Foreground Detection Based on Low-rank and Block-sparse Matrix Decomposition

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Summary

1. Introduction and motivations
2. Adapted Norm
3. RPCA-LBD and ALM
4. Results
5. Conclusion
# Introduction and motivations

## Purpose

- **Foreground detection**: Segmentation of moving objects in video sequence acquired by a fixed camera.
- **Background modeling**: Modelized all that is not moving object.

## Involved applications

- Surveillance camera
- Motion capture

## On the importance

- **Crucial Task**: Often the first step of a full video surveillance system.

## Strategy used

- Eigenbackground decomposition.
Find an « ideal » subspace of the video sequence, which describes the best as possible the (dynamic) background.

**Fig. 1** The common process of background subtraction via PCA (Principal Component Analysis). At final step, an adaptative threshold is used to get a binary image.

Without a robust framework, the moving object may be absorbed in the model!
First, we consider a video sequence as a matrix $A \in \mathbb{R}^{n \times m}$

- $n$ is the amount of pixels in a frame ($\sim 10^6$)
- $m$ is the number of frames considered ($\sim 200$)
RPCA formulation

- **PCA** with fixed rank is:
  \[
  \min_{L,S} \|S\|_F \\
  \text{s.t.} \quad \text{Rank}(L) = k \\
  A = L + S
  \] (1)

- **R**(obust)**PCA** is *(Non convex and NP-hard)*:
  \[
  \min_{L,S} \|\sigma(L)\|_0 + \lambda\|S\|_0 \\
  \text{s.t.} \quad A = L + S
  \] (2)

- Convex relaxed problem of (2) is **RPCA-PCP** proposed by Candès et al. [1]:
  \[
  \min_{L,S} \|\sigma(L)\|_1 + \lambda\|S\|_1 \\
  \text{s.t.} \quad A = L + S
  \] (3)

  *Where* \(\sigma(L)\) *means singular values of* \(L\).*

- A mix is **Stable PCP** of Zhou et al. [2] (both entry-wise and sparse noise):
  \[
  \min_{L,S} \|\sigma(L)\|_1 + \lambda\|S\|_1 \\
  \text{s.t.} \quad \|A - L - S\|_F < \delta
  \] (4)
In RPCA, residual error is sparse. Using the previous decomposition on a low-rank random matrix plus noise, the error looks like:

\[ \min \|S\|_2 = L - \text{Low-rank} + S - \text{Sparse} \]

\[ \min \|S\|_1 = L + S \]

But for our application, we want:

\[ \min \|S\|_2 = L + \text{Column outlier} \]

Same principle with video. Sparse noise (or outliers) are the moving objects.
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Let’s play with norms...

- For this application, the statistical distribution of outliers **differs** between rows and columns. An **adapted norm** surely provides a better estimate of $S$.

- **Notations**: We introduce a dissymmetric norm for matrices.

\[
||A||_{\alpha,\beta}, \text{ which means } \left( \sum_j \left( \sum_i A_{ij}^\alpha \right)^{\beta/\alpha} \right)^{1/\beta}
\]

- Note that: $||A||_{\alpha,\beta} \neq ||A||_{\beta,\alpha} \neq ||A^t||_{\alpha,\beta} \neq ||A^t||_{\beta,\alpha}$

- Quite confusing notation: Different of the common induced norm,

\[
||A||_{\alpha,\beta} = \max_x ||Ax||_\beta, \text{ s.t. } ||x||_\alpha = 1
\]
Let's play with norms...(2)

Varying the $\alpha, \beta$ norm $\rightarrow$ Different kind of recovering pattern error.

Suppose $A = L + S$, and $\text{Rank}(L) = 80\%$ of $\text{Rank}(A)$

Problem non-convex (hope the local minimum is the global min !).
Some issues

- What is the best appropriate norm for background modeling?
- How to find it?
- *Is our initial assumption of $||.||_{2,1}$ justified?*
If ideal eigenbackgrounds are that, best norm must be ...

Let’s denote $L_{opt}$, the ideal low-rank subspace which outliers do not contribute to PCA.
Experimental results

Let’s denote $L_{\alpha,\beta}$, the low-rank recovered matrix with a $\|\cdot\|_{\alpha,\beta}$-PCA. The plot shows the error between $\|L_{\text{opt}} - L_{\alpha,\beta}\|_F$ for parameters $\alpha$ and $\beta$ chosen freely. The darkest value means that the error is the smallest here.

- $\|S\|_{2,1}$ is not optimal, but for convenience we use it.
- The benefit of the ad hoc block-sparse hypothesis is confirmed by testing its efficiency directly on video dataset.

Experimentation done on dynamic category of dataset change detection workshop 2012: http://www.changedetection.net/
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We use the RPCA formulation provided by Tang et al. called LBD [3]

\[
\begin{align*}
\min_{L,S} & \quad \||\sigma(L)||_1 + \lambda \mu \|S\|_{2,1} + \lambda (1 - \mu) \|L\|_{2,1} \\
\text{s.t.} & \quad A - L - S = 0
\end{align*}
\]

(1)

The third term with balanced parameter $\mu$ enforces that the recovered matrix $L$ has zeros columns of outliers.

- Solve with Convex optimization method (Augmented Lagrangian Multipliers ALM)

- AL function is:

\[
\mathcal{L}(L, S, Y; \mu) = (1) + \sum_{ij} Y_{ij} \circ (2) + \frac{\mu}{2} \|(2)\|_F^2
\]

- Solve with alternating scheme:

\[
\begin{align*}
L^{(k+1)} & = \arg\min_L \mathcal{L}(L, S^{(k)}, Y^{(k)}; \mu^{(k)}) \\
S^{(k+1)} & = \arg\min_S \mathcal{L}(L^{(k+1)}, S, Y^{(k)}; \mu^{(k)})
\end{align*}
\]
Algorithm as an iterative SVD

- Each iteration needs a SVD, but usually do not requires a full SVD computation.
- PROPACK package is used for compute a partial SVD with only the first largest few singular values on large dataset. ([http://soi.stanford.edu/~rmunk/PROPACK/](http://soi.stanford.edu/~rmunk/PROPACK/)).
- For experimentation, we choose $\lambda = 1.1$ and $\mu = 0.6$
- In most cases, time consumption do not exceed 10 Full SVD’s.
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Experimental Protocol

- Optimal threshold is chosen for maximizing **F-measure** criterion which is based $2 \times 2$ histogram of True/false/positive/negative:

\[
D_R = \frac{T_P}{T_P + F_N}, \quad Prec = \frac{T_P}{T_P + F_P}, \quad F = \frac{2 \ D_R \ Prec}{D_R + Prec}
\]

- Good performance is then obtained when the F-measure is closed to 1
- Time consumption is not taken into account in the evaluation process.

- RPCA-LBD is compared with the following two Robust methods:
  - Robust Subspace Learning (RSL) De La Torre et al. [4]
  - Principal Component Pursuit (RPCA-PCP) [1]
Quantitative Results

From top to bottom: original image ground truth, PCA, RSL, RPCA-PCP, RPCA-LBD. From left to right: MO (985), TD (1850), LS (1865), WT (247), C (251).

From top to bottom: original image, ground truth, RSL, RPCA-PCP, RPCA-LBD. From left to right: Wallflower dataset: B (2832), FA (449). I2R dataset: shopping mall (1980), water surface (1594), curtain (22772).

Table 1. F-measure on the Wallflower dataset

<table>
<thead>
<tr>
<th></th>
<th>RSL</th>
<th>RPCA-PCP</th>
<th>RPCA-LBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD</td>
<td>75.73</td>
<td>81.18</td>
<td>71.43</td>
</tr>
<tr>
<td>LS</td>
<td>28.36</td>
<td>70.86</td>
<td>71.64</td>
</tr>
<tr>
<td>WT</td>
<td>89.69</td>
<td>86.40</td>
<td>86.16</td>
</tr>
<tr>
<td>C</td>
<td>91.78</td>
<td>75.43</td>
<td>96.78</td>
</tr>
<tr>
<td>B</td>
<td>69.38</td>
<td>74.4</td>
<td>74.9</td>
</tr>
<tr>
<td>FA</td>
<td>74.37</td>
<td>72.07</td>
<td>88.46</td>
</tr>
<tr>
<td>Average</td>
<td>71.55</td>
<td>76.73</td>
<td>79.90</td>
</tr>
</tbody>
</table>

Table 2. F-measure on the I2R dataset

<table>
<thead>
<tr>
<th></th>
<th>RSL</th>
<th>RPCA-PCP</th>
<th>RPCA-LBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shopping mall</td>
<td>58.64</td>
<td>76.07</td>
<td>77</td>
</tr>
<tr>
<td>Water surface</td>
<td>34.42</td>
<td>45</td>
<td>87.69</td>
</tr>
<tr>
<td>Curtain</td>
<td>91.04</td>
<td>90.96</td>
<td>91.09</td>
</tr>
</tbody>
</table>
Qualitative analysis

Advantages

- Experiments on video surveillance datasets show that this approach is more robust than RSL and RPCA-PCP in presence of dynamic backgrounds and illumination changes.
- Well suited for video with **spatially spread** and **temporarily sparse** outliers.

Disadvantages

- Not efficient on **Bootstrap sequences** (e.g. outliers always presents).
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Conclusion & Future Works

Conclusion

- We have presented a foreground detection method based on low-rank and block-sparse matrix decomposition.

Future Works

- Further research consists in developing an incremental RPCA-LBD to update the model at every frame and to achieve the real-time requirements.
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References


Advertising : Call for Paper

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