\end{table}