An inventory control system for products with optional components under service level and budget constraints
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Problem

Inventory management plays an important role in supply and value chains, respectively. Globalization, customization, quick response and rapid technology changes are the important issues faced by the enterprises. Thus enterprises must increasingly offer differentiated products in quality, function… and etc to target customers. At the same time, enterprises must design an inventory management decision to support above service objectives and to reduce the forecasting errors. An enterprise thus must be efficient and effective on business processes to adapt itself to the trend of these characterizations.

Modularization and postponement, are the form of risk-pooling, and are the key approaches to better match the issues in product and process design. And information sharing, has gained broad consideration, can reduce the effect of demand variability and operation costs in supply chain. Modularization implies a product design approach whereby the product is assembled from a set of standardized constituent units. At the same time, postponement means to assemble the different components (or modules) in the assembly processes closer toward the point of purchase. Business can get a competitive advantage in developing more effective production and inventory management policies by utilizing shared information. Hence, the structure of multi-item inventory with necessary components and optional components is the problem of business.

The \((Q, r)\) model is a heuristic approximating method for the fixed reorder quantity policy with backorders. Traditionally, the average annual variable costs discussed in \((Q, r)\) model include procurement costs, inventory carrying (holding) costs, and stockout costs. In \((Q, r)\) model, inventory carrying cost is defined as the cost of carrying a unit in inventory and is directly proportional to the length of time for which the unit remains in inventory; the backorder cost is inherently extremely difficult to measure and can be assumed as a small proportion of the number of backorders. The \((Q, r)\) model usually requires the operation of a large scale data processing system to update all records automatically and to provide the decision maker with the inventory control information in real time. Because more powerful and cheaper IT is applied, the \((Q, r)\) model can easily explore the inventory problems. By examining the above costs, one can find that those costs cannot represent the real cost when the service level (defined as the probability that a stockout will not occur within the lead time) is considered. This has drawn the attention of some researchers. For example, Swaminathan [9] has assumed that inventory carrying costs are directly related to service levels in the assembly sequences.

In practice, service level is defined as the probability that a stockout will not occur during an order is placed and
the ordered quantity is received. Thus different reorder point indicates different service level on inventory management. In fact, higher service level results in larger heuristic approximating error because it is necessary to increase the costs of labor, facilities, and related services to achieve the higher service level. The cost associated with these additional heuristic approximating errors is called the service cost. Some scholars have introduced different aspects of cost to be incorporated into their models. Thonemann takes the complexity cost (the cost in indirect functions cased by component variety) into account in the component design problem. Swaminathan and Dogramaci introduce a design cost associated with flexibility in the assembly sequence and the design cost for a component design problem, respectively. The production cost and R&D cost are encountered in Thomas’s component design problem. Similarly, the service cost will be incorporated into the proposed model.

The traditional multi-item inventory models with independent demand under resource constraints are available in the literature, and substantial researches have been done on \((Q, r)\) models. However, there exist interaction effects to be overlooked in the multi-item inventory with independent demand. That is, additional demands could depend on one item due to the increasing or decreasing demand of other. Thus, the \((Q, r)\) model which considering the service cost and the interaction among those different components is seldom to discuss.

Research Objectives

The purpose of this research is to find the optimal decision for the multi-item inventory control system that based on considering the service cost and the interactions among those different components. Specifically, the detailed objectives are described of this research as follows:

1. In this study, the service cost (the cost associated with these additional heuristic approximating errors) will be considered. In practice, the service cost is used to reduce the risk of overestimating the decision variables \(Q\) and \(r\).
2. Under service level and budget constraints, exploring the optimal decision in the multi-item inventory control system for products with optional components
3. These considerations based on the assumptions and the demand distributions make the model more complicated and the closed form solutions cannot be obtained. Thus, iterative algorithm (the B-H-W-1 procedure) is proposed to find a solution close to the optimal solution.

Proposed method

From the above discussion, one can find that the greater the reorder point, the higher the service level (and thus higher the service cost) is. This implies that the service cost is proportional to (or is positively related to) the service level. Thus, one can evaluate the service cost via the following definition:

Definition 1. The service cost is proportional to the service level. That is,

\[
SC = \kappa F(r)
\]

where \(\kappa\) : the service cost rate

\(F(r)\): the function of service level (the cumulative distribution of reorder point \(r\)).

The bill of material for the proposed model is shown in the following Figure:
According to the definition of service cost, the bill of material, assumptions on the proposed model and research objectives, the model can then be formulated as following:

\[
\text{Minimize } EAV (Q, r) = EAV_1(Q_V, r_V) + EAV_2(Q_o, r_o)
\]

\[
\text{Subject to } C_V Q_V + \sum_{j=1}^{m} (C_o Q_{oj}) + k_1 F_V (r_V) + \sum_{j=1}^{m} k_{o} f_{r_j|x} (r_{oj} | x = r_V) \leq \beta
\]

\[
Q_V \geq 0
\]

\[
r_V \geq 0
\]

\[
Q_{oj} \geq 0, \text{ for } j=1,2,\ldots,m.
\]

\[
r_{oj} \geq 0, \text{ for } j=1,2,\ldots,m.
\]

\[
EAV_1(Q_V, r_V) = \frac{A_V D_V}{Q_V} + C_V D_V + h_V (\frac{Q_V}{2} + r_V - \mu_V) + \frac{p_V D_V}{Q_V} \int_{r_V}^{\infty} (x - r_V) f_X(x, y) dx
\]

\[
EAV_2(Q_o, r_o) = \sum_{j=1}^{m} \left[ \frac{A_o D_{oj}}{Q_{oj}} + C_o D_{oj} + h_o (\frac{Q_{oj}}{2} + r_{oj} - \mu_{oj}) + \frac{p_o D_{oj}}{Q_{oj}} \int_{r_{oj}}^{\infty} (y - r_{oj}) f_{r_j|x} (y | x = r_{oj}) dy \right]
\]

Equations (1), (8), (7) and (2) indicate that the expected average annual cost of total components, the expected average annual cost of necessary components, the expected average annual cost optional components, and the budget (the order cost plus the service cost) is less than or equal to \(\beta\), respectively.

Parameters for the necessary component, optional components, and budget are shown in the following Tables:
The following Table lists the data solved by the N-R procedure and by the hybrid B-H-W-1 procedure, respectively:

<table>
<thead>
<tr>
<th>item(j)</th>
<th>$A_{oj}$</th>
<th>$C_{oj}$</th>
<th>$D_{oj}$</th>
<th>$h_{oj}$</th>
<th>$p_{oj}$</th>
<th>$\kappa_{oj}$</th>
<th>$\mu_{oj}$</th>
<th>$\sigma_{oj}$</th>
<th>$p_{j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>3</td>
<td>4000</td>
<td>0.7</td>
<td>1.0</td>
<td>200</td>
<td>100</td>
<td>15</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>2</td>
<td>6000</td>
<td>0.4</td>
<td>0.7</td>
<td>150</td>
<td>170</td>
<td>20</td>
<td>0.8</td>
</tr>
</tbody>
</table>

From the illustrated example, The results indicate that the $EAV(Q, r)$ solved by the hybrid B-H-W-1 procedure is better than that solved by the Newton-Raphson iteration procedure.

**Conclusion**

In this research, the proposed multi-item inventory control system with the interactions among those different items under the budgetary constraint (including the service cost) is better to approximate the practice than others. The proposed model can reduce the risk of overestimating the decision variables $Q$ and $r$ for all the models derived from $(Q, r)$ model. On the other hand, these considerations based on the assumptions and the demand distributions make the model more complex and the closed form solutions cannot be obtained.