A Novel Approach for Designing Cosine-Modulated Transceivers

Pilar Martin-Martín, Fernando Cruz-Roldán, and Tapio Saramäki

Dpto. Teoría de la Señal y Comunicaciones
Universidad de Alcalá
Alcalá de Henares, Madrid, Spain

e-mail: p.martin@uah.es, fernando.cruz@uah.es

Institute of Signal Processing
Tampere University of Technology
P. O. Box 553, FIN-33101 Tampere, Finland

e-mail: ts@cs.tut.fi

Abstract

This paper introduces a novel approach for designing critically sampled cosine-modulated transceivers (CMTs) for communications applications. The key idea in this approach is to design the prototype filter in such a manner that it provides a properly optimized tradeoff between the inter-symbol and inter-channel interferences for CMTs without directly concentrating on designing the prototype filter according to any predetermined criteria. In order to generate a very fast technique for testing this approach, the windowing technique is used for optimizing the prototype filter and the window function is a generalized Blackman window in the sense that this window function has the fourth additional term. Using this windowing approach results in a very fast unconstrained optimization problem with only three unknowns involved for optimally determining the above-mentioned tradeoff for a CMT, independently of the order of the prototype filter and the number of channel of the CMT. Several examples are included showing that CMTs with very good properties can be achieved very quickly by using the proposed technique.

1. Introduction

The discrete multitone technique can be analyzed from the point of view of a critically sampled M-channel transceiver system as shown in Fig. 1, where the filters in the transmitting and the receiving filter banks are IDFT and DFT filters, respectively. It is well known that the selectivity of the transmitting and receiving filters in such an overall system is rather limited due to the fact that their stop-band attenuation is only approximately 13 dB. The main advantages of this multi-carrier system are a low complexity and the ability to combat the main inter-symbol and inter-channel interferences (ISI), by adding a cyclic prefix [1, 2]. In order to get higher attenuation for the channel filters in Fig. 1, the systems based on the use of the transceivers with overlapping-block transmission are often used [1–3]. In this case, the discrete wavelet multi-tone technique (DWMT) replaces the DFT by a discrete wavelet transform in both the transmitting and receiving banks. A very attractive way for implementing this DWMT type of filter banks is to use cosine-modulated transceivers (CMTs) [2]. In this case, both higher stop-band attenuations and lower side-lobes for the filters in both banks of Fig. 1 can be achieved when compared to the use of the complex modulation. This is because when using the cosine modulation, the sub-filter orders are usually selected to be \( N = 2KM−1 \), where \( M \) is the number of sub-channels and \( K \) is the overlapping factor, whereas the filter orders is only \( N = M−1 \) for the complex modulation. Furthermore, CMTs can be designed quite quickly because all the filters are derived from a single prototype filter, and there exist computationally efficient structures for their implementation.

![Figure 1. Critically sampled M-channel transceiver over a noisy channel.](image)

The main problem in CMTs is that they suffer from a severe ISI and the equalization techniques are more complicated. In this paper, a novel approach is proposed for designing prototype filters for generating CMTs in such a manner that the ISI is considerably decreased when compared to other existing approaches [4].

Generating this approach is motivated by the following two facts. First, perfect-reconstruction (PR) CMFs are not worth using since in real communications environments this property is not needed due to the errors caused by the transmission channel. Therefore, it is more beneficial to use nearly PR (NPR) CMTs as long as the errors caused by the use of NPR CMTs are significantly smaller than those caused by the transmission channel. By properly exploiting this fact results either in CMTs with better filter banks performances or in CMTs meeting the same performance with a reduced arithmetic complexity and a lower overall filter bank delay. Second, and most importantly, most of the existing proposed approaches concentrate on designing the prototype filters in such a manner...
that its stop-band response is optimized in the least-mean-square sense, in the minimax sense, or in the peak-constrained least-mean-square sense. When generating a proper tradeoff between the ISI and the inter-carrier interference (ICI) for NPR CMTs, it is not essential how to design the prototype filter for generating NPR CMTs. The most important is to achieve this tradeoff with very small values of the ISI and ICI.

The key idea in the proposed approach is to design the prototype filter for a CMT in such a manner that the above-mentioned tradeoff is optimally achieved without considering the way of designing the prototype filter. The main goal is to generate this tradeoff in such a manner that the values of ISI and ICI become as small as possible. For testing this approach, a windowing technique proposed by the authors of this paper in [5] is used for designing prototype filters for NPR CMTs. In this technique, a generalized Blackman window is used. This is because of the fact that it has been observed in [6] that among the well known window functions the Blackman gives the best results for CMTs when regarding the values of ISI and ICI. Therefore, it was worth studying in [5] whether including the fourth term in the original Blackman window with adjustable parameters further improves the results. This turned out to be true as has been reported in [5].

When using the above-mentioned windowing technique for designing prototype filters for NPR CMTs, first, there are only three unknown parameters to be optimized. Second, importantly, the optimization problem for finding a proper tradeoff between the values of ISI and ICI becomes an unconstrained optimization problem that can be solved by very quickly.

The remaining of the paper is organized as follows. In Section 2, it is shown how to generated CMTs. In addition, the quality measurements are introduced. Section 3 briefly reviews how to design prototype filters for NPR CMTs using a windowing technique with the aid of the generalized Blackman window. Section 4 states the optimization problem and describes an efficient algorithm for solving it. Simulation results are considered in Section 5 that show that the proposed approach results in CMTs with good overall performance. Finally, the concluding remarks are drawn in Section 6.

2. Cosine-Modulated Transceivers (CMTs)

This section briefly reviews how to construct critically sampled CMTs as shown in Fig. 1, where the signal is transmitted over a noisy channel.

2.1 Modulation Technique

For such a system, based on an $N$th-order linear-phase FIR prototype transfer function given by

$$H_p(z) = \sum_{n=0}^{N} h_p[n] z^{-n}, \quad (1)$$

where $h_p[n] = h_p[N-n]$ for $n = 0, 1, \ldots, N$, the impulse-response coefficients of all the transmitting and receiving sub-channel filters, denoted by $f_i[n]$ and $h_i[n]$ for $k=0,1,\ldots,M-1$, respectively, are generated as follows:

$$f_i[n] = 2Mh_p[n]\cos\left[(2k+1)\frac{\pi}{2M} \left(n - \frac{N}{2}\right) - (-1)^k \frac{\pi}{4}\right] \quad (2)$$

and

$$h_i[n] = 2h_p[n]\cos\left[(2k+1)\frac{\pi}{2M} \left(n - \frac{N}{2}\right) + (-1)^k \frac{\pi}{4}\right] \quad (3)$$

for $n = 0, 1, \ldots, N$. Compared to the conventional construction of the $f_i[n]$’s (see, e.g., [7]), an additional constant $M$ is included in Eq. (2). This is because of the following reason. For the prototype filter as well as for the transmitting and receiving sub-channel filters resulting when applying the proposed technique, the maximum amplitude value in the pass-band is approximately equal to unity. Therefore, this constant is needed in order to preserve the signal energy when interpolating by a factor of $M$ before using the transmitting sub-channel filters.

2.2 Input-Output Relations

When $C(z) = 1$ and assuming that there is no noise in the channel in Fig. 1, the $z$-transform of the output signal of the $l$th channel, denoted by $\hat{X}_l(z)$, is expressible in terms of the $z$-transforms of the input signals, denoted by $X_k(z)$ for $k = 0, 1, \ldots, (M-1)$, as [7]

$$\hat{X}_l(z) = \sum_{t=0}^{M-1} T_{il}(z) \cdot X_t(z), \quad (4)$$

where $T_{il}(z)$ is the transfer function between the output of the $l$th channel and the input of the $k$th channel. This transfer function can be written as

$$T_{il}(z) = \sum_{m=0}^{M-1} H_l\left(z^{(M)} W^m\right) F_i\left(z^{(M)} W^m\right), \quad (5)$$

where $W = e^{-j2\pi/M}$.

2.3 Performance Evaluations

The performance of the CMTs as shown in Fig. 1 can be evaluated using the following three signal-to-interference ratios (SIRs). The first SIR, denoted by $\text{SIR}_{l\omega}$ for later use, is the ratio between the power of the received signal in the $l$th sub-channel and that due to the other input signals, called the inter-channel interference (ICI). This SIR is given by

$$\text{SIR}_{l\omega}(l) = \frac{\frac{1}{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{2}} \left| T_{il}(e^{j\omega}) \right|^2 \cdot d\omega}{\frac{1}{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{2}} \sum_{\nu=0}^{M-1} \left| T_{i\nu}(e^{j\omega}) \right|^2 \cdot d\omega}, \quad (6)$$

where

$$T_{il}(e^{j\omega}) = \sum_{m=0}^{M-1} H_l\left(e^{j(M-2\pi m)/M}\right) F_i\left(e^{j(M-2\pi m)/M}\right) \quad (7)$$

is the frequency response between the $l$th output and the $k$th input in the CMT of Fig. 1.
The second SIR, denoted by $\text{SIR}_{\text{ICI}}$, is due to the inter-symbol interference (ISI). This SIR is defined as the ratio between the power of the received signal in the $l$th sub-channel due to the $l$th input signal and the reduction in this power due to ISI. This SIR is given by

$$\text{SIR}_{\text{ICI}}(l) = \frac{\frac{1}{\pi} \int_0^\pi \left( |T_a(e^{j\omega})|^2 \right) d\omega}{\frac{1}{\pi} \int_0^\pi \left( |T_a(e^{j\omega})|^2 - 1 \right) d\omega}.$$  \hspace{1cm} (8)

The total interference, denoted by $\text{SIR}$, is the ratio between the power of the received signal in the $l$th subchannel due to the $l$th input signal and the total interference in this sub-channel, and is given by

$$\text{SIR}(l) = \frac{\frac{1}{\pi} \int_0^\pi \left( |T_a(e^{j\omega})|^2 \right) d\omega}{\frac{1}{\pi} \int_0^\pi \left( |T_a(e^{j\omega})|^2 - 1 \right) d\omega + \sum_{k=0}^{N-1} \frac{M}{2} |T_a(e^{j2\omega k})|^2}.$$  \hspace{1cm} (9)

### 3. Design of Prototype Filters for NPR CMT Using Generalized Blackman Windows

This section reviews how to generate the prototype filter of order $N$ for NPR CMTs consisting of $M$ channels using the windowing technique proposed by the authors of the contribution in [5] with the aid of the original and generalized Blackman windows.

#### 3.1 Original and Generalized Blackman Windows

The original Blackman window function used for generating NPR CMTs, where the order of the channel filters is $N$, is given by

$$w[n] = 0.42 - 0.5 \cos \left( \frac{2 \pi n}{N} \right) + 0.08 \cos \left( \frac{4 \pi n}{N} \right).$$  \hspace{1cm} (10)

The generalized Blackman window function, in turn, is given by

$$w[n] = a - b \cos \left( \frac{2 \pi n}{N} \right) + c \cos \left( \frac{4 \pi n}{N} \right) - d \cos \left( \frac{6 \pi n}{N} \right),$$  \hspace{1cm} (11)

where $a$, $b$, $c$, and $d$ are adjustable parameters subject to the following condition:

$$d = 1 - a + b - c.$$  \hspace{1cm} (12)

#### 3.2 Prototype Filter Design

When applying the windowing technique proposed in [8] for designing an $M$th-order linear-phase FIR prototype filter for a CMT with $M$ channels using either the Blackman window or a generalized Blackman window with fixed values of $a$, $b$, and $c$, the impulse-response coefficients of this prototype filter are expressed as

$$h_p[n] = w[n] h_{\text{ideal}}[n]$$

for $n = 0, 1, \ldots, N$, where $w[n]$ is the Blackman window or the modified Blackman window satisfying $w[n] = w[N-n]$ for $n = 0, 1, \ldots, N$, and $h_{\text{ideal}}[n]$ is the infinite-duration ideal brick-wall filter as given by

$$h_{\text{ideal}}[n] = \sin \left( \pi (n - N / 2) \right) / \left[ \pi (n - N / 2) \right].$$  \hspace{1cm} (11)

Here, $\omega$, the cutoff frequency of the ideal filter, is the adjustable parameter. Its value is determined using a simple line search so that the magnitude response of the prototype achieves the value of $1/\sqrt{2}$ at $\omega = \pi / (2M)$, that is, $|H_\omega(e^{j2\pi 2M})| = 1/\sqrt{2}$. When using this prototype filter, a NPR CMT is achieved [5], [6], [8].

### 4. Statement of the Optimization Problem and an Efficient Algorithm for Solving the Problem

This section states the optimization problem and describes an efficient unconstrained optimization algorithm for solving it.

#### 4.1 Objective Function to Minimized and the Adjustible Parameter Vector

As mentioned in the Abstract and in the Introduction, the main purpose of this paper is to optimize the prototype filter for a NPR CMT in such a manner that the weighted sum of the ISI and ICI is minimized. For this purpose, the windowing technique of Subsection 3.2 is used with the window function being the generalized Blackman window, as given by Eqs. (11) and (12). Due to Eq. (11), only $a$, $b$, and $c$ are adjustable in the optimization. Therefore, the adjustable parameter vector in the optimization is given by

$$\mathbf{x} = [a, b, c].$$  \hspace{1cm} (15)

The objective function under consideration, in turn, is expressible as

$$\tilde{\phi}(\mathbf{x}, \alpha) = \alpha \cdot \text{SIR}_{\text{ICI}} + (1 - \alpha) \cdot \text{SIR}_{\text{ICI}}.$$  \hspace{1cm} (16)

It worth pointing out when assuming that $C(z) = 1$ and there is no noise in the channel in Fig. 1, all the $\text{SIR}_{\text{ICI}}(l)$’s, as given by Eqs. (6) and (7) [SIR$_{\text{ICI}}(l)$’s, as given by Eqs. (8) and (7)] are the same and are, therefore, are simply denoted by $\text{SIR}_{\text{ICI}}[\text{SIR}_{\text{ICI}}]$.

In the above equation, $\alpha$ is a parameter controlling the importance between the levels of the ICI and ISI. As mentioned in the Introduction, it worth selecting the value of $\alpha$ to be very close to zero for generating NR CMTs because this emphasizes the importance of the ISI.

In order to use an unconstrained optimization technique for solving the optimization problem that will be stated in the next subsection, instead of maximizing the objective function, as given by Eq. (16), it more beneficial to minimize the following objective function:

$$\phi(\mathbf{x}, \alpha) = [\alpha \cdot \text{SIR}_{\text{ICI}} + (1 - \alpha) \cdot \text{SIR}_{\text{ICI}}]^{-1}.$$  \hspace{1cm} (17)

This is because most unconstrained optimization problems concentrate on minimizing the objective function.
4.2 Statement of Optimization Problem

This contribution concentrates on the following optimization problem: Given $M$, the number of channels, the integer $K$ specifying the order of the prototype to be $N = 2KM - 1$, find the adjustable parameter vector $\mathbf{x} = [a, b, c]$ in the generalized Blackman window, as given by Eqs. (11) and (12), in such a way that applying the windowing technique of Subsection 3.2 results in the prototype filter that minimizes the objective function, as given by Eq. (17).

4.3 An efficient Optimization Technique

For solving the above unconstrained optimization problem, there exists various alternatives. For this purpose, the following two-step procedure is proposed. At the first, in order to generate a proper start-up solution for the stated optimization problem, the coefficients of the original Blackman window are used in the windowing technique of Subsection 3.2, that is, $\mathbf{x} = [0.42, 0.5, 0.08]$ according to Eq. (10).

At the second step, the objective function, as given by Eq. (17), is minimized by utilizing the well known quasi-Newton method by using the solution of the first step as a start-up solution. For this purpose, the function fminunc from the Optimization Toolbox for Matlab provided by MathWorks, Inc. [9] is used. There exist closed-form expressions for evaluating the partial derivatives of the objective function, as given by Eq. (17), with respect the three unknowns in the adjustable parameter vector, as given by Eq. (15). These expressions make the convergence to the optimum solution very quick.

5. Simulation Results

In order to illustrate the benefits of the proposed design scheme, various 32-channel CMTs ($M = 32$) with different values of $K$ that result in the order of the prototype filter being $N = 2KM - 1$ were designed. Furthermore, the optimization was carried for the generalized Blackman in the $\alpha = 1$ case, that is, the optimization was concentrated the maximizing the $SIR_{IS}$ according to Eq. (16). The comparisons between CMTs designed using the original Blackman window and generalized Blackman window are summarized in Tables 1 and 2, where Table 2 also gives the optimized values of the generalized Blackman window. When comparing these tables, it can be observed that the values ISI that is measured in terms of $SIR_{IS}$, as given by Eq. (8), and the overall interference that is measured in terms overall $SIR$, as given by Eq. (9), are significantly decreased when using the generalized Blackman window at the expense of an increased value of ICI that is measured in terms of $SIR_{IC}$, as given by Eqs. (6) and (7).

The main reason for concentrating on the value of ISI is that it is usually the main problem in the most CMTs. The value of ICI, in turn, is not so important because its effect on the overall $SIR$ is always limited by ISI. Figure 2 compare the Blackman window and the generalized Blackman window for 32-Channel CMTs in the $K = 5$ and $N = 319$ case. The prototype filter and analysis filter banks in both cases are shown in Figs. 3 and 4.

Table 3 compares the proposed a 32-Channel CMT in the $K = 5$ and $N = 319$ case with the CMT that is achieved using a prototype filter designed in [4]. This prototype filter has been optimized in the least-mean-square sense for the cosine-modulated filter bank subject the maximum allowable amplitude error of 0.01 and the maximum aliasing error of 80 dB. The roll-off factor is unity. Figure 5 shows for this design the magnitude response of the prototype filter and those for the first analysis filters. As seen from Table 3, the $SIR$ is higher for the proposed design, as can be expected.

6. Conclusions

This paper has proposed a novel approach for designing NPR CMTs in such a manner that the objective function under consideration is a weighted sum of the values of ISI and ICI. The novelty lies in the fact that when optimizing this objective function the prototype filter is not directly included in the optimization, that is, there is no prescribed criterion for its design. For testing this approach, an efficient windowing method has been utilized based on the use of the generalized Blackman window. This window contains only three adjustable parameters for optimizing the above-mentioned weighted sum, independently of the number of channels and the order of the prototype filter. This enables one to find the optimum solution very quickly. Simulations have indicated that the resulting CMTs have very good properties. Future work is devoted to studying other alternatives of applying this approach. A very promising alternative is to optimize directly the prototype filter coefficients in order to provide tradeoffs between the values of ISI and ICI. It is expected that this technique results in CMTs having better performances than CMTs that can be achieved by using other existing techniques.

![Table 1. 32-Channel CMTs Designed Using the Original Blackman Window](image-url)
**Table 2. 32-Channel CMTs Designed Using the Proposed Technique With the Aid Of the Generalized Blackman Window**

<table>
<thead>
<tr>
<th>K, N</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>SIR (dB)</th>
<th>SIRICI (dB)</th>
<th>SIRISI (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 191</td>
<td>0.4648</td>
<td>0.3647</td>
<td>0.1600</td>
<td>0.0105</td>
<td>32.7454</td>
<td>33.5277</td>
<td>40.5750</td>
</tr>
<tr>
<td>4, 255</td>
<td>0.5851</td>
<td>0.2806</td>
<td>0.1199</td>
<td>0.0145</td>
<td>45.5632</td>
<td>46.0768</td>
<td>55.0891</td>
</tr>
<tr>
<td>5, 319</td>
<td>0.5185</td>
<td>0.3940</td>
<td>0.0191</td>
<td>0.0684</td>
<td>49.5913</td>
<td>57.1112</td>
<td>50.4374</td>
</tr>
<tr>
<td>6, 383</td>
<td>0.4554</td>
<td>0.4921</td>
<td>0.0499</td>
<td>0.0026</td>
<td>69.2292</td>
<td>90.0076</td>
<td>69.2657</td>
</tr>
<tr>
<td>7, 447</td>
<td>0.3914</td>
<td>0.4999</td>
<td>0.1062</td>
<td>0.0025</td>
<td>68.4967</td>
<td>86.3175</td>
<td>68.5690</td>
</tr>
<tr>
<td>8, 511</td>
<td>0.4196</td>
<td>0.5001</td>
<td>0.0792</td>
<td>0.0010</td>
<td>71.3525</td>
<td>95.0047</td>
<td>71.3713</td>
</tr>
</tbody>
</table>

**Table 3. Comparison Between Two 32-Channel CMTs in the K = 5 and N = 319 Case**

<table>
<thead>
<tr>
<th>Method</th>
<th>SIR (dB)</th>
<th>SIRICI (dB)</th>
<th>SIRISI (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method in [4]</td>
<td>49.5913</td>
<td>57.1112</td>
<td>50.4374</td>
</tr>
<tr>
<td>Method in [4]</td>
<td>43.0018</td>
<td>79.5700</td>
<td>43.0027</td>
</tr>
</tbody>
</table>

![Figure 2. Window functions in the K = 5 and N = 319 case in Tables 1 and 2. (a) Blackman window. (b) Proposed window.](image)

![Figure 3. Magnitude responses of prototype filters in the K = 5 and N = 319 case in Tables 1 and 2. (a) Blackman window. (b) Proposed window.](image)
Figure 4. Magnitude responses for the first analysis filters in the $K = 5$ and $N = 319$ case in Tables 1 and 2. (a) Blackman window. (b) Proposed window.

Figure 5. Some characteristics of the CMT designed using the technique described in [4]. (a) Magnitude response for the prototype filter. (b) Magnitude responses for the first analysis filters.

7. References


