Topological reduction of information systems

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Accepted 15 September 2004

Abstract

The main task of the present work is to apply some methods for knowledge reductions in the case of knowledge based on equivalence relations. Topological techniques are applied to construct knowledge bases via general relations. Topological structures are used to obtain discernibility matrix and discernibility function for knowledge reduction and decision making. Topology plays a significant role in quantum physics, high energy physics, and superstring theory. Reduction of attributes can be applied in the process of compactification of space time dimensions.

1. Introduction

Information play an important role in our life. The need for discovering knowledge from information increases with the forward rapid development of the recent civilization. El-Naschie [6] pointed to the uncertainty of information in quantum space-time. Reduction of information via the suggested method was applied in [1]. The process of knowledge discovery takes place along mathematical treatments, among them is the reduction of information systems. One of the most useful methods for this process is the rough set theory approach which depends on partitioning the universe of objects via equivalence relations. The works due to The Wheeler about importance of information analysis in physics can be found in [5]. Topologically we can say the mathematical treatments depend on a special class of topological spaces in which every open set is closed, in this class of spaces many recent topological class of subsets concedes with the class of open and closed sets and this in turn limits the application. The purpose of this work is to generalize the tool to a general binary relation in the place of the equivalence relations. We establish topological structures associated with this general relations and introduce a view for obtaining discernibility matrix and a method for knowledge reduction. However our approach is the same as Pawlak’s one [10,11] if the relation is an equivalence relation. The proposed topological structure we give have will open the way for using new topological results in the process of knowledge discovery for information systems whose basic classes are not pairwise disjoint. In this paper, we discuss the effect of information reduction and specialize of information classification (resolution) on the accuracy of approximation. In physics, this appear clearly for example see [9, pp. 8–9] . Relations of topology and physics have been appeared in [7,8,12].

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2. Basic concepts

2.1. Knowledge base

The approximation space \([10,11]\) is a pair of \((U, R)\), where \(U\) is a non-empty finite set of objects (states, patients, digits, cars, ... etc) called a universe and \(R\) is an equivalence relation over \(U\) which makes a partition for \(U\), i.e. a family \(C = \{X_1, X_2, X_3, \ldots, X_n\}\) such that \(X_i \subseteq U, X_i \neq \emptyset, X_i \cap X_j = \emptyset\) for \(i \neq j, i, j = 1, 2, 3, \ldots, n\) and \(\bigcup X_i = U\), the class \(C\) is called the knowledge base of \((U, R)\).

The universe \(U\) of objects with relation \(R\) play an important role in converting data into knowledge which use \(R\) as a tool of a mathematical model for dealing with members and subsets of \(U\). Thus we can say that \(R\) changes \(U\) from just being a set to a mathematical model.

The following is an example of an approximation space and its associated knowledge base.

**Example 2.1.** Let \(U = \{1,2,3,4,5,6,7,8,9\}\) and relation \(R\) which partition \(U\) into two classes even and odd numbers, then knowledge base is:

\[
U/R = \{\{1,3,5,7,9\}, \{2,4,6,8\}\}, \quad X_1 = \{1,3,5,7,9\}, \quad X_2 = \{2,4,6,8\}, \quad X_1 \cap X_2 = \emptyset, \quad X_1 \cup X_2 = U
\]

2.2. Information systems

An information system \(S [3,4]\) can be defined as \(S = (U, A, p, V)\), where \(U\) is a non-empty finite set of objects called a universe and \(A\) is a non-empty finite set of attributes (features, variables, characteristic conditions, leds, ... etc).

\[
S = (U, A, p, V)
\]

Any subset \(X \subseteq U\) will be called a concept or a category in \(U\). Each attribute \(a \in A\) can be viewed as a function that maps elements of \(U\) into a set \(V_a\), where the set \(V_a\) is called the value of set of the attribute \(a\).

\[
p : U \times A \rightarrow V_a
\]

In the following, we reformulated an example for information system using example given in [10].

**Example 2.2.** The seven segment display which gives us the numbers from 0 to 9 as shown below:

![Seven Segment Display Diagram](image-url)

The structure of each digit is shown in the information system Table 1:

The objects are digits 0,1,2,3,...,9 and attributes are leds \(a,b,c,d,e,f, g\) of display. We find that \(V_a = V_b = V_c = V_d = V_e = V_f = V_g = \{0,1\} \quad \& \quad 1 \equiv \text{ON}, \quad 0 \equiv \text{OFF}\). and \(a(5) = 1, \quad b(5) = 0, \quad g(9) = 1\).

<table>
<thead>
<tr>
<th>(U:A)</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
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<th>(f)</th>
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</tbody>
</table>
2.3. Mathematical model in information system

To extract knowledge from information system, it is necessary to find mathematical models using the given data.

2.3.1. Indiscernibility relation

For a subset \( B \subseteq A \), the indiscernibility relation [3,11] is:

\[
\text{IND}(B) = \{(x, y) \in U^2 / \forall a \in B, a(x) = a(y)\}
\]

Which groups the objects that possess the same features (the values of the attributes) with respect to \( B \), i.e. the objects that are indiscernible.

The \( \text{IND}(B) \) is an equivalence relation that partitions \( U \) and divides it into equivalence classes, and

\[
\text{IND}(B) = \bigcap_{a \in B} \text{IND} \{ \{a\} \}, \quad [10]
\]

An equivalence class of \( \text{IND}(B) \) is denoted by:

\[
[x]_B = \{ y \in U / (x, y) \in \text{IND}(B) \}
\]

Thus the family of all equivalence classes with respect to \( B \) is denoted by \( U/\text{IND}(B) \). Where:

\[
U/\text{IND}(B) = \{ [x]_B : \forall x \in U \}
\]

Example 2.3. Continued from Example 2.2. Given a set of attributes \( B = \{a, b, c\} \). Then from Table 1, we get:

\[
\text{IND}(B) = \{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9), (0, 3), (3, 0), (0, 7), (7, 0),
\]

\[
(0, 8), (8, 0), (0, 9), (9, 0), (1, 4), (4, 1), (3, 7), (7, 3), (3, 8), (8, 3), (3, 9), (9, 3), (5, 6), (6, 5),
\]

\[
(7, 8), (8, 7), (7, 9), (9, 7), (8, 9), (9, 8)\}
\]

\[
[0]_B = \{0, 3, 7, 8, 9\}, [1]_B = \{1, 4\}, [2]_B = \{2\}, [5]_B = \{5, 6\}
\]

And \( U/\text{IND}(B) = \{ \{0, 3, 7, 8, 9\}, \{1, 4\}, \{2\}, \{5, 6\} \} \)

2.3.2. Discernibility matrices

An information system \( S \) defines a matrix \( M_A \) called discernibility matrices. Each entry \( M_A(x, y) \subseteq A \) consists of a set of attributes that can be used to discern between objects \( x, y \in U \) [3]:

\[
M_A(x, y) = \{ a \in A / a(x) \neq a(y) \}
\]

\( M_A \) is an \( |U| \times |U| \) matrix, in [4] the discernibility matrix has the form:

\[
M_{ij} = \{ a \in A / a(x_i) \neq a(x_j) \}, \quad i, j \in [1, n], \quad n = |U|
\]

Example 2.4. Continued from Example 2.2. The discernibility matrix of Table 1 is shown in Table 2, in this table we neglect the row of object 9 and the column of object 0 for simplicity, not only for objects 0 and 9 but also for any two different objects, one from rows and the other from columns:

<table>
<thead>
<tr>
<th>Table 2</th>
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<tbody>
<tr>
<td>UxU</td>
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<tr>
<td>0 1 2 3 4 5 6 7 8 9</td>
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</table>
2.3.3. Discernibility function

The discernibility function is a Boolean function representation of the discernibility matrix [4].

\[ f_A(a_1, a_2, \ldots, a_n) = \wedge \{ \vee M_{ij} : 1 \leq j < i \leq n, M_{ij} \neq \phi \} \]

which use the product of sums (P.O.S.), \( n = |U| \).

This expression is used for reduction of attributes.

In [3]

\[ f_A(x) = \wedge_{a \in U} \{ \forall a : a \in M_A(x, y) \neq \phi \} \]

which is used for decision making.

2.4. Reduction of knowledge

Reduction of knowledge is divided to reduction of objects and reduction of attributes.

2.4.1. Reduction of objects

We can reduce the objects by using the discernibility matrix, where if we find an empty set (\( \phi \)) between two objects in discernibility matrix, then we can eliminate one of them.

2.4.2. Reduction of attributes

Let \( B \subseteq A, a \in B \), then \( a \) is superfluous attributes [10] in \( B \) if:

\[ U/\text{IND}(B) = U/\text{IND}(B - \{a\}) \]

The set \( M \) is called a minimal reduct of \( B \) iff:

(i) \( U/\text{IND}(M) = U/\text{IND}(B) \),

(ii) \( U/\text{IND}(M) \neq U/\text{IND}(M - \{a\}), \forall a \in M \)

which is shown in the Fig. 1 as an example:

\[ B = \{a, b, c, d\} \]

Then \( \text{RED}(B) = \{\{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{c,d\}, \{b,c,d\}\} \)

But minimal reduct is \( M = \{\{a,b\}, \{c,d\}\} \).

The core is the set of all characteristic of knowledge, where can not be eliminated from knowledge at reduct of knowledge.

\[ \text{CORE}(B) = \cap \text{RED}(B) = \{a, b\} \cap \{c, d\} = \phi \]
Example 2.5. Continued from Example 2.2. For object reduction, there is not reduction of objects because we find that no empty set between any two objects in discernibility matrix. For attribute reduction we get:

\[
\begin{align*}
U/\text{IND}(a) &= \{\{0, 2, 3, 5, 6, 7, 8, 9\}, \{1, 4\}\} \\
U/\text{IND}(b) &= \{\{0, 1, 2, 3, 4, 7, 8, 9\}, \{5, 6\}\} \\
U/\text{IND}(c) &= \{\{0, 1, 3, 4, 5, 6, 7, 8, 9\}, \{2\}\} \\
U/\text{IND}(d) &= \{\{0, 2, 3, 5, 6, 8, 9\}, \{1, 4, 7\}\} \\
U/\text{IND}(e) &= \{\{0, 2, 6, 8\}, \{1, 3, 4, 5, 7, 9\}\} \\
U/\text{IND}(f) &= \{\{0, 4, 5, 6, 8, 9\}, \{1, 2, 3, 7\}\} \\
U/\text{IND}(g) &= \{\{0, 1, 7\}, \{2, 3, 4, 5, 6, 8, 9\}\}
\end{align*}
\]

\[
U/\text{IND}(A) = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}\}
\]

We find that:

\[
\begin{align*}
U/\text{IND}(A) &\neq U/\text{IND}(A - \{a\}), \\
U/\text{IND}(A) &\neq U/\text{IND}(A - \{b\}), \\
U/\text{IND}(A) &= U/\text{IND}(A - \{c\}), \\
U/\text{IND}(A) &= U/\text{IND}(A - \{d\}), \\
U/\text{IND}(A) &\neq U/\text{IND}(A - \{e\}), \\
U/\text{IND}(A) &\neq U/\text{IND}(A - \{f\}), \\
U/\text{IND}(A) &\neq U/\text{IND}(A - \{g\}).
\end{align*}
\]

Then attributes \(c\) and \(d\) are superfluous attributes and we get:

\[
U/\text{IND}(A) = U/\text{IND}(A - \{c, d\})
\]

Then the reduct of \(A\) is:

\[
\text{RED}(A) = \{\{a, b, d, e, f, g\}, \{a, b, c, e, f, g\}, \{a, b, e, f, g\}\}
\]

The minimal reduct of \(A\) is

\[
M = \{a, b, e, f, g\}
\]

The core of \(A\) is:

\[
\text{CORE}(A) = \{a, b, e, f, g\}
\]

That information system can be reduced into the following Table 3:

\[
\begin{array}{cccccc}
U: A & a & b & e & f & g \\
0 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
2 & 1 & 1 & 1 & 0 & 1 \\
3 & 1 & 1 & 0 & 0 & 1 \\
4 & 0 & 1 & 0 & 1 & 1 \\
5 & 1 & 0 & 0 & 1 & 1 \\
6 & 1 & 0 & 1 & 1 & 1 \\
7 & 1 & 1 & 0 & 0 & 0 \\
8 & 1 & 1 & 1 & 1 & 1 \\
9 & 1 & 1 & 0 & 1 & 1
\end{array}
\]

Note: we can get the minimal reduct in one step by using discernibility function as shown in the following example.
Example 2.6. We can get the same result in Example 2.5 by using discernibility function as follows:

\[ f_d(a,b,c,d,e,f,g) = \{a + d + e + f\}.\{c + f + g\}.\{e + f + g\}.\{a + d + e + g\}.\{b + e + g\}.\{b + g\}.\{d + e + f\} \]

Then RED(A) = \( f_d(a,b,c,d,e,f,g) = \{a,b,e,f,g\} \) which is the minimal reduct of A, and CORE(A) = \{a,b,e,f,g\}.

2.5. Decision making

In this section we are going to illustrate by example the decision making which is used to give us a decision table and find all minimal decision algorithms associated with the table. Discernibility matrix and discernibility function are used for decision making.

Example 2.7. Continued from Example 2.5 and Table 3 we can make the following discernibility matrix in Table 4.

And by using the discernibility function we get to the decision table which is shown in Table 5.

From Table 4, we take the attributes which are found in row and column for each object.

\[ f_d(0) = \{a + e + f\}.\{f + g\}.\{e + f + g\}.\{a + e + g\}.\{b + e + g\}.\{b + g\}.\{e + f\}.\{g\}.\{e + g\} = \{g\}.\{e + f\} = g.e + g.f \]

\[ f_d(1) = \{a + e + f\}.\{a + e + g\}.\{a + g\}.\{f + g\}.\{a + b + f + g\}.\{a + b + e + f + g\}.\{a\}.\{a + e + f + g\}.\{a + f + g\} = \{a\}.\{f + g\} = a.f + a.g \]

\[ f_d(2) = \{f + g\}.\{a + e + g\}.\{e\}.\{a + e + f\}.\{b + e + f\}.\{b + f\}.\{e + g\}.\{f\}.\{e + f\} = e.f \]

\[ f_d(3) = \{e + f + g\}.\{a + g\}.\{e\}.\{a + f\}.\{b + f\}.\{b + e + f\}.\{g\}.\{e + f\}.\{f\} = e.g.f \]

\[ f_d(4) = \{a + e + g\}.\{f + g\}.\{a + e + f\}.\{a + f\}.\{a + b\}.\{a + b + e\}.\{a + f + g\}.\{a + e\}.\{a\} = \{f + g\}.\{a\} = a.f + a.g \]

\[ f_d(5) = \{b + e + g\}.\{a + b + f + g\}.\{b + e + f\}.\{b + f\}.\{a + b\}.\{e\}.\{b + f + g\}.\{b + e\}.\{b\} = e.b \]

\[ f_d(6) = \{b + g\}.\{a + b + e + f + g\}.\{b + f\}.\{b + e + f\}.\{a + b + e\}.\{e\}.\{b + e + f + g\}.\{b + e\} = e.b \]

\[ f_d(7) = \{e + f\}.\{a\}.\{e + g\}.\{g\}.\{a + f + g\}.\{b + f + g\}.\{b + e + f + g\}.\{e + f + g\}.\{f + g\} = a.g.\{e + f\} = a.g.e + a.g.f \]

\[ f_d(8) = \{g\}.\{a + e + f + g\}.\{f\}.\{e + f\}.\{a + e\}.\{b + e\}.\{b\}.\{e + f + g\}.\{e\} = b.e.f.g \]

\[ f_d(9) = \{e + g\}.\{a + f + g\}.\{e + f\}.\{f\}.\{a\}.\{b + e\}.\{f + g\}.\{e\} = a.b.e.f \]

<table>
<thead>
<tr>
<th>( U x U )</th>
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<th>3</th>
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<tbody>
<tr>
<td>0</td>
<td>a,e,f</td>
<td>f,g</td>
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</table>
Then we get:

\[
\begin{align*}
&g_0 e_1 \lor g_0 f_1 \rightarrow 0 \\
&a_0 f_0 \land a_0 g_0 \rightarrow 1 \\
&e_1 f_0 \rightarrow 2 \\
&g_0 g_1 f_0 \rightarrow 3 \\
&a_0 f_1 \land a_0 g_1 \rightarrow 4 \\
&g_0 b_0 \rightarrow 5 \\
&e_1 b_0 \rightarrow 6 \\
&a_1 g_0 e_0 \lor a_1 g_0 f_0 \rightarrow 7 \\
&b_1 e_1 f_1 g_1 \rightarrow 8 \\
&a_1 b_1 e_0 f_1 \rightarrow 9
\end{align*}
\]

From the above, we can get the decision making for objects 0, 1, 2, ..., 9. As an example the digit 1 appears iff \( V_A(1) = 0 \) & \( V_f(1) = 0 \) or \( V_A(1) = 0 \) & \( V_g(1) = 0 \), and the digit 6 appears iff \( V_A(6) = 1 \) \& \( V_b(6) = 0 \), and so on.

Which is shown in many tables, one of them is shown in Table 5:

<table>
<thead>
<tr>
<th>Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U:A )</td>
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<tr>
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3. Reduction of knowledge by topology

The condition of equivalence relation in the approximation space limits the range of applications. The purpose of this article is to use a generalized approximation space \((U, R)\) based on a general binary relation. Consider a binary relation as a general relation and by using the class of after sets which is formed by this relation \( R \) as a subbase \( S \) for topology \( \tau \) on \( U \).

If \( U \) is a finite universe and \( R \) is a binary relation on \( U \), then:

An after set: can be defined as \( xR = \{y: xRy\} \)

To get the topology on \( U \) needed for the construction of rough set we start by the class \( S_R = \{xR: x \in U\} \) as a subbase of \( \tau \), anda subset \( S_x \in S \) where \( S_x = \{G : xR : x \in G\} \). Each attribute in Pawlak space make an equivalence relation (indiscernibility relation).

Definition 3.1. Let \( U \) be a nonempty set of objects and \( R \) be a class of general binary relations on \( U \), \( R = \{r_1, r_2, r_3, \ldots, r_n\} \), then \((U, R)\) is called a generalized approximation space.

Definition 3.2. Let \( U \) be a nonempty set of objects and \( R \) be a class of general binary relations on \( U \), each \( r \in R \) yields a class \( S_r = \{xR: x \in U\} \) which called a subknowledge base.

So that we use the topological concepts (subbase and base of topology [2]) for reduction.

Definition 3.3. Let \( R \) be a class of general relations, Then a subbase for \( \tau \) for all \( R \) is

\( S_R = \cup_{r \in R} S_r \)
Definition 3.4. Let $R$ be a class of general relations, then a base for $\tau$ for all $R$ is:

$$\beta_R = \cap_{x \in M} S_x, \quad \forall x \in U$$

Definition 3.5. Let $P \subseteq R$, $r \in P$, where $R$ be a class of general relations, then $r$ is superfluous relation in $P$ if:

$$\beta_P = \beta_{(P-\{r\})}$$

The set $M$ is called a minimal reduct of $P$ iff:

(i) $\beta_M = \beta_P$,

(ii) $\beta_M \neq \beta_{(P-\{r\})}, \quad \forall r \in M.$

Example 3.1. Let $U = \{1,2,3,4,5\}$, and

$$S_r = \{\{1,2\}, \{2,3,4\}, \{4,5\}\}$$

$$S_p = \{\{1,2,3\}, \{3,4\}, \{5\}\}$$

$$S_q = \{\{1,2\}, \{3,4\}, \{4,5\}\}$$

Then we get: the subbase for topology for all relations is:

$$S_r = \{\{1,2\}, \{2,3,4\}, \{4,5\}, \{1,2,3\}, \{3,4\}, \{5\}\}$$

And the base for topology for all relations is:

$$\beta_R = \{\{1,2\}, \{2,3,4\}, \{4,5\}\},$$

which don’t make a partition for reduction we get:

$$S_{(R-\{r\})} = \{\{1,2\}, \{3,4\}, \{5\}\}$$

$$\beta_{(R-\{r\})} = \{\{1,2\}, \{2\}, \{3,4\}, \{4,5\}\}$$

$$S_{(R-\{p\})} = \{\{1,2\}, \{2,3,4\}, \{4,5\}\}$$

$$\beta_{(R-\{p\})} = \{\{1,2\}, \{2\}, \{3,4\}, \{4,5\}\}$$

$$S_{(R-\{q\})} = \{\{1,2\}, \{2,3,4\}, \{4,5\}\}$$

$$\beta_{(R-\{q\})} = \{\{1,2\}, \{2\}, \{3,4\}, \{5\}\} = \beta_R$$

we find that $q$ is only superfluous relation in $R$.

Then $\text{RED}(R) = \{r,p\}$,

And $\text{CORE}(R) = \{r,p\}$

Definition 3.6. Let $R$ be a class of general relations, which make subbases on an universe $U$. Then discernibility matrix can be defined as:

$$M_R(x,y) = \{r \in R/\{x,y\} \not\in G, \ G \in S_r\}, \quad x,y \in U$$

Other definition as:

$$M_R(x,y) = \{r \in R/xr \neq yr\}, \quad x,y \in U$$

Example 3.2. Let $U = \{1,2,3,4,5\}$, and $S_r = \{\{1,2\}, \{2,3,4\}, \{4,5\}\}$, $S_p = \{\{1,2,3\}, \{3,4\}, \{5\}\}$, $S_q = \{\{1,2\}, \{3,4\}, \{4,5\}\}$.

Then the discernibility matrix is: Table 6

<table>
<thead>
<tr>
<th>$UXU$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>–</td>
<td>$r,q$</td>
<td>$r,p,q$</td>
<td>$r,p,q$</td>
</tr>
<tr>
<td>2</td>
<td>$q$</td>
<td>$p,q$</td>
<td>$r,p,q$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>–</td>
<td>$r,p,q$</td>
<td>$p$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Definition 3.7. Let $R$ be a class of general relations, which make subbases on an universe $U$. Then discernibility function can be defined as: For each $x,y \in U$, then

$$f_R(r_1,r_2,\ldots,r_n) = \land \{\forall r \in M_R(x,y) \neq \phi\}$$

Which used for reduction of general relations as shown in the following example.

Example 3.3. Continued from Example 3.2. The discernibility function is:

$$f_R(r,p,q) =\{r + q\} \cdot \{r + p + q\} \cdot \{r + p + q\} \cdot \{p\} = p.q$$

Then the reduction is $\text{RED}(R) = \{p,q\}$, and $\text{CORE}(R) = \{p,q\}$.

Example 3.4. Continued from Example 3.2. After eliminating the superfluous objects which its discernibility matrix equals $\phi$. Then the new discernibility matrix has the form in Tables 7–10 as shown below:

The discernibility function respectively for each above table is:

(1) $f_R(r,p,q) =\{r + q\} \cdot \{r + p + q\} \cdot \{r + p + q\} = r + q$
(2) $f_R(r,p,q) =\{p + q\} \cdot \{r + p + q\} \cdot \{p\} = p$
(3) $f_R(r,p,q) =\{r + p + q\} \cdot \{r + p + q\} \cdot \{p\} = p$
(4) $f_R(r,p,q) =\{q\} \cdot \{r + p + q\} \cdot \{r + p + q\} = q$

Then the reduction for Table 7 has $\text{RED}(R) = \{\{r\}, \{q\}\}$ the reduction for Table 8 has $\text{RED}(R) = \{p\}$, the reduction for Table 9 has $\text{RED}(R) = \{p\}$, and the reduction for Table 10 has $\text{RED}(R) = \{q\}$.

We find that a different reductions according to what superfluous object is eliminated.

Definition 3.8. Let $R$ be a class of general relations, which make subbases on an universe $U$. Then discernibility function can be defined as: For each $x \in U$ then:

$$f_R(x) = \land_{y \in U}\{\forall r \in M_R(x,y), x \neq y\}$$

Which used for decision making after reduction of relations as shown in the following example.

<table>
<thead>
<tr>
<th>Table 7</th>
<th>$U \times U$</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>r.q</td>
<td>r.p.q</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>r.p.q</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 8</th>
<th>$U \times U$</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>p.q</td>
<td>r.p.q</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>p</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 9</th>
<th>$U \times U$</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>r.p.q</td>
<td>r.p.q</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>p</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 10</th>
<th>$U \times U$</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>q</td>
<td>r.p.q</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>r.p.q</td>
</tr>
</tbody>
</table>
Example 3.5. Continued from Example 3.3. The discernibility matrix after reduction of relations can be viewed as in Table 11.

Then we can find the decision for each object as follows:

\[ f_R(1) = \emptyset, \quad f_R(2) = \emptyset, \quad f_R(3) = \emptyset, \quad f_R(4) = \emptyset, \quad f_R(5) = \{p + q\}, \{p + q\}, \{p\}, \{p\} = p \]

From this example we find that: Only object 5 can be discernible from all objects by relation \( p \).

4. Summary

In this work we explain the reduction of information systems in Pawlak’s approach [10] and give examples for this methods using equivalence relations. To enlarge the range of application we generalize the approach to the setting of general binary relation. We initiate an algorithm for knowledge reduction using classes of topological structure wider than the associated ones in Pawlak’s approach.

5. Conclusion

The approach we suggest in this work is a method for topologizing any information system, that is, to embed it in a topological space. This embedding help in using topological knowledge reduction. In the general class of topological structures it is possible to use the new classes of near open sets such as semiopen sets, preopen sets, . . . etc, and the topological concepts based on them. For example we can construct several methods for distinguishing individuals by the use of separation properties and the various types of boundary regions. In our future work we aim to start by a method for constructing discernibility matrix and use it in the reduction process and the judgment for our method is to find the conditions under which the reduction using discernibility function concedes with that obtained by finding minimal reduct as in Definition 3.5.

References