A Feedback Update Control Scheme for Limited Feedback Multiple Antennas Systems.

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Abstract—Allowing the receiver in a multiple antenna wireless system to send a limited amount of channel state information (CSI) feedback is an effective way to enable channel adaptive signaling. This paper addresses the problem of controlling the feedback update period and feedback rate of limited feedback multiple antennas systems in temporally correlated channels. The challenge in our problem is how to assign the feedback update period and feedback rate subject to a constraint on feedback overhead. The presented approach analyzes the required CSI feedback rate and feedback update period by maximizing a lower bound on the average normalized effective signal-to-noise ratio. By imposing the channel evolution structure and employing a random quantization argument, we are able to determine a closed-form solution. This result leads to a bound on the feedback rate that characterizes when the proposed feedback update control scheme outperforms the conventional feedback scheme. Both analytical and numerical results demonstrate that the proposed feedback control strategy improves the average effective SNR with a relatively small amount of feedback overhead.

I. INTRODUCTION

The demand for higher data transmission rates in practical multiple antenna wireless communications has motivated the design and analysis of a variety of channel adaptive techniques. The adaptation is accomplished by providing the transmitter with partial information about the channel propagation conditions, often referred to as channel state information (CSI). Most work on partial CSI adaptation has focused on either transmitter adaptation to channel statistics or quantized (or noisy) instantaneous CSI. Past work in statistical approaches have focused on adapting to long term statistics such as channel mean (e.g., Rician K-factor) or channel correlation (either in space, time, or frequency) [1]. However, unless the channel is heavily correlated, the data rate attained by the statistical adaptation is restricted. Because of their ability to overcome the drawback and increase the data rate, adaptations to the instantaneous channel realization with the use of limited rate feedback are growing in popularity [1].

When the partial CSI is delivered to the transmitter through the limited rate feedback link, optimally designed transmission schemes that use limited feedback show improved performance. Early works on limited feedback focused on beamforming systems in block-to-block independently fading channels [2]–[4]. However, if the system experiences underspread fading or poor spatial scattering, codebook designs based on the independent block fading model come with a non-negligible performance loss.

This drawback has triggered efforts to design hybrid limited feedback schemes that adapt feedback to long term and short term statistics. Hybrid adaptations dealing with spatial correlation are proposed [5], [6]. In [5], the beamforming codebook is adapted to a spatially correlated channel by projecting the codebook onto the subspace of the channel correlation matrix. Systematic codebook adaptation based on the rotation and scaling of the codebook has been proposed in [6].

There have been several different types of hybrid approaches leveraging temporal correlation to improve the quantization [7]–[13]. Explicit adaptation of the beamforming codebook to the temporally correlated channel has been the focus of the works in [7]–[11]. A feedback assisted stochastic gradient approach to track the channel subspace is studied in [7]. Quantized CSI is modeled as a point on a geodesic of the Grassmannian manifold [8], and a geodesic prediction on the Grassmannian manifold to improve the quantization is proposed in [9] for Multiuser MIMO systems. A codebook switching algorithm using a codebook chosen from a supercodestat depending on the level of time correlation has been proposed in [10]. The significance of differential feedback applied directly to quantizing the full channel vector has been investigated in [11]. In [12], insights from finite state Markov chain analysis are employed to model the time evolution of the feedback states, and the effect of feedback delay on the data throughput is studied. In a recent work [13], it is shown that scaling the feedback update period proportional to the amount of temporal correlation gives benefits in terms of the average feedback overhead and average performance. This reveals that infrequent high resolution feedback can sometimes be preferable to frequent low resolution feedback in temporally correlated environment.

In this work, motivated by the insight in [13], we consider a feedback strategy controlling the feedback update period and feedback load together as a function of the amount of temporal correlation. This feedback control scheme can vary the tradeoff between the required feedback update period and feedback rate and poses an interesting optimization problem on how to assign these feedback resources so that the performance is maximized. The optimal allocation of the feedback update period and feedback rate is found and the benefits of the proposed feedback control scheme are studied using large system limits.

The paper is organized as follows. In Section II, the feedback model and problem statement are presented. Our main result is stated and proved in Section III. Some simulation results are given in Section IV, and concluding remarks are followed in Section V.
II. System Overview and Problem Formulation

The discrete-time baseband signal for a limited feedback multiple-input and single-output (MISO) beamforming system is defined by

\[ y_m = \sqrt{p} h_m g_m s_m + z_m \]  \hspace{1cm} (1)

where the subscript \( m \) (with \( m = 0, 1, \ldots \)) denotes the channel instantiation index, \( s_m \in \mathbb{C} \) is the transmitted symbol with \( E[|s_m|^2] = 1 \), \( y_m \in \mathbb{C} \) is the corresponding received signal, \( z_m \in \mathbb{C} \) is the noise process where the sequence \( \{z_m\} \) is independent and identically distributed (i.i.d.) with \( z_m \sim \mathcal{CN}(0,1) \), \( h_m \in \mathbb{C}^{M_t \times 1} \) is the channel vector, and \( p \) denotes the signal to noise ratio (SNR). A unit norm beamforming vector (or beamformer) \( g_m \in \mathbb{C}^{M_t \times 1} \) is chosen in a codebook \( \mathcal{G} = \{g_i\}_{i=1}^{|\mathcal{G}|} \) and \( B \) bits of index describing \( g_m \) is fed back to the transmitter.

The channel \( h_m \) is assumed to be spatially uncorrelated, but temporally correlated, that fading channel with i.i.d. entries distributed according to \( h_{m,i} \sim \mathcal{CN}(0,1) \). In a temporally correlated environment, the current and previous channel realizations, i.e., \( h_m \) and \( h_{m-1} \) are jointly Gaussian with a correlation variable \( \epsilon \). Here, \( \epsilon \) quantifies the amount of the correlation between elements \( h_{m-1,i} \) and \( h_{m,i} \), and we assume all the elements have the same correlation coefficient \( \epsilon \). Then, the time evolution of \( h_m \) is reasonably modeled by Gauss-Markov process

\[ h_m = \epsilon h_{m-1} + \sqrt{1 - \epsilon^2} v_m \]  \hspace{1cm} (2)

where \( v_m \in \mathbb{C}^{M_t \times 1} \) has i.i.d. entries distributed according to \( \mathcal{CN}(0,1) \) and \( E[|v_m|^2] = M_t \) with \( 0_{M_t} \) denoting \( M_t \times M_t \) zero matrix. The noise process \( z_m \) in (1) is assumed to be independent of \( h_0 \) and \( v_m \). The correlation coefficient \( \epsilon \) is generated using Jake’s model [14] with \( \epsilon = J_0(2\pi f_D T) \), where \( J_0(\cdot) \) is the zeroth order Bessel function, \( T \) denotes the channel instantiation interval, and \( f_D = \frac{v}{c} \) denotes the maximum Doppler frequency where \( v \) is the terminal velocity, \( f_c \) is the carrier frequency, and \( c = 3 \times 10^8 \) m/s. Perfect CSI is available at the receiver side, while the transmitter uses \( B \) bits of partial CSI attained from the reverse link. The long term statistic \( \epsilon \) is perfectly known at the transmitter and receiver. Throughout the paper, the direction of \( h_m \) is denoted by \( h_m = \hat{h}_m \).

In current standards such as IEEE 802.16m [15] and 3GPP LTE [16], the channel instantiation interval is fixed to \( T = 5ms \) or \( 10ms \). In every channel realization \( m \) (or channel duration \( T \)), the transmitter accesses the reverse link to attain \( g_m \) and sends \( s_m \) in the direction of \( g_m \) through the forward link. Thus, in current standards, the reverse and forward links are symmetrically deployed in one channel use.

We consider a slight variation of the feedback strategy in [15], [16] and refer to the modified scheme as feedback update control scheme. In the feedback update control scheme, the transmitter accesses forward link in every channel instance \( m \), while the reverse link is only available in a predefined period \( nT \). For notational simplicity, the channel duration \( T \) is normalized to 1 and \( n \) is called the feedback update period.

When \( n = 1 \), the modified system becomes a conventional closed loop system in [15], [16]. When \( n \neq 1 \), the transmitter uses the CSI obtained at channel instance \( kn \), for \( k = 0, 1, \ldots \), to transmit data during channel instances \( kn \leq m \leq (k+1)n-1 \). Specifically, for any \( m \in [kn, (k+1)n-1] \), the beamformer is fixed as

\[ g_m = g_{kn} \]

where \( g_{kn} \) is chosen such that \( g_{kn} = \arg\max_{g \in \mathcal{G}} |h_{kn}^T g|^2 \). For \( m \in [kn, (k+1)n-1] \), the average normalized effective SNR is expressed as \( E[|h_{kn}^T g_{kn}|^2] \). The feedback update control strategy has an opportunity to adapt the feedback update period \( n \) and feedback load \( B \) to optimize some criterion related to the amount of time correlation.

Throughout the paper, to facilitate the analysis, we employ a random beamforming codebook \( \mathcal{G} = \{g_i\}_{i=1}^{|\mathcal{G}|} \). At every \( kn \)-th channel instance, the random codebook \( \mathcal{G} \) is realized by choosing \( 2^B \) vectors independently from the uniform distribution on the \( M_t \)-dimensional unit sphere.

To measure the feedback load by accounting the number of forward channel uses, we define the average feedback overhead \( \gamma = \frac{B}{n} \). With the preceding assumptions, we raise the following problem: given a fixed average feedback overhead \( \gamma \), what are the contributions of \( B \) and \( n \) to the performance of the proposed feedback update control scheme?

III. Optimization and Performance Analysis

In this section, to investigate the above casted problem, we first attempt to optimize the normalized average effective SNR performance of the proposed feedback update control scheme and study the benefits of the proposed scheme in terms of the feedback load and average performance.

Without loss of generality, we consider the case of \( k = 0 \). When \( m \in \{0, 1, \ldots, n-1\} \), the normalized average effective SNR of the feedback update control scheme is

\[ E \left[ |h_{kn}^* g_0|^2 \right] = M_t E \left[ |\hat{h}_{kn}^* g_0|^2 \right]. \]

The closed-form expression of \( E[|\hat{h}_{kn}^* g_0|^2] \) is given by [13]

\[ E \left[ |\hat{h}_{kn}^* g_0|^2 \right] = 2^m \left( M_t - 2\beta \right) \left( M_t - 1, 2^B \right) + 1. \]  \hspace{1cm} (3)

The function in (3) indicates that with fixed \( B \) and \( \epsilon \), \( E[|\hat{h}_{kn}^* g_0|^2] \) is a decreasing function of \( m \), while with fixed \( m \) and \( \epsilon \), \( E[|\hat{h}_{kn}^* g_0|^2] \) is an increasing function of \( B \). This observation motivates us to consider a tradeoff point between \( B \) and \( n \) such that given the average feedback overhead constraint \( \gamma = B/n \), what are the values of the feedback bits \( B \) and the feedback update period \( n \) that maximize \( E[|\hat{h}_{kn}^* g_0|^2] \).

Our solution is based on a lower bound on the normalized average effective SNR. The bound allows us to compute the optimal amount of \( B \) and \( n \).

A. Bounds and Optimization

In this subsection, a tight lower bound on (3) is first derived. Then, given a constraint \( \gamma = \frac{B}{n} \), the lower bound is maximized as a function of \( B \) and \( n \).

We now review bounds in [17] regarding the quantization error of the random vector codebook, which is useful in later derivations. First, note that

\[ \frac{M_t}{M_t - 1} 2^{-\frac{B}{M_t - 1}} \leq 2^B \beta \left( \frac{M_t}{M_t - 1} \right) \left( \frac{M_t}{M_t - 1} \right) \leq \Gamma \left( \frac{M_t}{M_t - 1} \right) 2^{-\frac{B}{M_t - 1}}. \]  \hspace{1cm} (4)

From (4), a lower bound of (3) easily follows as

\[ E \left[ |\hat{h}_{kn}^* g_0|^2 \right] \geq 2^m \left( M_t - 1, \Gamma \left( \frac{M_t}{M_t - 1} \right) \right) 2^{-\frac{B}{M_t - 1}} + \frac{1}{M_t}. \]

Note that the lower bound in (5) is not necessarily a positive quantity. When it becomes negative quantity, it fails to characterize
the average effective SNR (i.e., it is not well defined quantity). Thus, we need the assumption

\[
\frac{M_t - 1}{M_t} - \Gamma \left( \frac{M_t - 1}{M_t - 1} \right) 2^{- \frac{\beta}{M_t - 1}} \geq 0.
\] (6)

However, it can be readily shown that most of the feedback scenarios satisfy the condition in (6).

**Lemma 1:** If \( B \) is chosen such that \( B \geq \log_2(\epsilon) \approx 1.443 \), the bound in (6) always holds.

**Proof:** Using \( \Gamma \left( \frac{M_t}{M_t - 1} \right) \leq 1 \), we further bound (6) as \( \frac{M_t - 1}{M_t} - 2^{- \frac{\beta}{M_t - 1}} \geq 0 \), which is equivalent to

\[
B \geq (M_t - 1) \log_2 \left( 1 + \frac{1}{M_t - 1} \right)
\] (7)

Note that the right hand side (r.h.s.) of (7) is an increasing function of \( M_t \). Taking \( \lim_{M_t \to \infty} \) to both sides of (7) and using the fact

\[
\lim_{M_t \to \infty} \left( 1 + \frac{1}{M_t + 1} \right)^{M_t + 1} = e
\]

lead to the bound \( B \geq \log_2(\epsilon) \).

**Remark 1:** It is not difficult to verify that the bound (5) is achieved as \( B, M_t \to \infty \) and the ratio \( B/M_t \) converges to a bounded value. This is because the lower and upper bounds in (4) coincide and \( \epsilon = \frac{B}{n} \), respectively. In the following, to derive an analytical solution, \( B/M_t \to \infty \) while maintaining \( B/M_t \) to be bounded.

With \( n = m + 1 \) (i.e., \( n \geq 1 \)) and \( \gamma = \frac{B}{n} \), the closed-form expression of \( E[\| \hat{v}_n \|^2] \) in (3) and the lower bound in (5) are expressed in terms of \( B \) and \( \gamma \) and denoted by

\[
F_{B, \gamma} = \epsilon^2 \left( \frac{\beta}{M_t - 1} \right) \left( \frac{M_t - 1}{M_t} - 2^{\beta} \left( \frac{M_t}{M_t - 1} \right) \ln 2 \right) + \frac{1}{M_t}
\] (8)

and

\[
G_{B, \gamma} = \epsilon^2 \left( \frac{\beta}{M_t - 1} \right) \left( \frac{M_t - 1}{M_t} - \ln \left( \frac{M_t - 1}{M_t} \right) 2^{- \frac{\beta}{M_t - 1}} \right) + \frac{1}{M_t}
\] (9)

respectively. In the following, to derive an analytical solution, \( B \) and \( n \) are assumed to be positive real numbers and to satisfy the condition in Lemma 1 we assume that \( \gamma \geq \log_2(\epsilon) \). Finding the maximizer of \( G_{B, \gamma} \) over \( B \) is the contents of the following lemma.

**Lemma 2:** Given an average feedback overhead constraint \( \gamma = \frac{B}{n} \geq \log_2(\epsilon) \), the optimal \( B \) that maximizes \( G_{B, \gamma} \) is given by

\[
B_{opt} = (M_t - 1) \log_2 \left( 1 + \frac{1}{M_t - 1} \left( 1 + \frac{\ln 2}{\ln 2} \right) \right)
\] (10)

**Proof:** With the objective function \( G_{B, \gamma} \), consider an optimization problem

\[
B_{opt} = \arg \max_B G_{B, \gamma}
\] (11)

To solve this, we need to examine the behavior of \( G_{B, \gamma} \). It is not difficult to conclude that \( G_{B, \gamma} \) is neither of convex or concave function of \( B \). Differentiating \( G_{B, \gamma} \) once with respect to \( B \) yields

\[
\frac{dG_{B, \gamma}}{dB} = \epsilon^2 \left( \frac{\beta}{M_t - 1} \right) \left( \frac{M_t - 1}{M_t} \right) \ln 2 \epsilon
\]

\[
+ \frac{M_t}{M_t - 1} 2^{- \frac{\beta}{M_t - 1}} \left( \ln 2^{- \frac{\beta}{M_t - 1}} \ln 2 \right)
\] (12)

Equating \( \frac{dG_{B, \gamma}}{dB} = 0 \) and after some algebraic manipulation, the equality \( \frac{d^2 G_{B, \gamma}}{dB^2} = 0 \) uniquely determines a single value

\[
\hat{B} = (M_t - 1) \log_2 \left( \Gamma \left( \frac{M_t}{M_t - 1} \right) \left[ 1 + \frac{M_t}{M_t - 1} \left( 1 + \frac{\ln 2}{\ln 2} \right) \right] \right)
\] (13)

We claim (13) is the maximizer of \( G_{B, \gamma} \). In order to verify this, we first observe that \( G_{0, \gamma} \leq \frac{1}{\Gamma} \lim_{B \to \infty} G_{B, \gamma} = \frac{1}{\Gamma} \) and \( G_{B, \gamma} \geq \frac{1}{\Gamma} \) for \( \hat{B} \geq \log_2(\epsilon) \) where the last inequality comes from (6) and Lemma 2. Thus, if

\[
\frac{dG_{B, \gamma}}{dB} \bigg|_{B=0} \geq 0,
\] (14)

the unique zero differential point in (13) is the maximizer. Otherwise, there is no single zero differential point. The condition in (14) can be readily seen by plugging \( \hat{B} = 0 \) in (12) and observing

\[
\frac{dG_{B, \gamma}}{dB} \bigg|_{B=0} = \epsilon^2 \left( \frac{\beta}{M_t - 1} \right) \left( \ln 2^{- \frac{\beta}{M_t - 1}} \right)
\]

The facts \( \ln 2 \frac{\beta}{M_t - 1} < 0 \) and \( \left( \frac{\ln 2}{\ln 2} \right) \geq M_t - 1 \) prove the claim. Now, the quantity in (13) is the optimal \( B_{opt} \) in (11).

With (10), the optimal feedback update period easily follows that \( n_{opt} = B_{opt}/\gamma \). However, depending on \( \epsilon, M_t \), and \( \gamma \) values, there is a chance that the found \( B_{opt} \) returns \( n_{opt} < 1 \) due to the assumption that \( n_{opt} \) is a positive real number. Note that \( n_{opt} < 1 \) implies \( m < 0 \), which is impossible. In this trivial case, we choose \( n = 1 \) and \( B = \gamma \), i.e., it becomes conventional feedback scheme.

The solutions \( B_{opt} \) and \( n_{opt} \) are based on the lower bound. Thus, depending on particular \( \epsilon, M_t \), and \( \gamma \) values, there is a chance that the obtained \( B_{opt} \) and \( n_{opt} \) yields smaller average effective SNR gain than the trivial case of \( n = 1 \) and \( B = \gamma \). This event may happen when the true maximal value of \( F_{B, \gamma} \) in (8) is close to the performance of the conventional feedback scheme (with \( n = 1 \) and \( B = \gamma \)).

In this regard, given \( \epsilon, M_t \), and \( \gamma \), we speculate what is the condition for \( B_{opt} \) (or \( n_{opt} \)) that ensures a larger average SNR gain of the proposed feedback scheme than that of the conventional feedback scheme and at the same time, guarantees \( n_{opt} \geq 1 \). Let us define the quantization error of the random vector codebook with size \( 2^\gamma \) as

\[
\alpha \gamma \triangleq 2^\gamma \left( \frac{M_t}{M_t - 1} \right)
\] (15)

With the average feedback overhead constraint \( \gamma \), (15) quantifies the quantization error of the conventional feedback scheme.

**Theorem 1:** The proposed feedback update control strategy with \( B_{opt} \) in (10) outperforms the conventional feedback scheme and satisfies \( n_{opt} \geq 1 \) if

\[
\gamma \leq B_{opt} \leq \frac{\ln \Phi}{\ln 2 \frac{\beta}{M_t - 1} - \frac{1}{\gamma} \ln 2^\epsilon}
\] (16)

where

\[
\Phi = \left( \frac{\gamma}{M_t - 1} \right) \left( \frac{M_t}{M_t - 1} \right) \ln 2 \left( \frac{\ln 2}{\ln 2} - \epsilon^2 \right)
\]
Proof: Note that with $B_{opt}$ in Lemma 2, $G_{B_{opt},\gamma}$ in (9) is smaller than $F_{B_{opt},\gamma}$ in (8). Thus, if $G_{B_{opt},\gamma} \geq 1 - \alpha_{\gamma}$, $F_{B_{opt},\gamma} \geq 1 - \alpha_{\gamma}$ is free.

We first show the upper bound of $B_{opt}$ in (16). Equating $G_{B_{opt},\gamma} \geq 1 - \alpha_{\gamma}$ gives

$$
\epsilon \left( \frac{B_{opt}}{\gamma} - 1 \right) \left( \frac{M_t - M_{t-1}}{M_t} - \Gamma \left( \frac{M_t}{M_{t-1}} - 1 \right) \right) \geq \frac{M_t - M_{t-1}}{M_t} - \alpha_{\gamma}. \quad (17)
$$

Directly working with (17) to extract a bound of $B_{opt}$ does not provide much insight. We use the first order condition in Lemma 2. Equating $\frac{dG_{B_{opt},\gamma}}{dB_{opt}}|_{dB_{opt}=0}$ in (12) yields an equality

$$
2 \epsilon \left( \frac{B_{opt}}{\gamma} - 1 \right) \left( \frac{M_t - M_{t-1}}{M_t} - \Gamma \left( \frac{M_t}{M_{t-1}} - 1 \right) \right) = \left( \frac{\epsilon}{2} \right)^{2 \left( \frac{M_t}{M_{t-1}} - 1 \right)} B_{opt} \Gamma \left( \frac{M_t}{M_{t-1}} \right) \ln 2 \left( \frac{\epsilon}{2} \right).
$$

Plugging (18) in (17) and solving for $B_{opt}$ gives the upper bound in (16). The lower bound follows from the fact that $\gamma = \frac{B_{opt}}{\gamma}$, the condition $n_{opt} \geq 1$ is equivalent to $\gamma \leq B_{opt}$. This concludes the proof.

The theorem offers that if $B_{opt}$ in (10) falls in the bound in (16), the performance of the proposed scheme always outperforms the conventional feedback scheme with non-trivial feedback update period.

Remark 2: From the expression in (10), it is easy to check that limiting $\gamma, M_t \to \infty$ such that $\frac{\gamma}{M_t} = \kappa$ (with $\kappa$ is bounded) implies limiting $B_{opt}, M_t \to \infty$ such that $\frac{B_{opt}}{M_t}$ bounded. Therefore, from Remark 1, it is obvious that $B_{opt}$ in (10) converges to the quantity $B_{opt}$ in (8) as $\gamma, M_t \to \infty$ such that $\frac{\gamma}{M_t} = \kappa$. This also implies that $G_{B_{opt},\gamma}$ approaches to the maximum of $F_{B,\gamma}$ and indeed $G_{B_{opt},\gamma} \to F_{B_{opt},\gamma}$.

B. Asymptotic Feedback Load and Performance Analysis

Motivated by Remark 2, in order to further investigate benefits of the proposed adaptive feedback update scheme, we perform a large system analysis in this subsection. First, the growth rate of $B_{opt}$ and $n_{opt}$ as the average feedback overhead $\gamma$ increases is identified. Then, the convergence of the average effective SNR of the proposed scheme as $\gamma, M_t \to \infty$ with fixed ratio $\frac{\gamma}{M_t} = \kappa$ is studied.

We first specify the corresponding growth rate for $B_{opt}$ and $n_{opt}$ as $\gamma$ tends infinity.

Theorem 2: For any $\epsilon \in (0, 1)$, $B_{opt} = O \left( \log_2(\gamma) \right)$ and $n_{opt} = O \left( \log_2(\gamma/\gamma) \right)$ as $\gamma \to \infty$.

Proof: We first show $B_{opt} = O \left( \log_2(\gamma) \right)$. From (10), $B_{opt} \log_2(\gamma)$ can be rewritten

$$
\log_2 \left( \Gamma \left( 1 + M_t - \frac{M_{t-1}}{M_t} \right) + \log_2 \left( 1 + \gamma \frac{\ln 2 \left( \frac{\gamma}{M_t} \right)}{\ln \epsilon} \right) \right)
$$

(19)

and taking $\lim_{\gamma \to \infty}$ to (19) results in

$$
\lim_{\gamma \to \infty} \frac{B_{opt}}{\log_2(\gamma)} = \lim_{\gamma \to \infty} \frac{(M_t - 1) \log_2 \left( 1 + \gamma \frac{\ln 2 \left( \frac{\gamma}{M_t} \right)}{\ln \epsilon} \right)}{M_t - 1} = \frac{\log_2(\gamma)}{M_t - 1}
$$

where the last equality follows from the fact that $\ln \gamma = \gamma / M_t$ is a finite positive constante regardless of $\gamma \to \infty$. The growth rate of $n_{opt}$ directly follows from the relation $n_{opt} = \frac{B_{opt}}{\gamma}$.

For the conventional feedback strategy, since $\gamma = B_{opt}, B$ scales linearly with $\gamma$. However, Theorem 2 offers that the proposed adaptive feedback scheme, accounting temporal correlation in the feedback load, reduces the order growth of $B_{opt}$ to $O(\log_2(\gamma))$.

Next, the convergence of the average normalized effective SNR of the proposed feedback scheme is stated. From (10), as $\gamma, M_t \to \infty$ while maintaining $\frac{\gamma}{M_t} = \kappa$, the ratio $\frac{B_{opt}}{n_{opt}}$ converges as

$$
\gamma, M_t \to \infty \quad B_{opt} = \log_2 \left( 1 - \frac{\ln 2}{\ln \epsilon} \right) \leftrightarrow a \quad (20)
$$

where we define $\gamma, M_t \to \infty \quad B_{opt} = \log_2 \left( 1 - \frac{\ln 2}{\ln \epsilon} \right) \leftrightarrow a$. With the equality $\kappa = \frac{\gamma}{M_t}$, $B_{opt}$, taking $\gamma, M_t \to \infty$, leads to

$$
\kappa = \frac{a}{n_{opt}, \infty} \quad (21)
$$

where in (21), we define $\gamma, M_t \to \infty \quad n_{opt} \leftrightarrow n_{opt, \infty}$. As $\epsilon \to 1$, from (20) and (21), $a \to \infty$ and $n_{opt, \infty} \to \infty$. Indeed, $a$ and $n_{opt, \infty}$ increase with the same growth order.

Now, plugging (21) in (20) and solving for $n_{opt, \infty}$ gives

$$
n_{opt, \infty} = \frac{\ln 2 a^2}{1 - 2 a}. \quad (22)
$$

With the convergence results (20) and (22), we provide following theorem.

Theorem 3: For $\gamma, M_t \to \infty$ with the finite ratio $\frac{\gamma}{M_t} = \kappa$ and $\epsilon \to 1$,

$$
E \left[ \frac{\left[ h_{n_{opt} - 1}^{\gamma} \right]^{2}}{M_t} \right]_{\gamma, M_t \to \infty} = \left( 2^a \right) \Gamma \left( 1 - 2^{-a} \right) \epsilon \quad (23)
$$

$$
\epsilon \to 1. \quad (24)
$$

Proof: Note that $E \left[ \left[ h_{n_{opt} - 1}^{\gamma} \right]^{2} \right] = M_t F_{B_{opt},\gamma}$. From Remark 2, as $\gamma, M_t \to \infty$ with $\frac{\gamma}{M_t} = \kappa$, we have $\gamma, M_t \to \infty \quad n_{opt} = \lim n_{opt} \to \lim G_{B_{opt},\gamma}$. Then, the expressions in (9), (20), and (22) together yield

$$
F_{B_{opt},\gamma} \gamma, M_t \to \infty \quad \frac{2^{n_{opt} - 1} \left( 1 - 2^{-a} \right)}{M_t}. \quad (25)
$$

Then, plugging (22) in (25) leads to (23).

Now, as $\epsilon \to 1$, $a$ in (20) approaches $\infty$. Thus, the convergence in (24) follows from the fact that

$$
(2^a)^{\frac{1}{1 - 2^a}} \to \infty \quad (26)
$$

The convergence in (26) follows because the quantity $\left( 2^a \right)^{\frac{1}{1 - 2^a}}$ is upper bounded by $(2^a)^{\frac{1}{2}}$ and is lower bounded by $\left( (2^a)^{\frac{1}{2}} \right)^{2}$, and the both bounds converge to 1 as $a \to \infty$. This concludes the proof.

Theorem 3 implies that the proposed scheme achieves full CSI beamforming performance as $\epsilon \to 1$ with the finite average feedback overhead $\frac{\gamma}{M_t} = \kappa$. For the conventional feedback scheme (with $n = 1$ and $\gamma = B$), however, we only have

$$
E \left[ \frac{\left[ h_{n_{opt} - 1}^{\gamma} \right]^{2}}{M_t} \right] \gamma, M_t \to \infty \quad 1 - 2^{-\kappa}. \quad (27)
$$

This suggests compared to the conventional feedback scheme, huge performance benefit of the proposed feedback update control strategy is possible.
IV. NUMERICAL SIMULATION

We present numerical simulation results to corroborate the analysis done in the previous section and measure the performance of the proposed feedback update control scheme. The correlation variable $\epsilon$ is generated described in Section II.

Fig. 1 displays the upper bound and lower bound in (16) and the optimal feedback rate $B_{opt}$ in (10) with different values of $\gamma \in \{2, 4\}$. As can be seen from the figure, for $\gamma = 2$, if $\epsilon \geq 0.72$ $(15 km/h)$ the proposed feedback update control scheme with $B_{opt}$ in (10) shows improved performance than the scheme with $B = \gamma$ and $n = 1$. Similarly, for $\gamma = 4$, when $\epsilon \geq 0.68$ $(18 km/h)$ the proposed scheme shows benefits in terms of the average feedback overhead and performance compared to the conventional feedback scheme.

In Fig. 2, we demonstrate the average throughput performances of the proposed scheme and conventional scheme across different SNR ($\rho$) values, while maintaining the same average feedback overhead $\gamma = 6$ with $\epsilon = 0.946$ $(6 km/h)$ and $\epsilon = 0.9987$ $(1 km/h)$. For the proposed scheme, $B_{opt}$ are determined by (10). This figure shows the insight in Theorem 3 that with finite $\gamma$ the performance of the proposed scheme approaches the full CSI performance as $\epsilon \rightarrow 1$.

V. CONCLUSIONS

In this paper, we presented a feedback rate and feedback update period tradeoff for temporally correlated MISO channels. Interestingly, this tradeoff raised an optimization problem, and we proposed an optimal control strategy for the feedback rate and feedback update period given the average feedback overhead constraint. Both analytical and numerical results demonstrate that the proposed strategy provides improved performance.

REFERENCES


