An Efficient Approach for Designing Filter Banks for Multi-Carrier Transmission

Pilar Martín Martín
Alcalá University
Department of Teoría de la Señal y Comunicaciones
28871 Alcalá de Henares, Madrid, Spain
e-mail: p.martin@uah.es

Robert Bregović and Tapio Saramäki
Institute of Signal Processing
Tampere University of Technology
P.O Box 553, FIN-33101 Tampere, Finland
e-mail:bregovic@cs.tut.fi; ts@cs.tut.fi

Abstract—This paper proposes a fast design scheme for optimizing a novel family of Multi-Carrier Transmission (MCT) systems that are built up based on critically-sampled Cosine-Modulated Filter Banks (CMFBs). This approach extends an earlier-proposed windowing-method (WM)-based technique for optimizing prototype filters for CMFBs such that, in addition to the cut-off frequency of the ideal filter, three terms in a four-term window function are used as unknowns, thereby leading to an optimization problem with only four adjustable parameters. Such an optimization problem is very efficiently solvable also when long prototype filters and many subchannels are required. What makes the resulting MCT systems novel is that the optimization concentrates on minimizing directly a weighted sum of the inter-symbol and inter-channel interferences in these MCT systems, without directly considering the prototype filter. If the weight values in this sum are fixed, then the values of the unknowns can be optimized for the given overlapping factor. After tabulating these values, a closely optimum solution for any number of channels is obtained by simply using the WM together with these tabulated values.

I. INTRODUCTION

Modulated Transmultiplexers (MTs) are good candidates for efficient implementations of Filter Bank based Multi-Carrier Transmission (FB-MCT) systems since in the MTs the synthesis and analysis filters are generated from one or two prototype filters. However, the design of the prototype filter is still very difficult when the number of subchannel is increased due to the fact that the number of coefficients involved in the optimization process tends to be very large.

This paper concentrates on drastically speeding up the optimization of prototype filters for Nearly Perfect Reconstruction (NPR) critically-sampled FB-MCT systems. The Generalized Windowing Approach (GWA) proposed in this contribution extends a numerically efficient scheme proposed in [1]. In addition to the cut-off frequency of the ideal filter, three terms in a four-term window function are used as unknowns, independently of $M$, the number of subchannels and $N$, the prototype filter order. This approach also provides the following attractive features. First, it concentrates on minimizing directly a weighted sum of the inter-symbol and inter-channel interferences affecting the FB-MCT system at hand, without directly considering the prototype filter. This enables one to provide, in a very straightforward manner, a proper trade-off between the two interferences. Second, based on experiences on using the proposed approach, after knowing the roles of two interferences on the system in use, finding out a closely optimized prototype filter for integer-valued overlapping factors $K = (N + 1)/2M$ and for any value of $M$ can be performed very fast by using the Windowing Method (WM).

It is worth mentioning that, due to the relation among MTs, although the proposed design approach is based on Cosine-Modulated Transmultiplexers (CMTs), the optimized prototype filters can be applied equally well to Sine-Modulated Transmultiplexers [3] and Modified Discrete Fourier Transform Transmultiplexers [4].

This paper is organized as follows. Section II describes the proposed GWA approach. Section III provides a comparison with other design approaches. Finally, some concluding remarks are given in Section IV.

II. PROPOSED GENERALIZED WINDOWING APPROACH

In this section, critically sampled NPR CMTs are briefly reviewed, the proposed optimization problem is given, and Tables are provided, which contain the parameters that allow the design of the prototype filter by using the simple WM without any optimization.

A. Cosine-Modulated Transmultiplexers

In critically sampled CMTs, as depicted in Figure 1, the analysis and synthesis filters with the impulse response denoted by $h_k[n]$ and $f_k[n]$, respectively, can be easily generated by modulating an $N$th-order lowpass linear-phase prototype filter with impulse response coefficients $p[n]$ as follows [5]:

$$h_k[n] = 2p[n] \cos \left( k + \frac{1}{2} \right) \frac{\pi}{M} \left( n - \frac{N}{2} \right) + (-1)^k \frac{\pi}{4}$$

$$f_k[n] = 2p[n] \cos \left( k + \frac{1}{2} \right) \frac{\pi}{M} \left( n - \frac{N}{2} \right) - (-1)^k \frac{\pi}{4}$$

(1)

for $k = 0, 1, \ldots, M - 1$ and $n = 0, 1, \ldots, N$.

An extended version of this paper will appear in IEEE Transactions on Circuits and Systems I.

1 This paper concentrates only on filter banks with subchannel filters of order $N = 2KM - 1$ (see, e.g., [2]).
The transfer function between the $l$th input and $k$th output assuming that $C(z) = 1$ is given by

$$
T_{kl}(z^M) = \sum_{i=0}^{M-1} F_i(zW^i) H_k(zW^i),
$$

(2)

with $W^i = e^{-j\frac{2\pi i}{M}}$. For $l \neq k$, this function describes the crosstalk between the $l$th and $k$th subchannel, whereas for $l = k$, it describes the transfer function of the $k$th subchannel.

In order to evaluate the NPR property of the transmultiplexers, the inter-channel and the inter-symbol interferences have to be evaluated. The $ICI_k$ measures the crosstalk interference level in the $k$th subchannel and can be evaluated by

$$
ICI_k = \frac{1}{\pi} \int_0^{\pi} \left( \sum_{i=0, t \neq k}^{M-1} \left| T_{kl}(e^{j\omega}) \right|^2 \right) d\omega.
$$

(3)

The $ISI_k$, in turn, measures the inter-symbol interference level in the $k$th subchannel and is evaluated by

$$
ISI_k = \frac{1}{\pi} \int_0^{\pi} \left( 1 - \left| T_{kk}(e^{j\omega}) \right|^2 \right) d\omega.
$$

(4)

A global measure for evaluating the energy of the overall interference in the $k$th subchannel, denoted by $I_k$, can be defined as

$$
I_k = ICI_k + ISI_k.
$$

(5)

### B. Statement of the Optimization Problem

The GWA approach is based on the WM [1], [6]. The key idea of this approach is to design the prototype filter in such a manner that it provides transmultiplexers for communication applications with a properly optimized trade-off between the inter-channel and the inter-symbol interferences by using a very simple and consequently fast design algorithm. The window function under consideration in this paper is the following four-term generalized window function:

$$
w[n] = \sum_{i=0}^{3} (-1)^i A_i \cos \left( \frac{2\pi in}{N} \right)
$$

(6)

for $n = 0, 1, \ldots, N$. The $A_i$ values are the weights of the terms for $i = 0, 1, 2, 3$. Without loss of generality, this generalized window function is normalized, according to [6], [7], as

$$
\sum_{i=0}^{3} A_i = 1.
$$

(7)

The general problem formulation is much simpler than many previously reported formulations. The prototype filter order $N$, the number of subchannels $M$, and the compromise factor $\alpha$ between the inter-channel and the inter-symbol interferences are fixed before the optimization procedure is started, while the weights $A_i$ in the generalized window function and the cut-off frequency of the ideal lowpass filter $\omega_c$ are adjusted by minimizing the following objective function:

$$
\phi(x) = \alpha ICI(x) + (1 - \alpha) ISI(x).
$$

(8)

The factor $0 \leq \alpha \leq 1$ controls the weights of the inter-channel and the inter-symbol interferences in above objective function. When the requirements of the application are known beforehand, the value of $\alpha$ can be selected such that it appropriately emphasizes the importance of the interference that is more crucial for the application at hand. For instance, when emphasizing both interferences in the same way, $\alpha = 0.5$ is a good selection.

The condition given by (7) reduces the number of unknowns to only three unknowns $A_0$, $A_1$, and $A_2$ in the generalized cosine window function. Therefore, the adjustable parameter vector $x$ contains only four adjustable terms, independently of the subchannel filter order and the number of subchannels, namely, three weights of the generalized cosine window and $\omega_c$, that is, $x = [A_0, A_1, A_2, \omega_c]$.

### C. Efficient Algorithm for Solving the Optimization Problem

In order to solve the design problem stated in the previous subsection, based on the experimental data, it has turned out that a very good optimization algorithm for solving the proposed optimization problem, from the time consuming and proper final solution points of view, is the Nelder-Mead Simplex minimization algorithm [8]. The Nelder-Mead Simplex minimization algorithm uses only function values, that is, it is a direct search method that does not use numerical or analytic gradients [9]. For this purpose, the function `fminsearch` from the optimization toolbox provided by MathWorks, Inc. is used.

Additionally, for a given optimization problem, it is important to find a good starting point. In most cases this is not trivial. Fortunately, for solving the above proposed optimization problem, based on the experimental data, a good common starting point of the adjustable parameter vector that can be used for all the designs, i.e., independently of the number and order of the subchannels, is

$$
x = [0.42, 0.5, 0.08, \pi/2M].
$$

(10)

In this case, the four-term generalized cosine window function is initialized at the Blackman window function parameters and the cut-off frequency of the ideal lowpass filter is located at $\pi/2M$. 

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TABLE I

<table>
<thead>
<tr>
<th>K</th>
<th>A₀</th>
<th>A₁</th>
<th>A₂</th>
<th>ω_c</th>
<th>M</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>0.5232</td>
<td>0.9818</td>
<td>0.0784</td>
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<tr>
<td>3</td>
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<td>0.4199</td>
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<td>4</td>
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<td>5</td>
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</tr>
<tr>
<td>6</td>
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</tr>
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<td>7</td>
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<td>0.5000</td>
<td>0.1113</td>
<td>1.7928</td>
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TABLE II

<table>
<thead>
<tr>
<th>K</th>
<th>ICI (dB)</th>
<th>ISI (dB)</th>
<th>I (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>−19.68</td>
<td>−32.04</td>
<td>−19.68</td>
</tr>
<tr>
<td>3</td>
<td>−26.40</td>
<td>−316.2</td>
<td>−25.00</td>
</tr>
<tr>
<td>4</td>
<td>−60.08</td>
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<td>−50.08</td>
</tr>
<tr>
<td>5</td>
<td>−65.25</td>
<td>−88.89</td>
<td>−65.23</td>
</tr>
<tr>
<td>6</td>
<td>−85.65</td>
<td>−108.37</td>
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</tr>
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<td>7</td>
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<tr>
<td>8</td>
<td>−88.80</td>
<td>−84.37</td>
<td>−83.04</td>
</tr>
</tbody>
</table>

D. Tabulated Parameters

By using the proposed GWA approach for designing prototype filters for CMTs, it is straightforward to first provide a table with the required parameters and, then, to design, based on these parameters, the prototype filters using only the WM, without involving any time-consuming optimization. This simplified procedure is motivated by the following experimentally observed facts:

1) The optimized values of $A_i$ for $i = 0, 1, 2, 3$ in the generalized cosine window function, as given by (6), depend only on the integer parameter $K = (N + 1)/2M$. Table I shows their values in the $2 \leq K \leq 8$ cases for $\alpha = 0, \alpha = 0.5$, and $\alpha = 1$. The attractive property of the optimized window functions is that their shapes for the given values of $\alpha$ and $K$ remain practically the same as $M$ is varied.

2) The optimized value of the cut-off frequency of the ideal lowpass filter $\omega_c$ does not only depend on the number of subchannels $M$, but also on the parameter $K$. This is illustrated in Table I that provides the optimized values for $\omega_c$.

3) The interference level is practically independent of the number of subchannels. It depends only on the parameter $K$. Therefore, the results presented here are valid for any number of subchannels as long as $N = 2KM − 1$. Table II shows the $IC1$, $IS1$, and $I$ for the GWA-based CMTs in the $2 \leq K \leq 8$ cases for $\alpha = 0, \alpha = 0.5$, and $\alpha = 1$. It can be observed that values of the interferences depend on the selected $\alpha$ value. Selecting $\alpha = 0.5$ provides the CMTs with the best $I$ performance. Smaller (larger) values of $\alpha$ improve the $IS1$ ($ICI$) by emphasizing in the objective function that interference. It is worth noticing that in the $\alpha = 0$ case, the algorithm minimizes the objective function, as given by (8), as much as possible. However, there is always a limit for the $ICI$ that depends on the $K$ value, that is, on the stopband attenuation attained by the corresponding order of the prototype filter.

III. Design Simulations

In this section, in order to illustrate the usefulness of the design approach proposed in this paper, the performances of various 32-channel NPR CMTs are compared in the $K = 3$ case. The following design approaches are included in the comparison:

1) The proposed GWA approach for $\alpha = 0.5$.
3) The Kaiser Window Approach (KWA) proposed in [11]. For comparison purposes, the minimum stopband attenuation $A_s$ is chosen to be equal to the $A_s$ of the proposed GWA-based NPR CMT for the same number of subchannels and the same subchannel filter order.
4) The Mirabba–Martin approach (M–M) proposed in [12]. The impulse response coefficients of the prototype filters in the M–M approach depend on the overlapping factor $K$ and are given in Table I in [12].
5) A PR CMT with the prototype filter designed in the least-mean-square sense [2].

Figure 2 shows the resulting magnitude response of the prototype filters for these five design approaches. Table III
Fig. 2. Magnitude response of the prototype filters for generating 32-channel CMTs in the $K=3$ case.

reports the values of the $ICI$, $ISI$, and $I$ for the designs under consideration. Due to the duality relation between transmultiplexers and Subband Coding Filter Banks (SBC-FBs), Table III provides also the amplitude distortion $E_a$ and the total aliasing error $\delta_d$ for the corresponding filter banks, defined as follows [5]:

$$\delta_d = \max_{\omega \in [0, \pi]} \left| \sum_{k=0}^{M-1} F_k(e^{j\omega}) H_k(e^{j\omega}) \right| - 1 \quad (11)$$

and

$$E_a = \frac{1}{\pi} \int_{0}^{\pi} \left( \sum_{i=1}^{M-1} \sum_{k=0}^{M-1} F_k(e^{j\omega}) H_k(e^{j(\omega - 2\pi i/M)}) \right)^2 \, d\omega. \quad (12)$$

As seen in Table III, the values of $E_a$ and $\delta_d$ (measures in decibels) are very close to the $ICI$ and $ISI$ values, respectively. Therefore, the proposed approach can also be used for designing SBC-FBs.

Additionally, the Signal-to-Noise Ratio ($SNR$), a measure independent of the design procedure is worth using. If the input signals are assumed to be binary random sequences with values $\pm 1$, then the SNR is defined by

$$SNR = \frac{1}{M} \sum_{k=0}^{M-1} \left( \frac{\sum_{n} (x_k[n])^2}{\sum_{n} (\tilde{x}_k[n - D_T] - x_k[n])^2} \right). \quad (13)$$

Here, $D_T$ is the overall system delay, whereas $x_k[n]$ and $\tilde{x}_k[n]$ are the input and the reconstructed signals in the $k$th subchannel, respectively. Table III shows that the GWA approach provides among the four NPR approaches under consideration the CMT with the lowest values of $I$ for the given overlapping factor $K$. This is a direct consequence of the fact that this measure is optimized in the GWA approach. Furthermore, the GWA-based CMT also provides the highest value of $SNR$. These results show that optimizing the interference levels in the proposed manner results also in a maximized value of $SNR$.

IV. Conclusion

A simple and efficient design procedure for optimizing prototype filters for critically sampled NPR CMTs was proposed and analyzed. In this approach, based on the use of the windowing method, the levels of interferences are controlled during the optimization stage by the objective function. The value of the trade-off factor $\alpha$ selected during the optimization process is directly affecting to the inter-channel and the intersymbol interferences levels. It was observed that, for $\alpha = 0.5$, it provides the CMT with the best $I$ performance. Smaller (larger) values of $\alpha$ improve the $ISI$ ($ICI$). In order to design a prototype filter for any $M$-channel FB-MCT, Table I provides all needed parameters for determining the window function to be used in the WM for the given value of $\alpha$.

REFERENCES


TABLE III

PERFORMANCES OF 32-CHANNEL CMTS UNDER CONSIDERATION IN THE $K=3$ CASE.

<table>
<thead>
<tr>
<th>WMFB</th>
<th>GWA</th>
<th>KWA</th>
<th>M-M</th>
<th>PR</th>
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</thead>
<tbody>
<tr>
<td>$ICI$ (dB)</td>
<td>-35.77</td>
<td>-55.53</td>
<td>-49.50</td>
<td>-43.87</td>
</tr>
<tr>
<td>$ISI$ (dB)</td>
<td>-37.42</td>
<td>-79.45</td>
<td>32.21</td>
<td>-54.26</td>
</tr>
<tr>
<td>$I$ (dB)</td>
<td>-33.50</td>
<td>-55.51</td>
<td>-32.13</td>
<td>-43.49</td>
</tr>
<tr>
<td>$SNR$ (dB)</td>
<td>33.35</td>
<td>54.43</td>
<td>31.03</td>
<td>43.15</td>
</tr>
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<td>$E_a$ (dB)</td>
<td>-35.77</td>
<td>-55.06</td>
<td>-49.43</td>
<td>-43.88</td>
</tr>
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<td>$\delta_d$</td>
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<td>1.18e-4</td>
<td>2.45e-2</td>
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