Load Shaping Strategy Based on Energy Storage and Dynamic Pricing in Smart Grid

Tao Jiang, Senior Member, IEEE, Yang Cao, Liang Yu, and Zhiqiang Wang

Abstract—Load shaping is one of important and challenging issues in power grid. In this paper, we propose a novel load shaping strategy based on energy storage and dynamic pricing in smart grid. In the proposed strategy, a consumer is encouraged to draw a certain amount of energy (i.e., quota) from the grid. When the actual energy demand is deviated from the quota, the consumer is faced with a higher electricity price. With the help of energy storage, the consumer can draw less electricity from the grid at a lower price by discharging energy when the demand is higher than the quota and draw more electricity from the grid at a lower price by charging energy when the demand is lower than the quota. As a result, the utility can implement load shaping and consumers can save energy cost simultaneously. Moreover, the proposed strategy can be implemented with low complexity and in a distributed fashion, which offers scalability to large number of consumers. Simulations results show the effectiveness of the proposed load shaping strategy.

Index Terms—Demand side management, dynamic pricing, energy storage, load shaping, smart grid.

I. INTRODUCTION

N the power grid, demand side management (DSM) refers to the concept of a market party actively using options to modify electricity demand to increase consumer satisfaction and coincidentally produce desired changes in the electric utility’s load shape. Load shape changes pursue a reduction of system costs and an increase of reliability, both improving the efficiency of generation planning and scheduling. Generally, there are six load shaping objectives, including peak clipping, valley filling, load shifting, strategic conservation, strategic load growth, and flexible load shape [1]. Usually, these objectives could be divided into two categories, i.e., energy adjustments (including strategic conservation and strategic load growth) and power adjustments (including peak clipping, valley filling, load shifting and flexible load shape). In order to realize power adjustments in a short time, consumers can be motivated by dispatchable demand response programs (e.g., direct load control) and dynamic pricing.

Dynamic pricing is considered as one of effective motivation strategies for load shaping, especially in smart grid which has been attracted more and more attentions due to the rising cost of energy and the urgent need of reducing global carbon emission [2]. As the presentation in [3], smart grid will be characterized by a two-way flow of electricity and information, and will incorporate into the grid the benefits of distributed computing and communications to deliver real-time information and enable the near-instantaneous balance of supply and demand at the device level. Hence, load shaping based on dynamic pricing supported by real-time communications will play an important role in smart grid.

The existing dynamic pricing models can mainly be divided into two categories, time-differ pricing [4]–[7] and quantity-differ pricing. Several time-differ pricing models have been proposed, such as day-ahead pricing, time-of-use pricing, critical-peak pricing and real-time pricing. These pricing models encourage consumers to use less electricity when peak load appears and to use more electricity when valley load appears. Therefore, the load shaping is achieved and the fluctuation of load is mitigated. Quantity-differ pricing model is also proposed [8], called as conservation rates model with inclining block rates, in which, the electricity price is increased when the total consumed electricity quantity is beyond a threshold during the considered period of time. The quantity-differ pricing model encourages consumer not to use too much electricity during the considered period of time, hence, the peak load can be reduced. Moreover, in [9], the authors proposed a computationally feasible and automated optimization-based residential load control scheme in a retail electricity market with real-time pricing combined with inclining block rates.

Energy storage is also considered to reduce the peak load and the load fluctuation [10]–[12]. Most of the existing works focus on exploiting the dispatchable functionalities of the energy storage to relieve the intermittent nature of the renewable generation from the perspective of the utility grid. Some other works focus on energy storage to reduce the consumers’ electricity cost [13]–[15]. In [13] and [14], authors studied the total electricity cost minimization problem for an Internet-service
provider having multiple data centers with energy storage in
deregulated electricity markets and in smart microgrids, respec-
tively. In [15], the authors proposed a novel energy buffering
framework to reduce the financial cost of consumers with bursty
energy usage patterns that could be deferred. In above work,
when energy storage is considered, the scheduling of charge or
discharge is often formulated as a joint optimization among
all consumers. This is not suitable when the consumer is in-
dependent, noncooperative, and selfish. Hence, the distributed
strategy on scheduling of charge or discharge may be more
meaningful if energy storage is considered.

Recently, some works have proposed some distributed
scheduling strategies. For example, Low et al. [16] proposed a
method for the utility to determine time-varying prices. Under
such prices, when the consumers selfishly optimize their own
benefits, they automatically also maximize the social welfare.
Guo et al. [17] proposed a distributed algorithm to minimize the
total energy cost of multiple residential households in a smart
grid neighborhood sharing a load serving entity. Different from
the above work, we consider the objective of effective load
shaping. Specifically, we intend to implement peak clipping and
valley filling, i.e., reducing the deviation between the base-load
generation and the overall consumer demands. To this end,
we propose a novel load shaping strategy based on dynamic
pricing scheme and energy storage. The main contributions are
summarized as follows:

- We formulate two optimization problems to maximize the
  welfare for consumers and minimize the load fluctuations
  for the utility, respectively.
- We propose an effective load shaping strategy to solve the
  above problems and obtain the optimal closed-form
  solutions. Therefore, the proposed algorithm can be
  implemented with low complexity. Moreover, the proposed
  strategy can offer scalability to large number of con-
  sumers.
- Simulation results show that the utility can implement load
  shaping and consumers can save energy cost simultane-
  ously.

The rest of the paper is organized as follows. In Section II,
the system model is given. Problem formulation and algorithm
design are presented in Section III. Then, we evaluate the pro-
posed strategy via some simulation results in Section IV and
conclude this paper in Section V. Notations used in this paper
are listed in Table I.

## II. System Model

We consider a time-slotted smart grid system with $M$
distributed consumers. All consumers are independent and non-co-
operative. In this paper, we focus on the inelastic electricity
demands. Generally speaking, inelastic electricity demands are
from lights, computers, televisions, microwave ovens and so on.
Since consumers have the habits of using such equipments, we
can assume that inelastic demands in the same time of different
days follow the same distribution. The interaction between the
utility and each consumer is shown in Fig. 1. At the end of the
$(t-1)$th slot, the utility develops a pricing

<table>
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<th>TABLE I</th>
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<tr>
<td><strong>NOTATIONS</strong></td>
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<tr>
<td>$M$: the number of consumers</td>
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<tr>
<td>$A_m(t)$: the $m$-th consumer’s total quantity of electricity drawn from the power grid at slot $t$</td>
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<tr>
<td>$Q_m(t)$: the $m$-th consumer’s expected quantity of electricity drawn from the power grid at slot $t$</td>
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<td>$B_m(t)$: the quantity of electricity drawn from the power grid for the electric equipment at slot $t$</td>
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<tr>
<td>$C_m(t)$: the quantity of electricity drawn from the power grid for charging the energy storage at slot $t$</td>
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<td>$\bar{C}_m$: the upper bound of $C_m(t)$</td>
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<tr>
<td>$D_m(t)$: the quantity of electricity from its own energy storage for the electric equipment at slot $t$</td>
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<tr>
<td>$\bar{D}_m$: the upper bound of $D_m(t)$</td>
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<tr>
<td>$V_m$: the capacity of the $m$-th consumer’s energy storage</td>
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<td>$G(t)$: the total energy from the base-load generator</td>
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<td>$Q_m(t)$: the inelastic demand of the $m$-th consumer at slot $t$</td>
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<tr>
<td>$R_m(t)$: the quantity of electricity in the $m$-th consumer’s energy storage at the beginning of slot $t$</td>
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<tr>
<td>$P_m(t)$: the average price of the electricity in the $m$-th consumer’s energy storage at the beginning of slot $t$</td>
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<td>$P_m(t)$: the lowest price of the electricity in the grid for the $m$-th consumer at slot $t$</td>
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<tr>
<td>$\mu(t)$: a parameter in the auxiliary variable</td>
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<tr>
<td>$H_m(t)$: the expected value of demand distribution of the $m$-th consumer at slot $t$</td>
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strategy for each consumer, which is then sent to each consumer
correspondingly. After obtaining the pricing strategy, the $m$th
consumer, $m = 1, 2, \ldots, M$, determines an action policy, in-
cluding: 1) the quantity of electricity drawn from the power grid
for the electric equipment during this slot, denoted as $H_m(t)$; 2)
the quantity of electricity drawn from the power grid for
charging the energy storage during this slot, denoted as $C_m(t)$;
3) the quantity of electricity from its own energy storage for the
electric equipment during this slot, denoted as $D_m(t)$. Let
$Q_m(t)$ denote the inelastic electricity demand of the $m$th con-
sumer at slot $t$, then

$$Q_m(t) = B_m(t) + D_m(t)$$

(1)

where $B_m(t) \geq 0$ and $D_m(t) \geq 0$. Moreover, the $m$th con-
sumer’s total quantity of electricity drawn from the power grid
at slot $t$ is denoted by $A_m(t)$, and we have

$$A_m(t) = B_m(t) + C_m(t).$$

(2)

Usually, base-load generator (BLG) is always in operation,
and the cost of electricity from BLG is relatively low. When
the electricity provided by BLG at slot $t$, i.e., $G(t)$, is less
than the overall electricity drawn from the power grid, i.e.,
$\sum_{m=1}^{M} A_m(t)$, some peaking generator (PG) should be em-
ployed, and the cost of electricity from PG is relatively high.
When $G(t) > \sum_{m=1}^{M} A_m(t)$, the excess electricity also results
in a high cost [18].

Clearly, the utility expects a load shaping that $\sum_{m=1}^{M} A_m(t)$
approaches $G(t)$ with as small as possible standard deviation.
To this end, we propose a load shaping strategy based on dynamic pricing and energy storage.

### A. Load Differentiation Dynamic Pricing (LDDP) Model

In this subsection, we propose a novel dynamic pricing model, called load differentiation dynamic pricing (LDDP). To avoid the fluctuations of total demand, the \( m \)th consumer is encouraged to draw certain quantity of electricity (quota), which is denoted as \( Q_m(t) \), from the power grid. We set:

\[
Q_m(t) = G(t)H_m(t)/\sum_{m=1}^{M} H_m(t),
\]

where \( H_m(t) \) denotes the expected value of demand distribution of the \( m \)th consumer at slot \( t \). If \( Q_m(t) = Q_m(t) \), the price of the electricity in the grid is set to be the lowest, denoted as \( P_m(t) \). Otherwise, the price of the electricity in the grid increases as \( Q_m(t) \) increases. In addition, we can observe that, when \( Q_m(t) < Q_m(t) \), the \( m \)th consumer can increase \( D_m(t) \) (i.e., reduce \( A_m(t) \)) considering (1). The benefit is that the consumer can use electricity in the grid at a lower electricity price and the utility can implement load shaping since less electricity is drawn from the grid. Similarly, when \( Q_m(t) = Q_m(t) \), the \( m \)th consumer can increase \( C_m(t) \) (i.e., increase \( A_m(t) \)) considering (2). The benefit is that the consumer can charge energy at a lower electricity price and the stored energy can be discharged when the grid electricity price is high, resulting in lower energy cost for the consumer. Meanwhile, the utility can implement load shaping since more electricity is drawn from the grid.

For analysis simplicity but without loss of generality, we employ linear functions to formulate LDDP. Then, electricity price with respect to the \( m \)th consumer is

\[
f(A_m(t)) = \begin{cases} f_1(A_m(t)), & 0 < A_m(t) < Q_m(t) \\ f_2(A_m(t)), & Q_m(t) \leq A_m(t) \end{cases}
\]

where \( f_1(A_m(t)) = k_{1,m}(A_m(t) - Q_m(t)) + P_m(t) \) and \( f_2(A_m(t)) = k_{2,m}(A_m(t) - Q_m(t)) + P_m(t) \). The parameters \( k_{1,m} \) and \( k_{2,m} \) indicate the encouragement strength of using energy storage for the \( m \)th consumer, \( k_{1,m} < 0 \) and \( k_{2,m} > 0 \). Furthermore, to guarantee that the cost of using electricity, i.e., \( f(A_m(t)) \times A_m(t) \), is an increasing function of \( A_m(t) \) when

\[
A_m(t) > 0, \quad k_{1,m} \text{ must satisfy } k_{1,m} \geq -(P_m(t)/Q_m(t)).
\]

As an example, Fig. 2 shows \( f(A_m(t)) \) versus \( A_m(t) \) when \( m = 1, 2, k_{1,1} = k_{1,2} = -3, k_{2,1} = k_{2,2} = -3, Q_1(t) = 0.05, Q_2(t) = 0.15, P_1(t) = P_2(t) = 0.5 \).

### B. Energy Storage Model

Let \( V_m(t) \) denote the capacity of the \( m \)th consumer’s energy storage. Due to the physical characteristic of energy storage, \( C_m(t) \) and \( D_m(t) \) are upper bounded by \( \bar{C}_m \) and \( \bar{D}_m \), respectively, i.e.,

\[
0 \leq C_m(t) \leq \bar{C}_m \quad (4)
\]

\[
0 \leq D_m(t) \leq \bar{D}_m \quad (5)
\]

Let \( R_m(t) \) denote the quantity of electricity in the \( m \)th consumer’s energy storage at the beginning of slot \( t \). As in [13], we do not explicitly consider some practical issues, such as energy leakage in the battery or DC/AC conversion loss, which can be readily incorporated. Then, the update equation of \( R_m(t) \) is given by

\[
R_m(t) = \begin{cases} 0, & t = 1 \\ R_m(t - 1) + C_m(t - 1) - D_m(t - 1), & t > 1. \end{cases}
\]

(6)

Let \( P_m(t) \) denote the average price of the electricity in the \( m \)th consumer’s energy storage at the beginning of slot \( t \). Then, the update equation of \( P_m(t) \) is given by

\[
P_m(t) = \begin{cases} 0, & \Phi(t-1) + \Psi(t-1) R_m(t) \leq 0 \\ \frac{\Phi(t-1) + \Psi(t-1)}{R_m(t)}, & \text{otherwise}, \end{cases}
\]

(7)

where \( \Phi(t-1) \) is the cost of the remaining electricity in the energy storage after discharging in the \((t-1)\)th slot and \( \Psi(t-1) \) is the cost of the charging electricity in the \((t-1)\)th slot, which are given by

\[
\Phi(t-1) = P_m(t-1) (R_m(t-1) - D_m(t-1))
\]

(8)

\[
\Psi(t-1) = C_m(t-1) f(A_m(t-1)).
\]

(9)

Similar to [13], [14], we also assume that charging and discharging cannot be done simultaneously, i.e., \( C_m(t) \neq 0 \rightarrow \)
and $D_m(t) = 0$ and $D_m(t) \neq 0 \rightarrow C_m(t) = 0$, which can be formulated as

$$C_m(t)D_m(t) = 0.$$  \hfill (10)

Moreover, during each time slot, the quantity of electricity in the energy storage cannot be larger than the capacity of the storage, and cannot be smaller than zero. Therefore, $C_m(t)$ and $D_m(t)$ are also bounded by

$$0 \leq C_m(t) \leq V_m - R_m(t)$$  \hfill (11)

$$0 \leq D_m(t) \leq \min \{ R_m(t), Q_m(t) \}.$$  \hfill (12)

### III. Problem Formulation

First, we introduce the concept of experience function of a consumer, denoted as $U(x, w)$, where $x$ is the electricity consumption level of the consumer, and $w$ is a parameter which may vary among consumers and also at different time durations. $U(x, w)$ represents the level of satisfaction obtained by the consumer as a function of its electricity consumption. In addition, when $Q_m(t) < R_m(t)$, the consumer may not draw electricity from the power grid, and only uses the electricity from its own energy storage. This case is not expected by the utility since the total valley load may be reduced further in this case. Thus, we introduce an auxiliary variable, which can be used by the utility to impact the operations of energy storage. Specifically, when the power demand is lower than the given quota for a consumer, increasing the parameter $\mu(t)$ in the auxiliary variable can encourage the consumer to draw more electricity from power grid at a lower electricity price by charging energy, and the stored energy can be discharged when the grid electricity price is high, resulting in the lower energy cost for the consumer. Meanwhile, the utility can implement load shaping since more electricity is drawn from the grid. In a word, auxiliary variable can help to achieve a win-win situation for both the utility and consumers.

We define the welfare function of the $m$th consumer as

$$W_m(t) = \underbrace{U(Q_m(t), w_m)}_{\text{profit if using electricity}} + \underbrace{R_m(t)P_m(t + 1) - R_m(t)P_m(t)}_{\text{profit due to the change of energy in the storage}} - \underbrace{B_m(t)f(A_m(t)) - C_m(t)f(A_m(t))}_{\text{cost if using electricity}} - \underbrace{\mu(t)D_m(t)P_m(t)}_{\text{auxiliary variable}},$$  \hfill (13)

where $\mu(t) \geq 0$ and its value can be dynamically designed online, which will be investigated later.

Substituting (6) and (7) into (13), the welfare function can be rewritten as

$$W_m(t) = U(Q_m(t), w_m) - B_m(t)f(A_m(t)) - (1 + \mu(t))D_m(t)P_m(t).$$  \hfill (14)

Each consumer aims to maximize its own welfare. Note that, one of the advantages of the proposed load shaping strategy is to guarantee the inelastic electricity demand of the consumers, i.e., $U(Q_m(t), w_m)$ is independent of the consumer’s action policy. Therefore, maximizing the welfare of the $m$th consumer can be written as

$$\min_{b_m(t), c_m(t), d_m(t)} \lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} \Theta(B_m(t), C_m(t), D_m(t))$$  \hfill (15)

s.t. \hspace{1cm} (1), (4), (5), (6)

$$\hspace{1cm} (7), (10), (11), (12).$$

where $\Theta(B_m(t), C_m(t), D_m(t)) = B_m(t)f(B_m(t)) + C_m(t) + (1 + \mu(t))D_m(t)P_m(t)$.

Obviously, the action policy in the $(t + 1)$th slot is related to the action policy in slot $t$ and the price information in the $(t + 1)$th slot. However, when the action policy in slot $t$ should be determined, the price information in the $(t + 1)$th slot is unknown, thus, it is not sure that how the action policy in slot $t$ will affect the action policy in the $(t + 1)$th slot. Therefore, it is very difficult to obtain the optimal solution to (15). Then, we implement an effective greedy strategy to obtain solutions to (16)–(19) in a slot-by-slot fashion, which are suboptimal but practical solutions to (15). The per-slot optimization problem can be formulated as

$$\min_{b_m(t), c_m(t)} \Omega(B_m(t), C_m(t))$$  \hfill (16)

s.t. \hspace{1cm} Q_m(t) - \beta \leq b_m(t) \leq Q_m(t)$$  \hfill (17)

$$0 \leq c_m(t) \leq \alpha$$  \hfill (18)

$$C_m(t)(Q_m(t) - B_m(t)) - \beta$$  \hfill (19)

where $\Omega(B_m(t), C_m(t)) = B_m(t)f(B_m(t) + C_m(t)) + (1 + \mu(t))P_m(t)(Q_m(t) - R_m(t))$, $\alpha = \min \{ C_m, V_m - R_m(t) \}$, $\beta = \min \{ D_m, R_m(t) \}$, $Q_m(t)$, and $D_m(t)$ is replaced by $Q_m(t) - B_m(t)$. The optimal closed-form solution to (16)–(19) can be obtained by using an analytic function, which is given in the following.

According to the last constraint of the optimization problem, all the feasible solution must satisfy $C_m(t) = 0$ or $B_m(t) = Q_m(t)$. Therefore, we can investigate these two cases respectively.

1) Case 1: $C_m(t) = 0$

When $C_m(t) = 0$, the optimization problem can be rewritten as

$$\min_{b_m(t)} B_m(t)f(B_m(t)) + (1 + \mu(t))P_m(t)(Q_m(t) - B_m(t))$$  \hfill (20)

s.t. \hspace{1cm} Q_m(t) - \beta \leq b_m(t) \leq Q_m(t).$$
If $Q_m(t) \leq \bar{Q}_m(t)$, as shown in Fig. 3(a), the corresponding optimal $B_m(t)$ is
\begin{equation}
B_m^1(t) = \begin{cases}
Q_m(t) - \beta, & I_m,1(t) > \frac{Q_m(t) - \beta + \bar{Q}_m(t)}{2}
Q_m(t), & I_m,1(t) \leq \frac{Q_m(t) - \beta + \bar{Q}_m(t)}{2}.
\end{cases}
\end{equation}

If $Q_m(t) - \beta \geq \bar{Q}_m(t)$, as shown in Fig. 3(b), the corresponding optimal $B_m(t)$ is
\begin{equation}
B_m^2(t) = \begin{cases}
Q_m(t) - \beta, & I_m,2(t) \leq Q_m(t) - \beta
Q_m(t), & I_m,2(t) \geq Q_m(t).
\end{cases}
\end{equation}

If $Q_m(t) - \beta < \bar{Q}_m(t) < Q_m(t)$, as shown in Fig. 3(c), we firstly analyze the value of $I_{m,1}(t)$. According to (21),
\begin{equation}
I_{m,1}(t) = \frac{(Q_m(t)/2) + ((1 + \mu(t))P_m(t) - P_m(t))/2k_{1,m}}{2k_{1,m}}.
\end{equation}

When $P_m(t) \neq 0$, i.e., $R_m(t) \neq 0$ (see the update equation of $P_m(t)$), we can obtain that $P_m(t) \geq \bar{P}_m(t) > 0$ since the price of the charging electricity in each slot is no less than $\bar{P}_m(t)$ (see the proposed load differentiation dynamic pricing model in Section II-A). Therefore, we can obtain that
\begin{equation}
(1 + \mu(t))P_m(t) - \bar{P}_m(t) \geq 0 \text{ due to } \mu(t) \geq 0 \text{ and } k_{1,m} < 0.
\end{equation}

Then, $L_{m,1}(t) - (Q_m(t)/2) + ((1 + \mu(t))P_m(t) - \bar{P}_m(t))/2k_{1,m} \leq 0$ due to $Q_m(t) - \beta \geq 0$. Therefore, as shown in Fig. 3(c), the corresponding optimal $B_m(t)$ can be obtained by the following equation:
\begin{equation}
B_m^3(t) = \begin{cases}
\bar{Q}_m(t), & L_{m,3}(t) \leq Q_m(t)
Q_m(t), & L_{m,3}(t) \geq Q_m(t)
\end{cases}
\end{equation}

2) Case 2: $B_m(t) = Q_m(t)$

When $B_m(t) = Q_m(t)$, the optimization problem can be rewritten as
\begin{equation}
\min_{C_m(t)} Q_m(t)f(Q_m(t) + C_m(t))
\text{s.t. } 0 \leq C_m(t) \leq \alpha.
\end{equation}

The objective function (25) is a piecewise continuous function, and each section is a linear function, as show in Fig. 4.

If $Q_m(t) + \alpha < \bar{Q}_m(t)$, as shown in Fig. 4(a), the corresponding optimal $C_m(t)$ is
\begin{equation}
C_m^1(t) = \alpha.
\end{equation}
If $Q_m(t) \geq \bar{Q}_m(t)$, as shown in Fig. 4(b), the corresponding optimal $C_m(t)$ is

$$C_m^*(t) = 0. \quad (27)$$

If $Q_m(t) < \bar{Q}_m(t) < Q_m(t) + \alpha$, as shown in Fig. 4(c), the corresponding optimal $C_m(t)$ is

$$C_m^*(t) = \bar{Q}_m(t) - Q_m(t). \quad (28)$$

Then, the optimal solution can be obtained by

$$(B_m^*(t), C_m^*(t)) = \arg \min_{(B_m(t), C_m(t)) \in \Delta} \Omega(B_m(t), C_m(t)). \quad (29)$$

where $\Delta$ denotes the set $\{(B_m^*(t), 0), (Q_m(t), C_m^*(t))\}$.

After the action policy in slot $t$ obtained, the state of the energy storage is updated by (6) and (7).

In addition, how to design $\mu(t)$ is a very important issue in the proposed strategy. As mentioned above, the utility expects that the total electricity drawn from the power grid approaches $Q_m(t)$ with small standard deviation, which can be formulated as

$$\min_{\mu(t)} \left( G(t) - \sum_{m=1}^{M} [B_m^*(t) + C_m^*(t)] \right)^2 \quad (30)$$

s.t. $\mu(t) \geq 0$.

where $B_m^*(t)$ and $C_m^*(t)$ are the optimal action policy of the $m$th consumer. To obtain the optimal $\mu(t)$, we introduce an important property firstly.

**Property 1**: $A_m^*(t) = B_m^*(t) + C_m^*(t)$ is a non-decreasing function with respect to $\mu(t)$, i.e., $\forall \mu_0(t) > \mu(t), A_m^*(t) \geq A_m^{*0}(t)$, where $A_m^{*0}(t) = B_m^{*0}(t) + C_m^{*0}(t)$ and $A_m^*(t) = B_m^*(t) + C_m^*(t)$ are the optimal action policies of the $m$th consumer under $\mu_0(t)$ and $\mu(t)$, respectively.

**Proof**: According to the discussion above, all the feasible solution must satisfy $C_m(t) = 0$ or $B_m(t) = Q_m(t)$. Therefore, we can also investigate these two cases, respectively.

When $C_m(t) = 0$, based on the discussion above, the corresponding optimal $B_m(t)$ could be obtained by (22), (23) or (24). Note that, $k_{1,m} < 0, k_{2,m} > 0$ and $P_m(t) > 0$ (when $P_m(t) = 0$, the analysis is given in the situation of $B_m = Q_m(t)$). Therefore, according to (21), $L_{m,1}(t)$ is a decreasing function with respect to $\mu(t)$, and $L_{m,2}(t)$ is an increasing function with respect to $\mu(t)$.

Then, in (22), $\forall \mu_0(t) > \mu(t)$, the corresponding $L_{m,1,0}(t) < L_{m,1}(t)$. Hence, there are three situations:

1) $L_{m,1,0}(t) < L_{m,1}(t) \leq (Q_m(t) - \beta + Q_m(t)/2) \quad (23)$ in which the corresponding $B_m^{*0}(t) = B_m^*(t) = Q_m(t)$.

2) $(Q_m(t) - \beta + Q_m(t)/2) < L_{m,1,0}(t) < L_{m,1}(t)$ in which the corresponding $B_m^{*0}(t) = B_m^*(t) = Q_m(t) - \beta$.

3) $L_{m,1,0}(t) \leq (Q_m(t) - \beta + Q_m(t)/2) < L_{m,1}(t)$ in which the corresponding $B_m^{*0}(t) = B_m^*(t) = Q_m(t) - \beta$ (note that $\beta \geq 0$).

Therefore, $\forall \mu_0(t) > \mu(t)$, the corresponding $B_m^{*0}(t) \geq B_m^*(t)$.

In (23), $\forall \mu_0(t) > \mu(t)$, the corresponding $L_{m,2,0}(t) > L_{m,2}(t)$. Hence, there are six situations:

1) $Q_m(t) - \beta \geq L_{m,2,0}(t) > L_{m,2}(t)$ in which the corresponding $B_m^{*0}(t) = B_m^*(t) = Q_m(t) - \beta$.

2) $Q_m(t) > L_{m,2,0}(t) > Q_m(t) - \beta \geq L_{m,2}(t)$ in which the corresponding $B_m^{*0}(t) = L_{m,2}(t) > B_m^*(t) = Q_m(t) - \beta$.

3) $L_{m,2,0}(t) \geq Q_m(t) \geq Q_m(t) - \beta \geq L_{m,2}(t)$ in which the corresponding $B_m^{*0}(t) = Q_m(t) \geq B_m^*(t) = Q_m(t) - \beta$.

4) $Q_m(t) > L_{m,2,0}(t) > L_{m,2}(t) > Q_m(t) - \beta$ in which the corresponding $B_m^{*0}(t) = B_m^*(t) = L_{m,2}(t)$.

5) $L_{m,2,0}(t) \geq Q_m(t) > L_{m,2}(t) > Q_m(t) - \beta$ in which the corresponding $B_m^{*0}(t) = Q_m(t) > B_m^*(t) = L_{m,2}(t)$.

6) $L_{m,2,0}(t) > L_{m,2}(t) \geq Q_m(t)$ in which the corresponding $B_m^{*0}(t) = B_m^*(t) = Q_m(t)$.

Therefore, $\forall \mu_0(t) > \mu(t)$, the corresponding $B_m^{*0}(t) \geq B_m^*(t)$.

In (24), the analysis is similar to that in (23). When $B_m(t) = Q_m(t)$, Based on the discussion above, the corresponding optimal $B_m(t)$ could be obtained by (26), (27) or (28), all of which is independent of $\mu(t)$. Therefore, $\forall \mu_0(t) > \mu(t)$, the corresponding $C_m^{*0}(t) - C_m^*(t)$.

Based on above discussions, we can conclude that the optimal action policy $A_m^*(t) = B_m^*(t) + C_m^*(t)$ is a non-decreasing function with respect to $\mu(t)$.

Therefore, the proof is completed.

Let $Y(t) = \sum_{m=1}^{M} (B_m^*(t) + C_m^*(t))$. Based on Property 1, $Y(t)$ is also a non-decreasing function with respect to $\mu(t)$. Moreover, the object function of (30) is a parabolic function with respect to $Y(t)$. Therefore, the optimal $\mu(t)$ is unique, and the approximate optimal $\mu(t)$ in each slot can be obtained by the following numerical algorithm.

**Algorithm 1** The pseudocode of obtaining the approximate optimal $\mu(t)$

**Procedure**:

1: Initialize $\mu(t) = 0, X_0 = \infty, X_1 = \infty, K_e = 0, K_e = 1$;

2: while $K_e = 1$

3: Obtain $B_m^*(t)$ and $C_m^*(t)$ according to (16), $m = 1, 2, \ldots, M$

4: $X_0 = X_1$

5: $X_1 = Y(t)$

6: if $X_1 \leq X_0$ and $K_e < K^{\text{max}}$

7: $\mu(t) = \mu(t) + \text{step}$

8: if $X_1 = X_0$

9: $K_e = K_e + 1$

10: else

11: $K_e = 0$

12: end if

13: else

14: $\mu(t) = \mu(t) - \text{step}$

15: $K_e = 0$

16: end if

17: end while

Note that $\text{step}$ is the step length when searching $\mu(t)$. In addition, as discussed above, $Y(t)$ may be kept unchanged as $\mu(t)$.
is increasing. In this situation, \( K_{\text{max}} \) is introduced to avoid infinite loop. In fact, when \( \text{step} \) is small enough and \( K_{\text{max}} \) are large enough, we can obtain the optimal \( \mu(t) \). However, small \( \text{step} \) and large \( K_{\text{max}} \) result in high computing complexity.

3) Remark: The timing of the relevant events in the proposed load shaping strategy can be described as follows: at the end of slot \( t - 1 \), the utility would broadcast \( Q_m(t) \), \( \overline{P}_m(t) \) and \( \mu(t) \) to all consumers (\( k_{1,m} \) and \( k_{2,m} \) are computed offline and are assumed to be known by consumers), then, all consumers decide their respective power demands individually based on the received parameters. After the decisions are made, consumers send demand information to the utility, which may update \( \mu(t) \) for better load shaping performance based on (30). If \( \mu(t) \) is updated, then, the above interaction between the utility and consumers would happen again. Since consumers can obtain the optimal closed-form solutions and send information to the utility in smart grid environment, we assume that the computation delay and communication delay are negligible when compared with the length of slot \( t \). In other words, consumer’s decisions about power demands are made online.

IV. PERFORMANCE EVALUATION AND DISCUSSION

In this section, we evaluate the performances of the proposed load shaping strategy with Monte Carlo searching. For all simulations, we consider that \( \overline{P}(t) = 0.5 \text{ RMB/kWh}, G(t) = 10 \text{ kWh}, k_1 = -3, \) and \( k_2 = 3 \). The duration of one slot is set to be 10 minutes. There are \( M = 100 \) consumers, which is divided into two classes. The statistical properties of \( Q_m(t) \) keeps unchanged in different slots. If the \( m \)th consumer belongs to class 1, \( Q_m(t) \) follows the truncated normal distribution with mean 0.05, variance 0.02, and range (0,0.2), \( \overline{C}_m = \overline{D}_m = 0.04 \text{ kWh}, V_m = V_{m,1} \). If the \( m \)th consumer belongs to class 2, \( Q_m(t) \) follows the truncated normal distribution with mean 0.15, variance 0.07, and range (0,0.6), \( \overline{C}_m = \overline{D}_m = 0.12 \text{ kWh}, V_m = 3V_{m,1} \). Note that the effectiveness of the proposed strategy is independent of the distribution type of \( Q_m(t) \). \( \text{step} \) and \( K_{\text{max}} \) are set to be 0.05 and 10, respectively.

Fig. 5 shows the performance of load shaping over time. Obviously, the fluctuation of the load is mitigated dramatically and the mean of the load approaches the total energy from the BLG, i.e., \( G(t) \). Moreover, we can find that \( \mu(t) \) intends to be larger when there is a load valley compared with the situation with load peak. The reason can be explained as follows: larger \( \mu(t) \) can encourage consumers to draw more electricity from the grid with lower prices by charging energy, which can contribute to fill the load valley and the stored energy can be discharged when the grid electricity price is high, resulting in lower energy cost for consumers.

We evaluate the impact of the energy storage capacity on the performance of the proposed load shaping strategy, as shown in Figs. 6–8. Fig. 6 illustrates the peak/valley load performance. Obviously, as \( V_{m,1} \) is increasing, more and more peak/valley load performance improvement can be obtained by using the proposed strategy. When \( V_{m,1} > 1 \text{ kWh} \), \( \overline{C}_m \) and \( \overline{D}_m \) mainly determine the performance of the proposed load shaping strategy, hence, the performance cannot be further improved as \( V_{m,1} \) keeps on increasing. Fig. 7 shows the standard deviation of the load versus \( V_{m,1} \), which is similar to the performance of the peak/valley load. Fig. 8 shows the performance of the consumer’s cost versus \( V_{m,1} \). The consumer’s cost results from purchasing electricity from the power grid. When we do not consider the energy storage, all the inelastic electricity demand must be provided by the power grid. Based on the discussion in Sections II and III, the time-averaged cost of the \( m \)th consumer is given by

\[
\Gamma_{w,m} = \lim_{T \to +\infty} \frac{1}{T - T_0} \sum_{t = T_0}^{T} Q_m(t)f(Q_m(t)) .
\]  

When we consider energy storage, the quantity of electricity drawn from the power grid is \( A_m(t) \) for the \( m \)th consumer at slot \( t \), and the time-averaged cost is given by

\[
\Gamma_{w,m} = \lim_{T \to +\infty} \frac{1}{T - T_0} \sum_{t = T_0}^{T} A_m(t)f(A_m(t)) .
\]  

Then, the percentage of savings with respect to the \( m \)th consumer’s cost is obtained by
the grid electricity price is low and discharging energy when
poral diversity of electricity price, i.e., charging energy when
larger charge/discharge limit, which can better utilize the tem-
sumer, since class 2 consumer has a larger demand variance and
2 consumer has more cost improvement than the class 1 con-
the grid electricity price is high.

Energy cost of consumers can be reduced. Speci-
cally, the class
consumer could decide its
action policy individually without interaction with other con-
sumers, i.e., the proposed strategy could be implemented in a
distributed fashion. Moreover, the optimal decision can be made
easily since the optimal closed-form solution of the optimization
problem is given. Simulation results showed that the proposed
load shaping strategy can implement load shaping for the utility
and reduce energy cost for consumers simultaneously.

V. CONCLUSION

In this paper, we proposed a novel load shaping strategy based
on energy storage and dynamic pricing in smart grid. In the pro-
posed load shaping strategy, each consumer could decide its
charging energy when the grid electricity price is low and discharging energy when the grid electricity price is high.

As shown in Fig. 8, when energy storage is considered, the energy cost of consumers can be reduced. Specifically, the class 2 consumer has more cost improvement than the class 1 consumer, since class 2 consumer has a larger demand variance and larger charge/discharge limit, which can better utilize the temporal diversity of electricity price, i.e., charging energy when the grid electricity price is low and discharging energy when the grid electricity price is high.

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