Space-Time Equalization for Asynchronous Multiuser Bit-Interleaved Coded OFDM

Taiwen Tang, Member, IEEE, and Teng Joon Lim, Senior Member, IEEE

Abstract—For a multiuser bit interleaved coded OFDM (BIC-OFDM) system, we study the methods of time domain equalization and per-subcarrier maximum likelihood (ML) frequency domain equalization when there is timing asynchronism among the users. For the time domain equalizer structure, a coded pairwise error probability (PEP) approximation is derived, which leads to a training-based space-time design method that takes the error rate performance into consideration. We compare the time domain equalization and per-subcarrier ML frequency domain equalization methods, and conclude that at high SNR's, the time domain equalization approach is preferable.

I. INTRODUCTION

Multiple antenna technology is being combined with orthogonal frequency division multiplexing (OFDM) in next generation wireless systems, because of the flexibility such a combination offers in creating various modes of operation offering a range of spectral efficiencies and reliabilities. We study a MIMO-OFDM uplink, in which multiple single-antenna transmitters send OFDM signals to a receiver with multiple receive antennas. The system is asynchronous, meaning that the received signals from the transmitters are not aligned in time at the receiver due to different propagation delays.

In conventional cellular systems, such asynchronism may be overcome by adjusting the transmission times at the transmitters. This requires a feedback timing adjustment mechanism from the receiver to the transmitters. For communications without such a feedback mechanism, we need to study suitable transceiver architectures. Also for applications where there are asynchronous interferers, we have to consider such asynchronism between the transmitters (including interferers) and the receivers.

Due to the asynchronism among the transmitters, it may not be possible to find an FFT time window that is free of inter-OFDM symbol interference (IOSI) and inter-carrier interference (ICI) for the transmitted signals. There are mainly two equalization methods to deal with such ISOI and ICI. The first method is frequency domain equalization. It has been shown in [1] that the per-subcarrier receive beamforming that only utilizes the signal and channel response on each subcarrier cannot completely cancel the ISOI and ICI caused by the asynchronism unless the number of antennas is greater than the number of time taps in the channel, which is not feasible in real systems. Another alternative is to place a multiple-antenna finite impulse response (FIR) equalizer at each subcarrier to deal with the ISOI and ICI [2] [3]. The drawbacks of this approach are that the design complexity is high since we need to design each multi-antenna FIR equalizer individually, and it requires a large memory to store the equalizer coefficients [3].

The method of time domain equalization (TDE) was introduced in [4]–[9]. The TDE approach uses one space-time equalizer per user before the FFT to (i) minimize multi-user interference, in case each subcarrier is occupied by more than one user in a spatial division multi-access (SDMA) manner, and (ii) shorten the channel response from the desired user to within the cyclic prefix. The equalization delay is adjusted optimally in the mean square error sense for each user. Then after the FFT, in the frequency domain, a per-subcarrier scalar equalization is applied. The design metrics for the space-time equalizer are mainly the signal-to-interference and noise power ratio or the mean square error between the equalized signal and the transmitted signal convolved with the desired channel response in these papers. The coded PEP has not been considered as the equalizer design metric.

Our main contribution in this paper is to derive a coded PEP approximation for the multiuser asynchronous OFDM uplink with bit interleaved coded modulation (BICM). We then minimize this approximate PEP expression to design the space-time equalizers. Our approach is different from the minimum bit-error rate design for space time equalization-based multiuser detection in [10]. The latter assumes a Gaussian mixture model after space time equalization to derive the uncoded error probability expression, which does not take error control coding into consideration. Also it may not be practical to OFDM systems due to the optimization complexity scales exponentially with the number of OFDM subcarriers.

We also derive a frequency-domain joint ML based equalization method that uses only the signal and the channel response on each subcarrier, and compare it with the TDE designs based on MMSE and PEP minimization. We find that an error floor caused by the ISOI and ICI due to asynchronism exists for the per-subcarrier ML receiver, and thus the space-time equalization method should be favored in the high SNR region.

The standard matrix notations are used in this paper, where $(\cdot)^T$ denotes transpose and $(\cdot)^H$ denotes conjugate transpose.

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II. SYSTEM MODEL

We consider an OFDM system where the receiver has $M_r$ receive antennas, and receives signals from $M_t$ users each with one transmit antenna. The channel is assumed to be frequency selective and constant over a block duration of multiple OFDM symbols and varies independently from block to block. We will define the data and training structure within each block later. The channel between the $m^{th}$ user and the $j^{th}$ receive antenna of the base station is denoted by $h_{j,m}(l)$ where $l = 0, ..., \nu - 1$ and $\nu$ denotes the number of channel taps. The absolute propagation delay of the link between the $m^{th}$ user and the $j^{th}$ receive antenna is denoted by $d_{j,m}$. We model the effects of propagation delays as part of the channel response by setting the first $d_{j,m}$ taps of the channel response to zero. Assume that the number of nonzero taps of the channel response $h_{j,m}(l)$ is $\nu_{j,m} + 1$. We have $\nu_{j,m} + d_{j,m} + 1 \leq \nu + 1$, where $\nu = \max_{j,m}(\nu_{j,m} + d_{j,m})$. We make the assumption that $\nu$ is less than the number of OFDM subcarriers $N$, but $\nu$ may be less than, equal to or greater than the cyclic prefix length $L_{CP}$. The discrete time signal model is

$$r_j(k) = \sum_{m=1}^{M} \sum_{l=0}^{\nu_{j,m}} h_{j,m}(l)s_m(k-l) + n_j(k),$$

(1)

where $r_j(k)$ is the received signal on the $j^{th}$ receive antenna at time $k$. We also assume that the noise term $n_j(k)$’s are i.i.d. circularly symmetric complex Gaussian distributed with zero mean and variance $\sigma_n^2$.

The system diagram for space-time equalization is illustrated in Fig. 1. Some notations are summarized in Table I. A bank of $M_t$ space-time equalizers is employed at each receive antenna to suppress the various sources of interference. The space-time equalizer at the $j^{th}$ receive antenna for the $i^{th}$ user of the equalizer is represented by a vector $w_{i,j} = [w_{i,j}(L), ..., w_{i,j}(0)]^T$ of $L+1$ taps. Each FIR equalizer $w_{i,j}$ may have a different equalization delay $\Delta_{i,j}$.

A packet of $B$ information bits is transmitted via $J$ coded OFDM symbols per user. In addition, $K$ training OFDM symbols are transmitted right before the coded OFDM symbols to train the space-time equalizer for each user. The channel is assumed to be constant over a duration of $J+K$ OFDM symbols. The frequency domain signal of the $j^{th}$ OFDM symbol of the $m^{th}$ user is denoted by $v_{m}(b) = (v_{m}(0,b), ..., v_{m}(N-1,b))^T$,

where $v_{m}(p,b)$ is the symbol on the $p^{th}$ subcarrier.

The transmitted information bit vector for each user is of a length $B$. For the coded OFDM symbols, the transmitted bits for each user are encoded with binary linear code $C$ with a rate $R_C$, then these bits are interleaved by a bit interleaver $\Pi$. The coded bits are denoted by the sequence $c_m$ for the $m^{th}$ user. The total number of coded bits is denoted by $B = B/R_C$. Note that, for simplicity, we assume that $B, C$ and $\Pi$ are the same for all users. After interleaving, the bits are mapped to frequency domain symbols by a Gray mapping rule $\chi$ using a QAM constellation $\gamma$ of size $2^{m_q}$ with the minimum distance $d_{mn}$. Note that, in this paper, we do not assume the knowledge of channel information at the transmitter, therefore, we do not adapt the modulation order and coding rate for the packets as in prior work, e.g., [11]. Effectively, the bit interleaver specifies a one-to-one correspondence between the bit location $t$ and the $u^{th}$ bit of the $p^{th}$ subcarrier of the $j^{th}$ OFDM symbol, i.e., $\Pi : t \rightarrow (u,p,b)$.

The time domain signal can be obtained by $s_m(b) = Q^H v_{m}(b)$, where $Q$ denotes the DFT basis matrix and $s_m(b) = (s_{m}(0,b), ..., s_{m}(N-1,b))^T$. For the sample $s_m(p,b)$, the relation between $b$, $p$ and $k$ is that $k = b(N + L_{CP}) + L_{CP} + p$ and $k$ denotes the universal time index in the system. We also make the assumption that only a portion of the subcarriers are used for data transmission because in practical OFDM systems, there are unused bands/guard bands for spectrum shaping. The set of used subcarriers is denoted by $N_m$. Assume that the cardinalities of the set $N_m$ are the same for all users and $|N_m| = N_s$. We denote the subcarrier selection matrix by a diagonal matrix $T_m$ for the $m^{th}$ user, which consists of only 0 and 1’s.

We can construct a Toeplitz channel matrix $H_{m,i}$ of a size $(L+1) \times (L+\nu+1)$ for the $m^{th}$ transmit antenna and the $j^{th}$ receive antenna constructed from $h_{j,m}(l)$. The transmitted signal of the $m^{th}$ user is stacked into a vector $s_{m}(k-L-\nu : k) = (s_{m}(k-L-\nu), ..., s_{m}(k))^T$, where $p : q$ denotes the $p^{th}$ to the $q^{th}$ sample of the signal in MATLAB notation. The time domain signal vector at the $j^{th}$ receive antenna is denoted by

![Fig. 1. Illustration of the transceiver block diagram. Two users send signals to a receiver with three receive antennas. For each user, there is an FIR equalizer bank at the receiver.](image-url)
$$r_j(k - L : k)$$ and we have

$$r_j(k - L : k) = \sum_{m=1}^{M_r} H_{j,m} s_m(k - L - \nu : k) + n_j(k - L : k),$$

(2)

where $$n_j(k - L : k) = [n_1(k - L), ..., n_j(k)]^T$$ is the noise vector at the $$j^{th}$$ receive antenna.

The equalizer bank performs the interference suppression and the output from the equalizer at time $$k$$ is denoted by $$x_i(k)$$ where

$$x_i(k) = \sum_{j=1}^{M_r} w_{i,j}^T r_j(k + \Delta_{i,j} - L : k + \Delta_{i,j}).$$

(3)

We define the post-equalization equivalent channel response for the $$i^{th}$$ user (i.e., between $$s_i(k)$$ and $$x_i(k)$$) as $$b_{i,i} = [b_{i,i}(L_{CP}) , ..., b_{i,i}(0)]^T$$ of a length $$L_{CP} + 1$$. The partial DFT basis is denoted by $$Q$$, formed from the first $$L_{CP} + 1$$ columns of $$Q$$, which transforms the time domain channel response $$b_{i,i}$$ into the frequency domain response $$f_{i,i} = [f_{i,i}(0), ..., f_{i,i}(N-1)]^T$$. The frequency domain equivalent model after the space-time equalization and FFT transformation is thus

$$y_i(p, b) = f_i(p)v_i(p, b) + e_i(p, b)$$

(4)

on the $$p^{th}$$ subcarrier, where $$e_i(p, b)$$ denotes the noise and interference on the $$p^{th}$$ subcarrier of the $$b^{th}$$ OFDM symbol.

Let $$v_i(u, p, b)$$ be the $$u^{th}$$ bit of the symbol at the $$p^{th}$$ subcarrier of the $$b^{th}$$ OFDM symbol of the $$i^{th}$$ user after the interleaving. The log likelihood ratio for this bit can be approximated as the following by using the max-log approximation [12]

$$\Lambda_i(u, p, b) = \min \left\{ \frac{|y_i(p, b) - f_i(p)v_i(p, b)|^2}{2\sigma_i^2(p)} \right\}$$

(5)

$$\begin{align}
\Lambda_i(u, p, b) &= \min_{v_i(p, b) \in \chi_i^u} \frac{|y_i(p, b) - f_i(p)v_i(p, b)|^2}{2\sigma_i^2(p)} \\
&- \min_{v_i(p, b) \in \chi_i^0} \frac{|y_i(p, b) - f_i(p)v_i(p, b)|^2}{2\sigma_i^2(p)}
\end{align}$$

where $$\sigma_i^2(p)$$ denotes the noise plus interference power on the $$p^{th}$$ subcarrier. The terms $$\chi_i^u$$ and $$\chi_i^0$$ denote the complex QAM signal set in $$\chi$$ whose $$u^{th}$$ bit is 1 and 0, respectively. Let us define a matrix $$W_{i,j}$$ of size $$N \times (N + L)$$ as

$$W_{i,j} = \begin{bmatrix}
w_{i,j}(L) & ... & w_{i,j}(0) & ... & 0 \\
0 & ... & w_{i,j}(L) & ... & w_{i,j}(0)
\end{bmatrix}.$$ 

(6)

The noise covariance matrix after the space-time equalization and DFT operation can be obtained as

$$G_i = \sigma_i^2 \sum_{j=1}^{M_r} Q W_{i,j} W_{i,j}^H Q^H.$$ 

(7)

For simplicity, we assume that the interference power after the space-time equalization is negligible, so that the noise and interference power on subcarrier $$p$$ for the $$i^{th}$$ user can be represented as $$\sigma_i^2(p) \approx G_i(p, p)$$, i.e., the $$p^{th}$$ diagonal element of the matrix $$G_i$$. Another reason is that for simplicity again, we do not estimate the residual interference power after the space-time equalization using the training sequences, hence the residual interference power is unknown to our algorithm. Overall, this leads to a slight loss in error rate performance. We denote the vector of approximate noise/interference powers on all subcarriers for the $$i^{th}$$ user as $$\Sigma_i = [\sigma_i^2(0), ..., \sigma_i^2(N-1)]^T$$. We use the Viterbi decoding algorithm to decode the data bits. The ML codeword decision is

$$\hat{c}_i = \arg\max_{c \in C} \sum_{t=1}^{L} \Lambda_i(\Pi(t)).$$

(8)

### III. EQUALIZATION METHODS

In this section, we first describe the mean square error based equalizer training technique. Then we describe the training sequence-based equalizer design technique that brings the bit error rate performance into consideration.

#### A. Mean Square Error Based Equalizer Design

We stack the received signal vector $$r_j(k + \Delta_{i,j} - L : k + \Delta_{i,j})$$ $$(k = 0, 1, ..., N - 1)$$ into a matrix $$R_j(b, \Delta_{i,j})$$ of size $$N \times (L + 1)$$. We stack the frequency domain signal for the $$i^{th}$$ user into a matrix $$V_i(b) = diag\{v_i(0, b), ..., v_i(N-1, b)\}$$. We transform the received signal after the space-time equalization to the frequency domain and obtain

$$y_i(b) = Q \sum_{j=1}^{M_r} R_j(b, \Delta_{i,j}) w_{i,j}$$

(9)

$$= V_i(b) Q b_{i,i} + e_i(b)$$

$$= V_i(b) f_{i,i} + e_i(b)$$

The design objective is to minimize the mean square error between the received signal and the transmitted signal conditioned with the desired post equalization channel response. Define the cost function $$g_i(\Delta_{i,j}, w_{i,j}, b_{i,i})$$ as the following

$$g_i = \sum_{b=0}^{K-1} ||T, Q \sum_{j=1}^{M_r} R_j(b, \Delta_{i,j}) w_{i,j} - T, V_i(b) Q b_{i,i}||^2.$$ 

(10)

Define $$w_i = [w_{i,1}, ..., w_{i,M_r}]$$. To lower the search complexity we assume that $$\Delta_{i,j} = \Delta_i$$, $$\forall j$$, the optimization problem is formulated as [9]

$$\min_{\Delta_i, w_i, b_{i,i}} g_i(\Delta_i, w_i, b_{i,i})$$

(11)

$$s.t. \|b_{i,i}\|^2 = 1.$$ 

The solution to this problem is given in [9].

#### B. PEP Based Space-time Equalizer Design

We first introduce the tight PEP upper bound for the bit-interleaved coded OFDM in [11]. We then obtain an approximation to the PEP from this result. Finally, we will discuss the training based equalizer design that considers the derived PEP result. The main result is given in equation (18).
Define the sequence of coded bits as \( \mathbf{c}_i = [c_{1,i}, \ldots, c_{2,i}, \ldots, c_{d,i}] \) before interleaving. We consider that the decoder is in favor of \( \hat{\mathbf{c}}_i = [\hat{c}_{1,i}, \ldots, \hat{c}_{t,i}, \ldots, \hat{c}_{B,i}] \), where \( \mathbf{c}_i \) and \( \hat{\mathbf{c}}_i \) differ by \( d \) bits. Without loss of generality, \( \mathbf{c}_i \) and \( \hat{\mathbf{c}}_i \) differ in the first \( d \) consecutive positions.

Let \( \mathbf{O}_{t,i} = (u_{t,i}, p_{t,i}, b_{t,i}) \) denote the \( u_{t,i}^{th} \) bit of the \( p_{t,i}^{th} \) subcarrier of the \( b_{t,i}^{th} \) OFDM symbol. Let \( \mathbf{O}_t \) denote the Cartesian product of \( d \) such tuples for \( [c_{1,i}, \ldots, c_{d,i}] \), thus

\[
\mathbf{O}_t = (u_{1,i}, p_{1,i}, b_{1,i}) \times \cdots \times (u_{d,i}, p_{d,i}, b_{d,i}).
\]

(12)

Given \( \mathbf{O}_t \), let \( \mathbf{F}_i = [f_{i,1}(p_{1,i}), \ldots, f_{i,d}(p_{d,i})]^T \) be the \( d \) dimensional vector whose elements are the complex channel gains selected by \( \mathbf{O}_t \) and define \( \Sigma_i = [\sigma_1^2(p_{1,i}), \ldots, \sigma_d^2(p_{d,i})]^T \), which represents the \( d \) dimensional vector whose elements are the noise plus interference power on a particular subcarrier selected by \( \mathbf{O}_t \). We also define

\[
\mathbf{X}^O_{\hat{c}} = \mathbf{X}^{u_{1,1}} \times \cdots \times \mathbf{X}^{u_{d,i}},
\]

(13)

\[
\mathbf{X}^O_{\hat{c}} = \mathbf{X}^{v_{1,1}} \times \cdots \times \mathbf{X}^{v_{d,i}},
\]

(14)

We assume that the noise/interference components in all subcarriers are independent to simplify the analysis. (After equalization the noise will become correlated, but due to the interleaving, the noise and interference terms may be considered to be uncorrelated. This is a standard approximation for analyzing the BICM systems.) Adapting the derivation of the pair-wise error probability (PEP) in [11] to the system of interest, we have the PEP result (after the expurgation [11]) as

\[
P(\mathbf{c}_i \rightarrow \hat{\mathbf{c}}_i|\mathbf{O}_t, \mathbf{F}_i, \Sigma_i) \\
\leq 2^{-d(M_Q - 1)} \sum_{\mathbf{v}_i \in \mathbf{X}^O_{\hat{c}}} P(\mathbf{v}_i \rightarrow \hat{\mathbf{v}}_i|\mathbf{O}_t, \mathbf{F}_i, \Sigma_i) \\
\times \sum_{\mathbf{v}_i \in \mathbf{X}^O_{\hat{c}}} Q \left( \sqrt{\frac{1}{d} \sum_{t=1}^d |f_{i,t}(p_{i,t})|^2 \eta_t d_{min}^2 / 2\sigma_t^2(p_{i,t})} \right) \\
\leq 2^{-d(M_Q - 1)} \sum_{\mathbf{v}_i \in \mathbf{X}^O_{\hat{c}}} Q \left( \sqrt{\frac{1}{d} \sum_{t=1}^d |f_{i,t}(p_{i,t})|^2 \eta_t d_{min}^2 / 2\sigma_t^2(p_{i,t})} \right)
\]

(15)

We can optimize the equalizer parameters with respect to the cost function in (16), however, this gives high computational complexity. We want to derive a bound for the PEP so that the cost function has lower computational complexity. Since the arithmetic mean is greater than or equal to the harmonic mean, we have the following inequality

\[
\frac{1}{d} \sum_{t=1}^d |f_{i,t}(p_{i,t})|^2 \eta_t d_{min}^2 / 2\sigma_t^2(p_{i,t}) \leq \left( \frac{1}{d} \sum_{t=1}^d |f_{i,t}(p_{i,t})|^2 \eta_t d_{min}^2 / 2\sigma_t^2(p_{i,t}) \right)^{-1}.
\]

(17)

Therefore, we arrive at

\[
P(c \rightarrow \hat{c}|\mathbf{f}_i, \Sigma_i) \leq (J N_s M_Q)^{-d} \\
\times \sum_{\mathbf{O}_t} Q \left( \sqrt{\frac{1}{d} \sum_{t=1}^d |f_{i,t}(p_{i,t})|^2 \eta_t d_{min}^2 / 2\sigma_t^2(p_{i,t})} \right)^{-1}.
\]

(18)

Since the subcarriers with large interference and noise to signal power ratio dominates the PEP performance, the approximation in the last step follows. The PEP approximation implies that the sum of the inverse of the per subcarrier SINR is a suitable criterion for the space-time equalizer design. Given a training sequence, the design objective is to minimize the sum of the inverse of the per-subcarrier SINR. This method has been used to design the space-time equalizer for unencoded MIMO-OFDM systems [14]. We elaborate on this design method below.

Define the interference and noise signal vector post-FFT and after the equalization as the following

\[
e_i(\Delta_i, b) = T_i Q \sum_{j=1}^M R_j(b, \Delta_i) w_{i,j} - T_i V_i(b) Q b_{i,i}.
\]

(19)

We define a diagonal matrix \( \Lambda_i \) as the following

\[
\Lambda_i = \text{diag} \{ |f_{i,0}|^{-2}, \ldots, |f_{i,N-1}|^{-2} \}
\]

(20)

where \( f_{i,i} = Q b_{i,i} \). The cost function which represents the summation of the inverse of the per subcarrier SINR can be written as

\[
g_2(\Delta_i, w_i, b_{i,i}) = \sum_{b=0}^{K-1} e_i(\Delta_i, b)^H \Lambda_i e_i(\Delta_i, b).
\]

(21)

Therefore, the design optimization problem can be written as the following

\[
\min_{\Delta_i, w_i, b_{i,i}} g_2(\Delta_i, w_i, b_{i,i}), \\text{s.t.} \| b_{i,i} \|^2 \neq 0.
\]

(22)

By fixing the first tap of \( b_{i,i} \) to be one (i.e., \( b_{i,i}(L_{CP}) = 1 \)), the problem can be solved by the steepest descent algorithm with the armijo step sizing after applying the technique of separation of variables [9].

To optimize the cost function over all delays is very costly. We hence first perform the MSE based method in III-A
and find the best delay and the post equalization channel response. Then use this channel response as the initial input to the optimizer that assumes the PEP based design method. The rationale here is that we want to maximize the post equalization channel response energy and then use this channel response for the optimization.

C. Joint Maximum Likelihood (ML) Frequency Domain Equalization

In this subsection, we derive the ML-based frequency domain equalization method that only utilizes the signal and channel response on each subcarrier for all users. It is different from the per-subcarrier equalization used in [3].

Define the cross product of the QAM constellation $\chi_{u,i}^{n, i} = \chi \times \cdots \times \chi_{i}^{n, i} \times \cdots \chi$ where the $u$th bit of the $i$th user is 1 and $\chi_{0,i}^{n, i} = \chi \times \cdots \chi_{i}^{n, i} \times \cdots \chi$ where the $u$th bit of the $i$th user is 0. From (1), we remove the cyclic prefix and transform the signal $r_j(k)$ into the frequency domain signal $\tilde{y}_j(p, b)$ of the $p$th subcarrier of the $b$th OFDM symbol. We can transform the time domain channel $h_{j,m}$ into the frequency domain channel response $\tilde{f}_{j,m}(p)$ on the $p$th subcarrier. We stack the received signal $\tilde{y}_j(p, b)$ into $\tilde{y}(p, b)$ and stack the channel responses $\tilde{f}_{j,m}(p)$ into a matrix $F(p)$ whose $(j, m)$th element is $\tilde{f}_{j,m}(p)$. We also stack the transmitted signal $v_m(p, b)$ into a vector $v(p, b)$ of size $M_t \times 1$. With perfect channel information, we can calculate the log likelihood ratio for each user based on the maximum likelihood criterion as the following

$$\bar{\Lambda}_i(u, p, b) = \min_{v(p, b) \in \chi_{u, i}^{n, i}, \ v(u, p, b) = 1} \frac{||\tilde{y}(p, b) - F(p)v(p, b)||^2}{2\sigma_n^2} - \min_{v(p, b) \in \chi_{0, i}^{n, i}, \ v(u, p, b) = 0} \frac{||\tilde{y}(p, b) - F(p)v(p, b)||^2}{2\sigma_n^2}, \quad (23)$$

where the effect of ICI caused by the asynchronism is not modelled into the denominator of the noise and interference component for simplicity. This is the max-log approximation to log likelihood ratio. When the effective channel response of all users lie within the cyclic prefix, the equation (23) accurately represents the ML based equalization method. The ML based approach offers very desirable error rate performance, but it has a high complexity because searching over the product space of the constellation is required.

IV. SIMULATION RESULTS

In our simulation, we use either QPSK or 16-QAM modulation. The number of subcarriers $N$ is fixed to be 128 and the cyclic prefix length is $L_{CP} = 16$. A block Rayleigh fading channel model is assumed in the simulation. The number of training OFDM symbols is one ($K = 1$) and the number of data symbols is 18 ($J = 18$) throughout the simulation. The channel is quasi-static fading, constant over blocks of 19 OFDM symbols but independent between blocks. For each channel realization, we use QPSK signals for the training. The training samples are generated randomly from the QPSK signal set. Convolutional coding with generator polynomial (133,171) and the IEEE 802.11a interleaving scheme are used in our simulations. We use the Viterbi decoder for the convolutional decoding. We assume that each user uses all subcarriers, thus, the selection matrix $T_j = I$. We also assume that the same power scaling is used for the training and the data transmission.

We first illustrate the effect of space-time equalization for one channel realization of a two user scenario with two receive antennas. The post-equalization channel responses are plotted in Fig. 2 and Fig. 3. We observe that for both the MSE and PEP based design, the equalizers suppress the other user’s channel and preserve the desired users’ channel.

We illustrate the bit error rate (BER) comparison of the MSE and PEP based space-time equalizer design for the synchronous case in Fig. 4, i.e., $d_{j,m} = 0$. We assume that there are two synchronous users in the system. The number of channel taps are 6 and 8 respectively for the first and second users. The number of receive antennas is fixed to be three. From the BER comparison, we observe 5-6 dB performance gain for the BER based equalizer design versus the MSE based equalizer design at BER = $10^{-4}$. We observe that the ML based equalization method significantly outperforms both MSE and BER based space-time equalization methods. The performance gain is about 11 dB at BER = $10^{-4}$, however, it requires higher implementation complexity.

For asynchronous cases, in Fig. 5, we assume that the delay of the first user is zero and the delay for the second user...
is 15. The number of receive antennas is 3. The 16-QAM modulation is adopted. The performance comparison shows that the PEP based approach outperforms the MSE based method by 4 dB at BER = $10^{-3}$. In high SNR region, the PEP based space-time equalization method performs better than the joint ML equalization method. Due to the ICI caused by the asynchronism, the BER curve of the joint ML equalization method has an error floor. In Fig. 6, we compare the case of two receive antennas and QPSK modulation. The joint ML based approach has very good performance in the low to medium SNR region, but due to the error floor, its performance is less desirable in the high SNR region.

V. CONCLUSION

In this paper, we studied the MSE and PEP based space-time equalizer design methods for the multiuser asynchronous bit interleaved coded OFDM system. The PEP based design significantly outperforms the MSE based design in terms of BER performance. We also compared the time domain equalization approach with the joint ML frequency domain equalization for the asynchronous case. From the simulations, we observe that in the high SNR region, time domain equalization should be favored since it suppresses the inter-OFDM symbol interference and the inter-carrier interference.

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