Switch Synchronizing Delayed Feedback Control for Piecewise Linear Systems

Yu Toyosaki
Department of Information Science and Intelligent Systems, Faculty of Engineering, Tokushima University, Japan
Email: toyo@is.tokushima-u.ac.jp

Tetsushi Ueta
Center for Advanced Information Technology, Tokushima University, Japan
Email: tetsushi@is.tokushima-u.ac.jp

Takuji Kousaka
Department of Electronic and Electrical Engineering, Faculty of Engineering, Fukuyama University, Japan
Email: takoji@fuee.fukuyama-u.ac.jp

Abstract—This paper proposes Switch Synchronizing Delayed Feedback Control (abbr. SSDFC) in the piecewise linear systems. This control method weeds out the unsuitable control input from the different piecewise continuous functions. As an illustrated example, we then apply this general methodology to a simple circuit containing state-period dependent switch. The unstable periodic orbit is stabilized effectively in the computer simulations. Finally, the performance of SSDFC is compared with Delayed Feedback Control (DFC).

I. INTRODUCTION

For many years, a great deal of effort has been made in the piecewise linear systems[1]. We can find the piecewise linear systems in many area such as robotics, biology, electrical systems, and so on. The chaotic attractors are also ubiquitously observed in these systems. Therefore, it is also important to stabilize the unstable orbit embedded in the chaotic attractor[2]. This directly indicates the controlling chaos is one of the most effective engineering application of chaotic attractor from the practical view point.

In previous studies, the controlling chaos of the piecewise systems have been developed extensively both numerically and experimentally. Saito[3] has showed a control method for the occasional proportional feedback method in the piecewise linear system. Poddar[4][5] proposed the control method of the converters. Although this kind of methods work effectively, they must construct the discrete mapping by utilizing the periodicity of the switching action. Moreover, the control schemes require the information about the unstable periodic orbits and their stability.

Pyragas proposed the Delayed Feedback Control (DFC) method for the controlling chaos[6]. The advantage of this method is that DFC does not require the information of the analytical models. However, little attention has been paid for the controlling chaos in the piecewise linear systems. Ref. [7] proposed a time-delayed impulsive feedback approach to stabilize the unstable periodic orbits in the hybrid chaotic systems. On the other hand, the unstable periodic orbit for the PWM current-mode H-bridge inverter is also stabilized by using the conventional time-delayed feedback control (TDFC) method[8]. This is an continuous control method via synchronization with a delayed signal, and the control input is proportional to the difference of the current state and the state at some earlier time. However, the piecewise linear systems are defined some subsystems described by the differential equations. When the current subsystem is different from the subsystem with earlier signal, this method will be unable to work effectively.

In this paper, DFC is developed into the Switch Synchronizing Delayed Feedback Control (abbr. SSDFC) method.

II. FORMULATION OF SWITCH SYNCHRONIZING DELAYED FEEDBACK CONTROL

Let us consider the chaotic piecewise system written in the form:

\[ \frac{dx}{dt} = f(t, x), \]  

where \( x \in \mathbb{R}^n \), \( f \) is described by the piecewise linear function. Assume that \( x(t) = \varphi(t, x(0)) \) is a solution of Eq. (1). In general, a UPO embedded in the chaotic attractor can be written as:

\[ x_0 = \varphi(0, x_0(0)) = \varphi(\tau, x_0), \]  

where \( \tau \) is the period of this UPO. The control input \( u(t) \) of the conventional DFC with Eq. (1) is added to the control parameter as a perturbation.

\[ u(t) = K D(t) = K(x(t - \tau) - x(t)). \]  

The control input \( u(t) \) converges to 0 by choosing a suitable gain \( K \), thus the chaotic attractor stabilizes to the \( \tau \)-periodic orbit. The system becomes an infinite dimensional system, therefore there is no theoretical evidence to ensure global stability of this control method. In fact, because the gain \( K \) (\( n \times n \) matrix) can be chosen as a diagonal matrix, the availability of DFC is confirmed easily from computer simulations.

On the other hand, the control input \( u(k) \) of the SSDFC with Eq. (1) is described as follows:

\[ u(t) = \begin{cases} 
K D(t) & \text{if } S(x(t)) = S(x(t - \tau)) \\
0 & \text{otherwise,}
\end{cases} \]  

where \( S(x(t)) \) shows a state of a switch with \( x(t) \). SSDFC is a method that the control input is generated only when a
III. SSDFC FOR A SIMPLE CHAOTIC SYSTEM

As an example, we apply SSDFC to a chaotic 1-dimensional piecewise linear system in the computer simulation.

A. A simple chaotic system

Figure 2 (a) shows a simple chaotic circuit which contains a state-period dependent switch.

Now we explain the dynamics of the circuit. Assume that the analog switch turned into the position A. The capacitor C charges by the source E so the voltage of capacitor $v_r$ rises. If the $v$ reaches the reference voltage $v_r$, the comparator sends a signal to the RS-FF, and the switch is replaced to the position B. Then the capacitor discharges so $v$ falls. On the other hand, the $T$ periodical reset pulse is impressed on the RS-FF. If the RS-FF receives the reset pulse, the switch is turned into the position A. After normalization as $t = t/RC$, we have a switched 1-dimensional ODE:

\[
\begin{align*}
\text{SW : A} & \quad \frac{dv}{dt} = -v + E, \\
\text{SW : B} & \quad \frac{dv}{dt} = -v.
\end{align*}
\]

When the parameters are set as follows, we can observe the chaotic attractor[10]. $v_r = 2.5[V]$, $C = 1[\mu F]$, $R = 51[k\Omega]$, $E = 3.0[V]$, $T = 0.606$. Figure 2(b) shows the behavior of $v$ in the computer simulation.

In the following we try to stabilize the UPO with $\tau = 1.212 = 2T$. Note that, $T$ is the period of the reset pulse added to the RS-FF in Fig. 2(a).

B. Apply SSDFC to the chaotic system

Now we apply SSDFC to this system. The control input is added to point c and d in Fig. 2 (a), and start the control with $t = 1.212 = 2T$. The SSDFC for Eq. (5) is described as follows:

\[
\begin{align*}
\text{SW : A} & \quad \frac{dv}{dt} = -v + E + u(t), \\
\text{SW : B} & \quad \frac{dv}{dt} = -v + u(t),
\end{align*}
\]

where $u(t)$ is a control input.

\[
u(t) = \begin{cases} K(v(t-\tau) - v(t)), & \text{if } S(v(t-\tau)) = S(v(t)) \\ 0, & \text{otherwise} \end{cases}
\]

where $S(v(t))$ means $S(x(t))$ of Eq. (4).

Figure 3 (a) shows the result of the computer simulation. In this control, we define a feedback gain as $K = 0.8$ and the current state $v(2T) = 2.0[V]$. $v(t)$ is stabilized to a $2T$-periodic orbit and the control input $u(t)$ converges to 0. $u(t)$ is removed when a current position of the switch is not the same as a position of the switch with $t = 2T$. Figure 3 (b) shows the results of the computer simulation applying DFC to this system with same parameters. These figures indicate that SSDFC can stabilize the UPO more quickly than DFC.
C. Comparison with SSDFC and DFC

Firstly, we compute the basin of attractions of these orbits to examine a controllable gain. In the following discussion, we start the control with $t = 1.212 = 2T$. Figure 4 (a) shows the basin of attraction with SSDFC. The horizontal axis shows the gain $K$, and the vertical axis shows the initial value $v(2T)$, respectively. The horizontal resolution of the figure is 0.05, and the vertical one is 0.01. The white region means that the control is failed. This figure indicates that the controllable gain regardless of the initial values with $0.25 \leq K \leq 3.4$. Figure 4(b) shows the basin of attraction with DFC. The controllable gain is given by $0.25 \leq K \leq 0.9$. These results indicate that SSDFC removes the unsuitable control input.

Secondly we compare the consumption energy of SSDFC and DFC. We define the integration value of $u(t)$ as the consumption energy $U$.

$$U = \int_{t_1}^{t_2} u(t)dt,$$

where $t_1 = 1.212 = 2T$, $t_2 = 150.0$. Figure 5 shows an example of the consumption energy $U(t)$ for the initial value $v(2T)$. Note that we compare the consumption energy of SSDFC and DFC at common controllable gain range $0.25 \leq K \leq 0.9$ shown in Fig. 4. These figures indicate that SSDFC can stabilize a UPO by less consumption energy compared with DFC.

Finally, SSDFC is compared with DFC in term of energy saving. For the minimum value $K_{min} = 0.25$ of $K$, calculate the consumption energy with 114 initial points in the interval $1.37 \leq v(2T) \leq 2.5$. After that, increase $K$ slightly, say by 0.05. As a result, SSDFC realizes the energy saving of approximately 34 percent than DFC.
Fig. 4. The basin of attraction. (a) SSDFC. (b) DFC for this system.

IV. CONCLUSION

In this paper, we have proposed the SSDFC for the piecewise linear systems. SSDFC is a method to remove a control input generated by the different piecewise continuous functions. We applied SSDFC to 1-dimensional piecewise linear system in the computer simulation, and confirmed the availability and the high-efficiency of SSDFC. As a future work, we try to realize the SSDFC in the electric circuit. We have already realized the DFC with the Digital Signal Processor(DSP)[9]. By using the multiplexer, we consider it easy to realize.

REFERENCES