Limit Cycle of Induction Motor Drive and Its Control

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SUMMARY Limit cycle oscillations of rotor speed are substantially caused by inverter’s dead time, when an induction motor (IM) drive operates in low frequency condition. In this paper, without any hardware modification, discontinuous PWM (DPWM3) modulate strategy possibly controls the unfavorable rotor speed limit cycle under no load operation condition. Simulated results are presented to demonstrate the effectiveness.

key words: IM drive, limit cycle, DPWM3, control of limit cycle

1. Introduction

With the introduction of solid-state inverters, three phase IM drive systems fed by V/f PWM voltage source inverter (VSI) become popular. The great majority of operating variable speed drives is of this type today. They have often experienced unexpected sustained oscillations in low speed range and in light load conditions [1]. Although these oscillations have been observed and analyzed [2]–[5], few papers deal with the essence of these oscillations and improving stability of the V/f controlled IM drive system.

In this paper, a nonlinear mathematical model of IM is set up, allowing for the effect of main magnetic circuit saturation. The study shows the oscillatory states depend on inverter’s dead time. In order to clarify the inherent relationship between sustained oscillations and dead time, the bifurcation theory is introduced to explain the mechanism of oscillations. The study indicates dead time can cause IM drive to appear Hopf bifurcation, and the rotor speed motion has limit cycles. Finally, preserving the merit of simplicity in IM drive, DPWM3 modulate strategy is proposed to control the limit cycles of rotor speed without any additional hardware modification. Simulated results demonstrate the effectiveness.

2. Mathematical Model of IM Drive

2.1 Model of IM

VSI is widely utilized in IM drive, as shown in Fig. 1. VSI generates output voltage with controllable magnitude and frequency and drives an IM. The magnetization curve of IM is assumed to have the characteristic described by the following formula:

\[ I_m = K_1 \lambda_m + K_3 \lambda_m^3 + K_5 \lambda_m^5, \]

where \( I_m \) denotes magnetizing current and \( \lambda_m \) magnetizing linkage. We can fit experimental datum shown in Fig. 2 and obtain coefficients \( K_1, K_3, \) and \( K_5. \)

The voltage matrix equation of IM, fixed in stator d-q reference frame, is given by [5]:

\[
\begin{align*}
\begin{bmatrix}
\dot{I}_1 \\
\omega_r
\end{bmatrix} &= -\begin{bmatrix}
Z_2^{-1} & Z_1
\end{bmatrix} \begin{bmatrix}
I_1 \\
\lambda
\end{bmatrix} + \begin{bmatrix}
Z_2^{-1}U
\end{bmatrix}, \\
p\omega_r &= n(T_e - T_l)/J, \\
T_e &= \frac{3}{2}n(I_{qs}\lambda_{md} - I_{ds}\lambda_{mq}),
\end{align*}
\]

Fig. 1 Circuit diagram of PWM-VSI.

Fig. 2 Magnetizing curve of IM.
where \( n \) denotes number of pole pairs, \( T_e \) electromagnetic torque, \( J \) moment of inertia of motor, \( T_l \) load torque, \( r \) resistor, \( L_r \) leakage impedance, \( \omega_r \) rotor electrical angular velocity, \( \lambda_{dn} \) d axial magnetizing linkage, and \( \lambda_{mq} \) q axial magnetizing linkage, respectively. Subscripts 1 and 2 correspond to stator and rotor. \( p \) depicts differential operator \( \frac{d}{dt} \).

Moreover,

\[
A = 2K_3^2\lambda_{mq}^2 + 4K_5^2\lambda_{mq}^2 + 4K_3\lambda_{mq}^2\lambda_{md}^2 + K_3^2\lambda_{md}^2 + K_5\lambda_{md}^2, \\
B = 2K_3\lambda_{mq} + 4K_5\lambda_{md}\lambda_{mq}^2 + 4K_5^2\lambda_{md}^2\lambda_{mq}, \\
C = 2K_3\lambda_{md}^2 + 4K_3\lambda_{md}^2 + 4K_5\lambda_{md}^2 + K_3\lambda_{md}^2 + K_5\lambda_{md}^2, \\
Y = K_3\lambda_{md}^2, \\
\lambda_{md}^2 = \lambda_{md}^2 + 2\lambda_{md}\lambda_{mq} + \lambda_{mq}^2, \\
\lambda_{md}^2 = \lambda_{md}^2 + 2\lambda_{md}\lambda_{mq} + \lambda_{mq}^2,
\]

are satisfied.

### 2.2 Model of Inverter

First, switch function concept is introduced to set up the model of VSI inverter. Sinusoidal modulate wave is compared with triangular carrier wave and the intersections define switch instants. Then switch functions \( S_1(\omega t) \), \( S_3(\omega t) \), and \( S_5(\omega t) \) are obtained and shown in Fig. 3. The phase voltages of inverter (output phase of inverter with respect to the fictional center point \( o \) of the dc supply) are figured by \( U_{ao} \), \( U_{bo} \) and \( U_{co} \), and \( U_D \) denotes the DC-link voltage. Then, the phase voltages are derived as follows [6]:

\[
\begin{align*}
U_{ao}(\omega t) &= U_p S_1(\omega t), \\
U_{bo}(\omega t) &= U_p S_3(\omega t), \\
U_{co}(\omega t) &= U_p S_5(\omega t),
\end{align*}
\]

Taking inverter’s dead time into consideration, the deviation of \( U_a \) from the exact potential \( U_{ao} \) becomes pulse-wise voltage shown in Fig. 4. The deviation voltage \( U_{ae} = U_{ao} - U_a \) has the following features:

1. Constant height is equal to dc source voltage \( U_D \),
2. Pulse width is always kept at dead time \( t_d \),
3. Polarity of pulse depends on the motor current’s polarity.

According to abovementioned three features, deviation voltage of inverter can be acquired and shown in Fig. 5.

Then voltage matrix \( U \) of Eq. (2) is acquired as follows:


Taking main magnetic circuit saturation of IM and dead time of inverter into consideration, formulas (2) and (3) address nonlinear mathematical model of IM drive controlled by SPWM modulate strategy.

### 3. Simulation Results

In the simulation of IM drive based on the obtained model, inverter’s carrier wave frequency is $f_c=900$ Hz, output frequency $f=10$ Hz, $t_d=20\,\mu$s and $U_D=513$ V. The IM parameters are listed in Table 1.

Figure 6 shows results of acceleration response in IM drive without dead time. The system is stable except a little pulsation of rotor speed, inexistence of distorted stator current and a little torque pulsation. Figure 7 shows results of acceleration response in IM drive with dead time, the system is unstable. The sustained rotor speed oscillation, distorted stator current and large-scale torque pulsation all shows the unstable states. A new sustained periodicity is generated.

### 4. Hopf Bifurcation in IM Drive

Simulation results indicate that dead time can cause IM drive to appear sustained oscillation of rotor speed. In order to explain the phenomenon physically, Hopf bifurcation theory is introduced to indicate the essence of these oscillations.

Using the reference frame fixed on stator, transient rotating variable formed by transient qualities of three phases is given by:

$$ F^2 = 2(F_a + \alpha F_b + \alpha^2 F_c)/3 \quad |\alpha = \exp(j2\pi/3)|, \quad (4) $$

The IM equation is expressed by:
\[
\begin{bmatrix}
U_1 \\
0
\end{bmatrix} = \begin{bmatrix}
 r_1 + L_{14} p & L_{12} p \\
L_{12}(p - j\omega_r) & r_2 + L_{22}(p - j\omega_r)
\end{bmatrix} \begin{bmatrix}
\vec{t}_1 \\
\vec{t}_2
\end{bmatrix},
\] (5)

where \(\vec{U}_1, \vec{t}_1,\) and \(\vec{t}_2\) denote spatial rotating variable of stator voltage, current and rotor current, respectively.

\[
\vec{t}_0 = \vec{t}_1 + (L_{22}/L_m)^2 \vec{t}_2,
\] (6)

From Eq. (5) and Eq. (6), we can obtain:

\[
\begin{bmatrix}
U_1 \\
0
\end{bmatrix} = \begin{bmatrix}
 r_1 + L_{14} p & L_0 p \\
-1/T_0 & 1/T_0 + (p - j\omega_r)
\end{bmatrix} \begin{bmatrix}
\vec{t}_1 \\
\vec{t}_0
\end{bmatrix},
\] (7)

where

\[
L_0 = L_{12}^2/L_{22},
\]
\[
L_1' = L_{14} - L_0,
\]
\[
T_0 = L_{22}/r_2.
\]

Torque equation of IM is expressed as:

\[
T_e = qL_0\omega_r (\vec{r} \times \vec{t}_0).
\]

Dynamic equation of IM is expressed as:

\[
(J/n)p\omega_r = T_e - T_i,
\]

\[
q = 3/2n
\]

On the supposition that harmonics of inverter output voltage are neglected, the stator voltage given by \(\vec{V}_1 = jU \exp (j\theta)\) drives an IM, then the corresponding magnetizing current is \(\vec{i}_0 = \exp \exp (j\theta + \phi_0)\), where \(\theta = 2\pi f t\) and \(\phi_0\) is the initial phase angle of magnetizing current.

Small deviations in the variables \((i_0, \phi_0, \omega_0)\) are assumed to be as \(i_0 = i_0 + \Delta i_0, \phi_0 = \phi_0 + \Delta \phi_0, \omega_0 = \omega_0 + \Delta \omega_0\), and \(\omega_0' = \Delta \omega_0', \) then the linear mathematical model of IM drive is given by [7]:

\[
\begin{bmatrix}
\begin{array}{cccc}
\alpha_{11} p^2 + \beta_{11} p + \gamma_{11} & -\alpha_{12} p - \beta_{12} & L_1'T_0 \theta \\
\alpha_{12} p + \beta_{12} & \alpha_{11} p^2 + \beta_{11} p + \gamma_{11} & -L_1'T_0 \theta - r_1 T_0 \\
-\beta_{12} & -2\beta_{12} & p + k_2
\end{array}
\end{bmatrix}
\begin{bmatrix}
\Delta i_0 \\
\Delta \phi_0' \\
\Delta \omega_0'
\end{bmatrix} = 0
\]

(10)

where

\[
\begin{align*}
\Delta \phi_0' &= i_0 \Delta \phi_0, \\
\alpha_{11} &= L_1'T_0, \\
\beta_{11} &= r_1 T_0 + L_1', \\
\gamma_{11} &= [r_1 - L_1'T_0 \theta (\theta - \omega_0)], \\
\alpha_{12} &= L_1'T_0 (2\theta - \omega_0), \\
\beta_{12} &= [r_1 T_0 (\theta - \omega_0) + \theta L_1'], \\
\Delta \omega_0' &= i_0 \Delta \omega_0, \\
k_1 &= 2k_0 (\theta - \omega_0) L_0 T_0 / J, \\
k_2 &= k L_0 T_0 \phi_0 / J, \\
k &= 3n / J.
\end{align*}
\]

The characteristic equation is derived as:

\[
\alpha_0 p^3 + \alpha_1 p^2 + \alpha_2 p^3 + \alpha_3 p^3 + \alpha_4 p + \alpha_5 = 0
\]

(11)

where

\[
\begin{align*}
\alpha_0 &= 1, \\
\alpha_1 &= 2A, \\
\alpha_2 &= uk_2 + A^2 + 2u_1 + \theta^2,
\end{align*}
\]

\[
\begin{align*}
\alpha_3 &= (Au + u_1)k_2 + 2Au_1 + 2u_1^2, \\
\alpha_4 &= (e^2 u + (u_1 + 2u_1)u_1)k_2 + u_1^2 + u_1^2 e^2, \\
\alpha_5 &= (e^2 u^2 + u_1^2)k_2,
\end{align*}
\]

\[
\begin{align*}
A &= u_0 + u, \\
1 &= L_{11}/(L_1'T_0), \\
1' &= 1/T_1', \\
L_1' &= L_1'/r_1, \\
u_1 &= r_1/(L_1'T_0).
\end{align*}
\]

Allowing for inverter’s dead time, if the fundamental wave of deviation voltage caused by dead time is considered, then the following expression is obtained:

\[
\vec{U}_{el} = \xi U_D \exp \{j(\theta + \phi_0)\},
\]

(12)

where \(\xi = 4f_1/\pi\), according to equivalent circuit of IM, the following representation is given:

\[
jU - \xi U_D \phi_0 = i_0 (r_1 + j\theta L_1') \phi_0,
\]

(13)

From Eq. (13), we know that dead time corresponds to the increment of equivalent stator resistance, which can be calculated by:

\[
\xi U_D \phi_0 = r_{eq1} \phi_0
\]

\[
r_{eq} = \frac{\xi U_D (r_1^2 + 2L_1')}{-\xi U_D + \sqrt{(\xi U_D r_1^2 + (r_1^2 + 2L_1')^2)(U^2 - \xi^2 U_D^2)}}
\]

(14)

Eq. (14) clearly shows that the existence of inverter’s dead time results in an increase of equivalent stator resistance.

If characteristic equation (11) has a single pair of purely imaginary eigenvalues \(\lambda(u^0) = \pm j\omega_0\) and no other eigenvalues with zero real part, and furthermore, \(d \text{Re}(\lambda(u)) / du |_{u=u^0} \neq 0\). The conditions for the existence of a pair of pure imaginary eigenvalues can be formulated for any dimension \(n\) in the following way [8]:

\[
H_{n-1}(u) = 0,
\]

\[
H_{n-2} > 0, \quad H_{n-3} > 0, \ldots, \quad H_1 (u) > 0, \quad p_0 (u) > 0,
\]

where \(H_i (u)\) stands for the \(i\) principal minor of the Hurwitz matrix of the characteristic polynomial and \(p_0(u)\) is the zero-order term of this polynomial.

Using the abovementioned model and analytical results, Hopf bifurcation set for dead time is predetermined and illustrated in Fig. 8. Taking dead time into consideration, the IM system results in an increase of equivalent stator resistance.

![Fig 8 Hopf bifurcation set with dead time.](image-url)
resistance and appears Hopf bifurcation, and then limit cycle oscillation of rotor speed is inevitable. The range of Hopf bifurcation set will expand with increase of dead time. When $t_d = 0 \mu s$ and $f = 10 \text{Hz}$ are satisfied, the system becomes stable, in same conditions, taking dead time into consideration, the system appears Hopf bifurcation and limit cycle oscillations of rotor speed will appear.

5. Control Limit Cycle of Rotor Speed

The phenomenon that Hopf bifurcation set will be expanded with increase of dead time is a clue to control limit cycle through inverter’s operation. As we know, when an inverter operates in the condition of discontinuous PWM (DPWM) modulate strategy, sectors of two $60^\circ$ have not switch in per phase and per cycle, therefore haven’t dead time effect. Therefore DPWM modulate strategy can be utilized effectively to improve stability of IM drive.

The potential difference between the three-wire load neural point and the center point of DC-link capacitor, $U_o$, is the zero-sequence voltage, and can be arbitrarily selected. The zero-sequence signal injecting technique block diagram is illustrated in Fig. 9. Assume that three phase sinusoidal modulate waves are expressed as:

$$
\begin{align*}
U_{a}^* &= M \cos(2\pi ft), \\
U_{b}^* &= M \cos(2\pi ft - 120^\circ), \\
U_{c}^* &= M \cos(2\pi ft + 120^\circ),
\end{align*}
$$

(15)

where $M$ denotes modulate factor. Then rotary transform is implemented to three phase sinusoidal modulate waves, according to power factor angle $\phi$ of load, the following is derived.

$$
U = U^* e^{i\phi}
$$

(16)

The zero-sequence signal of DPWM3 modulate strategy is given as follows:

$$
\begin{align*}
U_{\min} + U_{\max} &\geq 0 \Rightarrow U_o = U_i/2 - U_{\max}^*, \\
U_{\min} + U_{\max} &< 0 \Rightarrow U_o = -U_i/2 - U_{\min}^*, \\
U_{\max}^* &= \max(U_{a}^*, U_{b}^*, U_{c}^*), \\
U_{\min}^* &= \min(U_{a}^*, U_{b}^*, U_{c}^*),
\end{align*}
$$

(17)

$U_i$ corresponds to amplitude of triangular carrier wave.

When an IM drive has no load, power factor $\phi = 90^\circ$, we select $\phi' = 60^\circ$, then DPWM3 modulate strategy is brought and shown in Fig. 10. Figure 11 shows results for acceleration response of IM drive using DPWM3 modulate strategy. Limit cycle oscillation of rotor speed is effectively suppressed by DPWM3 modulate strategy.

6. Conclusion

In this paper, we have discussed that the sustained oscillation of rotor speed in IM drive is caused by inverter’s dead
time. Hopf bifurcation theory is introduced to explain the phenomenon physically and clarify the inherent relationship between dead time and sustained oscillation. Without any additional hardware modification, limit cycle oscillation of rotor speed caused by dead time can be effectively suppressed by DPWM3 modulate strategy.

References


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