Coordination of supply chains by option contracts: A cooperative game theory approach

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Abstract
Manufacturer–retailer supply chains commonly adopt a wholesale price mechanism. This mechanism, however, has often led manufacturers and retailers to situations of conflicts of interest. For example, due to uncertain market demand, retailers prefer to order flexibly from manufacturers so as to avoid incurring inventory costs and to be able to respond flexibly to market changes. Manufacturers, on the other hand, prefer retailers to place full orders as early as possible so that they can hedge against the risks of over- and under-production. Such conflicts between retailers and manufacturers can result in an inefficient supply chain. Motivated by this problem, we take a cooperative game approach in this paper to consider the coordination issue in a manufacturer–retailer supply chain using option contracts. Using the wholesale price mechanism as a benchmark, we develop an option contract model. Our study demonstrates that, compared with the benchmark based on the wholesale price mechanism, option contracts can coordinate the supply chain and achieve Pareto-improvement. We also discuss scenarios in which option contracts are selected according to individual supply chain members’ risk preferences and negotiating powers.

Keywords:
Supply chain management, Cooperative game, Option contract, Negotiating power, Channel coordination

1. Introduction

Manufacturer–retailer supply chains commonly adopt a wholesale price mechanism in practice. This mechanism, however, has often led manufacturers and retailers to situations of conflicts of interest, resulting in an inefficient supply chain. For instance, due to uncertain market demand and aversion to incurring inventory costs, retailers prefer to order flexibly from manufacturers so as to accommodate fluctuating market demand. On the other hand, in order to hedge against the risks of over- and under-production, manufacturers prefer retailers to place full orders as early as possible. This conflict between the retailer and the manufacturer can lead to actions that are not system-wide optimal, thereby resulting in an inefficient supply chain. Such problems have been encountered by companies such as Mattel, Inc., a toy maker (see, e.g., Barnes-Schuster et al., 2002). Motivated by this observation, we address in this paper the issue of whether an option mechanism is a viable alternative to achieving an efficient supply chain. The option mechanism is characterized by two parameters, namely the option price \( a \) and the exercise price \( c \). The option price, in essence, is an allowance paid by the retailer to the manufacturer for reserving one unit of the production capacity. The exercise price is to be paid by the retailer to the manufacturer for one unit of the product purchased by exercising the option. Option mechanisms have attracted substantial attention not only in the context of financial market, pioneered by Black and Scholes (1973), but also in the area of supply chain management (SCM) (please refer to Section 2). In fact, option mechanisms have been extensively applied in various industries such as fashion apparel (Eppen and Iyer, 1997), toys (Barnes-Schuster et al., 2002), and electronics (Billington, 2002; Carbone, 2001).

Relationships between manufacturers and retailers in supply chains have dramatically changed in recent years. One such change is evidenced by the fact that partnerships have become prevalent in supply chains. A typical example is the numerous operating modes based on cooperative relations that have been widely adopted in the practice of SCM, such as vendor managed inventory (VMI), holding cost subsidies, and consignment. These operating modes have significantly improved the overall performance of supply chains. Such industry environment motivates us to address the problem under study by way of a cooperative game approach. Again, much of the literature on supply chain coordination has focused on developing incentive contract forms, and did not take into
account issues concerning implementation of these contract forms. However, profit allocations differ under different coordinating contracts. The ultimate implementation outcome of a coordinating contract form inevitably depends on supply chain members’ individual risk preferences and negotiating powers. Therefore, given that an incentive contract form has been derived, two important questions remain: (i) What coordinating contracts are feasible? (ii) How do supply chain members’ individual risk preferences and negotiating powers impact the ultimate outcome?

Supply chain coordination is an important issue in SCM. Various contract types have been designed to align supply chain members’ incentives to drive the optimal action in the supply chain. However, it is clear that feasible coordinating contracts are those that leave each party to be at least as well off as that party would be without these contracts. Motivated by this, we propose a notion called the core of a contract set, which is a subset of the contract set consisting of all the contracts satisfying the coordination requirement. Again, noting that the wholesale price mechanism is prevalently used in practice, we take the present less than the wholesale price mechanism as a benchmark and derive the core of option contracts by taking a cooperative game approach. We demonstrate that option contracts can coordinate the supply chain and achieve Pareto-improvement. Hence, the manufacturer and the retailer will both have strong incentives to adopt the option mechanism rather than the wholesale price mechanism.

Even though every option contract in the core, compared with the wholesale price mechanism, can coordinate the supply chain with Pareto-improvement, the profit allocations between the retailer and the supplier differ for different option contracts in the core. As a result, the retailer and supplier will negotiate the option contract from the core to implement. Obviously, the outcome depends heavily on individual members’ negotiating powers, as well as their risk preferences. By taking these factors into consideration, we identify the option contract that the retailer and supplier both agree to implement according to Nash’s bargaining model and Eliasberg’s model, and explore analytically the effects of these factors on the ultimate outcome. The remainder of this paper is organized as follows: In Section 2 we review the related literature. In Section 3 we introduce the model. In Section 4 we analyze the model. In Section 5 we examine supply chain coordination with option contracts and discuss the related characteristics. We explore analytically in Section 6 the selection of option contracts and the sharing of the extra profit gained from coordination, taking into consideration the effects of members’ risk preferences and negotiating powers. We conclude the paper and suggest topics for future research in Section 7. We give all the proofs in the supplementary Appendix, which appears as an online electronic companion of the paper.

2. Literature review

This paper can be regarded as a study of supply chain contracts. To highlight our contributions, we review only the literature that is representative and particularly relevant to our study.

First, this paper is closely related to the literature on the use of options in SCM. Research on this aspect mainly focused on operations flexibility and economics efficiency derived from options. Barnes-Schuster et al. (2002) studied how options provide managerial flexibility in response to uncertain market changes and how to achieve channel coordination by options based on a model of two periods with correlated demand. Eppen and Iyer (1997) considered a backup agreement, which is essentially an option contract. They showed that backup agreements have substantial effects on expected profit and committed quantity. Wu et al. (2002) explored the optimal option contract bidding and contract-
negotiation powers (by a line aggregation rule) into the analysis of the issues concerning implementation of the option contract. We think doing so is important because the implementation outcome of supply chain contracts clearly depends on supply chain members’ negotiation powers on the negotiation table, as well as their risk preferences. In addition, our research approach can easily be extended to other types of supply chain contract popular in SCM research. Of course, we also discuss the limitations of our study. Despite the acknowledged limitations, we believe our paper has made a good contribution to the literature on the implementation aspect of supply chain contracts.

3. Model description

We consider a two-echelon supply chain consisting of a single manufacturer and a single retailer where the manufacturer’s product is sold via the retailer to end consumers. Market demand for the product is a stochastic variable $X$, which follows a strictly increasing distribution function $F(x)$ with $x > 0$. We assume the option mechanism is employed to facilitate the production and procurement in the supply chain, which is characterized by two parameters, namely the option price $o$ and the exercise price $e$. The option price is essentially an allowance paid by the retailer to the manufacturer for reserving one unit of the production capacity at the beginning of the production season. The exercise price is to be paid by the retailer to the manufacturer for one unit of the product purchased by exercising the option after the market demand is realized. In addition, considering the current industry environment in which many retailers are as powerful as, or even more powerful than, their manufacturers, we assume the manufacturer uses the “make-to-order” policy for production. The event sequence of our model can then be described as follows: At the beginning of the production season, the retailer reserves a quantity of the production capacity beforehand from the manufacturer, say $Q_{wr}$, at a unit price of $o$. Using the “make-to-order” production policy, the manufacturer produces $Q$ units of the product by following the quantity reserved by the retailer. During the selling season, according to the realized market demand for the product, the retailer purchases a quantity of the product up to $Q$ units from the manufacturer at the exercise price $e$ to satisfy the demand, and any unsatisfied demand is lost with no penalty cost. We assume demand information is symmetric between the manufacturer and the retailer. We let the marginal production cost of the manufacturer be $c$ and the salvage value per unit of unsold product be $v$ for both the manufacturer and the retailer. Let $p$ be the retailer’s retail price. We focus on the reasonable and non-trivial case where $p > c > v$, $0 < o < c - v$, and $e > v$. In fact, assuming $o < c - v$ avoids the unreasonable case where the manufacturer is risk-free for its production while assuming $e > v$ avoids the trivial case where the retailer always exercises all the options purchased by it.

To focus on the implementation issue of the option contract, as reviewed in Section 2, we consider a relatively simple option model. The primary purpose of our model is to explore the implementation issue of the coordinating option contract form, taking into account the effects of the supply chain members’ risk preferences and negotiating powers.

4. Basic option contract model

Since the wholesale price mechanism is commonly used in manufacturer–retailer supply chains in practice, we use the wholesale price mechanism as the benchmark against which we will compare the option mechanism developed in this study. With the wholesale price mechanism, the event sequence is identical to the option model except for the option mechanism, which should be replaced by the wholesale price mechanism. Then, the retailer’s expected profit function under the wholesale price mechanism is given by

$$EI_{wh}(Q_{wr}) = E[p \min(Q_{wr}, x) - wQ_{wr} + v \max(Q_{wr} - x, 0)],$$

where $Q_{wr}$ is the retailer’s order quantity and $w$ is the wholesale price. Obviously, in order to avoid the unreasonable cases, we require $p > w > c$. The first term of (1) is the retailer’s sales revenue, the second term is the order cost, and the third term is the salvage value. Hence, with the wholesale price mechanism, the retailer’s problem is to maximize the expected profit function (1) with respect to $Q_{wr}$, which yields the following proposition.

**Proposition 1.** With the wholesale price mechanism, the retailer will earn an expected profit of:

$$EI_{wh}(Q_{wr}) = E[p \min(Q_{wr}, x) - wQ_{wr} + v \max(Q_{wr} - x, 0)].$$


Similar to the proof of Proposition 1, we derive the manufacturer’s optimal production quantity as $Q_{wm} = F^{-1}(\frac{e}{p-c})$. All these results obtained with the wholesale price mechanism will be taken as benchmarks against which we will compare the option mechanism developed in this paper.

In what follows, we consider the option model. With the option contract mechanism, say $(o, e)$, the retailer’s expected profit function is given by

$$EI_{opt}(Q_{om}) = E[p \min(Q_{om}, x) - oQ_{om}].$$

where $Q_{om}$ is the retailer’s reserved quantity under the option contract mechanism. The first term of (4) is the retailer’s sales profit and the second term is the allowance payout for the reserved capacity. Similarly, when the manufacturer plans its production quantity for its own interest instead of using the “make-to-order” production policy, its expected profit function is given by

$$EI_{om}(Q_{om}) = E[p \min(Q_{om}, x) - cQ_{om} + v \max(Q_{om} - x, 0)].$$

Hence, with the option mechanism, the retailer’s problem is to maximize the expected profit function (4) and the manufacturer’s is to maximize (5), which lead to the following proposition.

**Proposition 2**

(i) Given $(o, e)$, the manufacturer’s optimal production quantity is $Q_{wm} = F^{-1}(\frac{e}{p-c})$ and the retailer’s optimal reserved quantity is $Q_{om} = F^{-1}(\frac{o}{p-c})$.

(ii) Given $o + e$, $EI_{om}(Q_{om})$ is decreasing in $o$ or increasing in $e$, whereas $EI_{om}(Q_{om})$ is increasing in $o$ or decreasing in $e$.

(iii) Only if the option contract satisfies $e = p - \frac{c}{2} (o < c - v)$ will the retailer’s optimal reserved quantity be just consistent with the manufacturer’s optimal production quantity.

By the option mechanism, we know that $o + e$ is the unit price for the quantity of the product purchased by the retailer. From Proposition 2, we see that, given $o + e$, the higher the retailer is
willing to pay for the option price, the higher is the optimal production quantity of the manufacturer. Besides, the retailer is prone to reserve more if it is allowed to pay a lower option price and a higher exercise price later. However, the converse is preferred by the manufacturer. These indeed coincide with the intuition from practice. Furthermore, in terms of the option contract that makes the retailer’s optimal reserved quantity just consistent with the manufacturer’s optimal production quantity, the exercise price is a negative linear function of the option price.

Comparing with the wholesale price mechanism, we have the following proposition.

**Proposition 3**

(i) \( Q_{wT} < Q_{ot} \) iff \( o < \frac{(p-v)(1-x)}{x} \)

(ii) \( Q_{wm} < Q_{om} \) iff \( o > \frac{(p-v)}{x} \)

**Proposition 3** shows that compared with the wholesale price mechanism, the option contract mechanism with an option price that is lower than \( \frac{(p-v)}{x} \) will induce the retailer to reserve more, whereas an option price that is higher than \( \frac{(p-v)}{x} \) will push the manufacturer to produce more.

5. Supply chain coordination with option contracts

To derive the system-wide optimal expected profit for the supply chain, we take the supply chain as a centralized entity. Let \( E \Pi(Q_c) \) denote the expected profit of the centralized entity when the production quantity is \( Q_c \). With some algebra, we have:

\[
E \Pi(Q_c) = E[p \min(Q_c, x) + v \max(Q_c - x, 0) - C_c Q_c] = (p - c) Q_c - (p - v) \int_0^{Q_c} F(x)dx.
\]

Similar to the proof of **Proposition 2**, we can show the strict concavity of \( E \Pi(Q_c) \) with respect to \( Q_c \). Therefore, by the first-order optimality condition, we find that the first-best production quantity for the supply chain is \( Q_{opt} = F^{-1}\left(\frac{p - c}{p - v}\right) \), and accordingly the system-wide optimal expected profit, denoted by \( \pi_c \), is

\[
\pi_c = E \Pi(Q_{opt}) = (p - c) Q_{opt} - (p - v) \int_0^{Q_{opt}} F(x)dx.
\]

It can be shown that \( Q_{opt} > Q_{wT} > Q_{om} > Q_{wm} \) (see the Supplementary Appendix). We discuss below supply chain coordination with the option contract. Obviously, an option contract that provides the retailer with an incentive to reserve as much as \( Q_{opt} = F^{-1}\left(\frac{p - c}{p - v}\right) \) will make the supply chain system achieve the system-wide optimal expected profit. This perspective yields the following proposition.

**Proposition 4**

(i) The system-wide optimal expected profit of the supply chain can be achieved under any option contract \((o, e)\) in the following set \( M \):

\[
M = \{(o, e) : o = \lambda (c - v), e = (1 - \lambda)p + \lambda v, \quad \text{where} \quad \lambda \in [0, 1]\}.
\]

Compared with the wholesale price mechanism, any option contract in \( M \) can increase the expected profit of the supply chain system by

\[
\Delta \pi = (p - c)(Q_{wm} - Q_{wT}) - (p - v) \int_{Q_{wT}}^{Q_{wm}} F(x)dx.
\]

(ii) Under any option contract \((o, e)\) in \( M \), we have \( Q_{wm} = Q_{om} = Q_{ot} \). Furthermore, with the option contract associated with \( \lambda \), the maximum expected profit received by the retailer, denoted by \( \pi_{om}(\lambda) \), is given by

\[
\pi_{om}(\lambda) = \lambda \pi_c,
\]

and the maximum expected profit received by the manufacturer, denoted by \( \pi_{om}(\lambda) \), is given by

\[
\pi_{om}(\lambda) = (1 - \lambda) \pi_c,
\]

where \( \pi_c \) given by (7), is the system-wide optimal expected profit of the supply chain.

The relationship between \( o \) and \( e \) can be derived from (8) as

\[
e = p - \frac{p - v}{c - p} o,
\]

which has the following implications: (i) the exercise price negatively correlates with the option price, and (ii) an increase in the option price by an unit will induce a decrease in the exercise price by \( \frac{1}{1 - \lambda} \). This coincides with the intuition in practice that the retailer will under the wholesale price mechanism. Since

\[
\lambda = \frac{c - p - v}{p - v},
\]

\( \lambda \) strictly increases in \( \lambda \), only the option contracts in \( M \) with \( \lambda \) satisfying \( 0 < \lambda < \frac{c - p - v}{p - v} \) will make the retailer strictly better off than that under the wholesale price mechanism. Any other option contract in \( M \) will make the retailer worse off and subsequently is unacceptable to it. We denote \( \lambda_{min} = \frac{c - p - v}{p - v} \). Similarly, we see from (11) that, with an option contract associated with \( \lambda = \frac{c - p - v}{p - v} \) in \( M \), the manufacturer will be just as well off as that it will under the wholesale
price mechanism. Again, since \((1 - \lambda)\pi_r\) strictly decreases in \(\lambda\), only the option contracts in \(M\) with \(\lambda\) satisfying \(0 \leq \lambda < \frac{1}{C_0}\) will make the manufacturer earn more profit than that under the wholesale price mechanism. Any other option contract in \(M\) is unacceptable to it because it will make the manufacturer worse off. We denote \(\lambda_{\text{max}} = \frac{1}{C_0}\). To summarize, hence, only the option contracts in the set \(M\) with \(\lambda \in [\lambda_{\text{min}}, \lambda_{\text{max}}]\) will make both the retailer and the manufacturer at least as well off as that under the wholesale price mechanism, and the remaining \(\lambda\)'s, even though making the supply chain achieve system-wide optimal profit, should be excluded from \(M\) because they are unacceptable by either the retailer or the manufacturer. This discussion leads to the following proposition, which gives the core of the option contract set \(M\).

**Proposition 5.** The core of the option contract set \(M\) is given by

\[
N = \{(o, e) : (o, e) \in M, \lambda \in [\lambda_{\text{min}}, \lambda_{\text{max}}]\},
\]

where

\[
\lambda_{\text{min}} = \frac{\pi_{\text{ar}}}{\pi_r} \quad \text{and} \quad \lambda_{\text{max}} = \frac{\pi_c - \pi_{\text{wm}}}{\pi_c}.
\]

Fig. 1 graphically illustrates the problem of profit allocation between the retailer and the manufacturer under the option mechanism. If there is no coordination, the retailer receives profit \(\pi_{\text{ar}}\) and the manufacturer receives profit \(\pi_{\text{wm}}\), represented in Fig. 1 as point “A”. Except the point “E” (denoted by a square), all the other points on the line segment D–E correspond to the profit allocations under the option contracts given as in \(M\). However, due to the fact that neither the retailer nor the manufacturer is willing to accept less after coordination is achieved than before it is achieved, the likely allocations of the profit will only correspond to the points on the line segment B–C, which correspond to the profit allocations under the option contracts in the core \(N\). The point “B” corresponds to the coordinating option contract under which the manufacturer captures all the additional profit gained from coordination, whereas the retailer just earns as much as before coordination is achieved. The converse is applicable to the point “C”. Other points in the line segment B–C correspond to those option contracts under which the retailer and the manufacturer both receive some benefits from coordination.

### 6. Selection of option contracts

The discussion at the end of Section 5 implies that any option contract in the core \(N\) can be exploited to coordinate the supply chain with Pareto-improvement, as compared with the wholesale price mechanism. Consequently, the manufacturer and retailer will both have strong incentives to shift to adopting an option contract in \(N\) from the wholesale price mechanism. However, for different option contracts in \(N\), the profit allocations between the manufacturer and retailer differ. The option contract with a larger \(\lambda\) is preferred by the retailer and the converse is preferred by the manufacturer. As a result, they will negotiate over which option contract to select from \(N\), or equivalently over selecting \(\lambda\) from \([\lambda_{\text{min}}, \lambda_{\text{max}}]\). In this section we discuss issues concerning selection of the option contract that the retailer and manufacturer both agree to implement and the corresponding profit allocation.

For ease of exposition, we denote:

\[
\begin{align*}
\Delta \pi_r(\lambda) &= \pi_{\text{ar}}(\lambda) - \pi_{\text{ar}} = \lambda \pi_r - \pi_{\text{ar}}, \\
\Delta \pi_m(\lambda) &= \pi_{\text{wm}}(\lambda) - \pi_{\text{wm}} = (1 - \lambda)\pi_r - \pi_{\text{wm}},
\end{align*}
\]

where \(\Delta \pi_r(\lambda)\) and \(\Delta \pi_m(\lambda)\) correspond to the additional profits split by the retailer and the manufacturer from \(\Delta \pi\), which are their own increased profits from coordination with the option contract associated with \(\lambda\) in \(M\). \(\Delta \pi\), given as (9), corresponds to the total extra profit from coordination. Clearly, \(\Delta \pi_r(\lambda) + \Delta \pi_m(\lambda) = \Delta \pi\) for all \(\lambda\). For simplicity, we will omit the argument \(\lambda\) hereafter and denote \(\Delta \pi_r(\lambda)\) and \(\Delta \pi_m(\lambda)\) as \(\Delta \pi_r\) and \(\Delta \pi_m\), respectively.

Since the demand is stochastic, \(\Delta \pi\) must be uncertain for any option contract in \(N\). We assume that such uncertainty is represented by the probability distribution of \(\Delta \pi\). In order to focus on

![Fig. 1. Allocations of the coordinating profit.](image-url)
the determination of the ultimate coordinating option contract and the allocation of the additional profit, we assume that the retailer is consistent with the manufacturer with respect to the probability distribution of $\Delta r$. Also, we suppose that both the retailer and the manufacturer have risk preferences towards the profit shared from $\Delta r$ and their preferences are represented by von Neumann and Morgenstern's (vN–M) utility functions (von Neumann and Morgenstern, 1953) with respect to $(\Delta r, \Delta m)$, which can be assessed by their preferences over lotteries involving $(\Delta r, \Delta m)$. Let $U(\Delta r_i, \Delta m)$ be the retailer's $v$N–M utility function and $U_m(\Delta r_i, \Delta m)$ is the manufacturer's. For more details about the assessment of utility functions, the reader is referred to, e.g., Fishburn (1970) and Keeney and Raiffa (1976). We also assume that for the supply chain system, there is a system utility function $U(\Delta r, \Delta m)$ that is based on individual members' utility functions in the supply chain and their bargaining positions, and is determined under the "linear aggregation rule". There are various forms of utility functions, among which the additive form is used extensively in realistic applications of decision analysis. A utility function is said to be additive if it has the following form:

$$U_i(\Delta r_i, \Delta m) = \gamma_i U_i(\Delta r_i) + \gamma_2 U(\Delta m),$$

(16)

where $U_i(\Delta r_i)$ is the conditional utility function of member $i (i = r, m)$ for $\Delta r_i$ (assumed to be a monotonic and increasing function of $\Delta r_i$). Similar implications are applicable to $U(\Delta m)$, and $\gamma_1$ and $\gamma_2$ are positive scaling constants. A utility function is additive if and only if $\Delta r_i$ and $\Delta m$ are additively independent, which means that the preferences over $(\Delta r_i, \Delta m)$ only depend on their marginal probability distributions (Keeney and Raiffa, 1976). Under the linear aggregation rule, the supply chain system utility function $U(\Delta r, \Delta m)$ is expressed as

$$U_i(\Delta r_i, \Delta m) = \delta U_i(\Delta r_i, \Delta m) + \epsilon_m U_m(\Delta r, \Delta m),$$

(17)

where $\delta_i$ and $\epsilon_m$ are the aggregation weights that reflect the relative negotiating powers of the retailer and the manufacturer on the bargaining table. Without loss of generality, we suppose $\delta_i + \epsilon_m = 1$. The general additive form of an utility function implies that an individual has preferences not only over his own share but also over his partner's share. However, a reasonable special case is the degenerate additive form, in which an individual only cares about his own share. This form has received extensive attention in the decision science literature (Ellaschberg, 1986; Raiffa, 1968). In our discussion, we focus only on the degenerate form of the individual's utility function. Thus, we simplify the retailer's utility function $U_i(\Delta r_i, \Delta m)$ as $U_i(\Delta r_i)$ and the manufacturer's utility function $U_m(\Delta r_i, \Delta m)$ as $U_m(\Delta m)$. Also, we define the Pratt–Arrow risk aversion functions (Pratt, 1964) as follows:

$$R(\Delta r_i) = \frac{U_i'(\Delta r_i)}{U_i''(\Delta r_i)}, \quad R_m(\Delta m) = \frac{-U_m''(\Delta m)}{U_m''(\Delta m)},$$

(18)

where the single prime indicates the first derivative of $U_i (i = r, m)$ and the double prime denotes the second derivative of $U_i$ (the same are true for these notations below). $R(\Delta r_i)$ represents the retailer's risk aversion measurement for $\Delta r_i$ and $R_m(\Delta m)$ represents the manufacturer's for $\Delta m$. In the following sections, taking into account supply chain members' risk preferences and negotiating powers, we will discuss issues concerning implementation of the option contract by Nash's bargaining model and Ellaschberg's model, and explore analytically how risk preferences and negotiation powers affect the ultimate implementation outcome. While it is difficult to obtain meaningful insights on these issues by assuming completely abstract utility functions, we consider in the following several concrete types of the utility function that have been used extensively in the decision science literature (see, e.g., Kohli and Park, 1989; Huang and Li, 2001; Huang et al., 2002). Despite this limitation, we are able to obtain some helpful insights on these issues. Furthermore, our approach to address these issues can be easily extended to any other type of the utility function.

6.1. The case of Nash's bargaining model

Nash's bargaining model predicts that the option contract that the retailer and manufacturer both agree to implement maximizes the product of each member's utility over their own disagreement point, which, herein, is represented by the profit level under the wholesale price mechanism. We consider below several cases for illustration.

Case 1. Consider a manufacturer–retailer supply chain in which the retailer's utility function is $U_i(\Delta r_i) = (\Delta r_i)^\gamma$ and the manufacturer's utility function is $U_m(\Delta m) = (\Delta m)^\gamma$, where $0 < \gamma < 1$ and $0 < \beta < 1$. By the Pratt–Arrow risk aversion functions, we know that $-U_i''(\Delta r_i) = -U_m''(\Delta m) = 1-\gamma$. Therefore, a smaller $\gamma$ indicates a more risk-averse retailer. The Nash's bargaining solution is obtained by solving the following programming problem:

$$P_1: \max_{\lambda \in [0,1]} \min_{(o,e)} U_m(\Delta m)U_i(\Delta r_i) = [\pi_{om}(\lambda) - \pi_{om}]^\gamma [\pi_{ar}(\lambda) - \pi_{ar}]^\gamma$$

s.t. $(o,e) \in N.$

(19)

The optimal solution for problem $P_1$ is clearly the option contract in $N$ with $\lambda$ solving the following programming problem:

$$P_2: \max_{\lambda \in [\lambda_{min}, \lambda_{max}]} \min_{\lambda} U_m(\Delta m)U_i(\Delta r_i) = [(1 - \lambda)\pi_{c} - \pi_{om}]^\gamma (\lambda \pi_{C} - \pi_{ar})^\gamma$$

s.t. $\lambda \in [\lambda_{min}, \lambda_{max}].$

(20)

where $\lambda_{min} = \frac{\pi_{C}}{\pi_{cr}}$ and $\lambda_{max} = \frac{\pi_{om}}{\pi_{ar}}$. Solving problem $P_2$, we give the following proposition.

**Proposition 6.** Based on Nash's bargaining model, for a manufacturer–retailer supply chain where the retailer's utility function is $U_i(\Delta r_i) = (\Delta r_i)^\gamma (0 < \gamma < 1)$ and the manufacturer's is $U_m(\Delta m) = (\Delta m)^\gamma (0 < \beta < 1)$, the share split by a member is inversely proportional to its degree of risk averse, and the option contract predicted is given by

$$(o,e) = ((\lambda^*(c - v), (1 - \lambda^*)p + \lambda^*v),$$

(21)

where $\lambda^* = \frac{\pi_{ar} - \pi_{om}}{\pi_{ar} + \pi_{om} - \pi_{cr}}$. Accordingly, the expected profit obtained by the retailer is $\pi_{ar}(\lambda^*) = \pi_{ar} + \frac{\pi_{ar}}{\pi_{cr}} \Delta r$ and the expected profit obtained by the manufacturer is $\pi_{om}(\lambda^*) = \pi_{om} + \frac{\pi_{ar}}{\pi_{cr}} \Delta r$. In particular, when the retailer and the manufacturer are equally risk-averse or both are risk-neutral, they will split the extra profit $\Delta m$ in equal proportions.

Case 2. Consider a risk-averse retailer with a strictly concave utility function $U_i(\Delta r)$ and a risk-neutral manufacturer with an utility function $U_m(\Delta m)$ = $\Delta m$. We suppose that $U_i(\Delta r)$ is increasing, twice differentiable, and $U_i(0) = 0$. Then, similar to Case 1, the Nash's bargaining solution can be obtained by solving the following programming problem:

$$P_3: \max_{\lambda \in [\lambda_{min}, \lambda_{max}]} U_i(\Delta r_i)U_m(\Delta m) = U_i(\lambda \pi_{C} - \pi_{ar})(1 - \lambda)\pi_{C} - \pi_{om})$$

s.t. $\lambda \in [\lambda_{min}, \lambda_{max}].$

(22)

Solving problem $P_3$, we obtain the following proposition.
**Proposition 7.** Based on Nash’s bargaining model, for a manufacturer–retailer supply chain consisting of a risk-averse member with a strictly concave utility function \(U(t)\) being increasing, twice differentiable, and \(U(0) > 0\), and a risk-neutral member, the risk-averse member will obtain a smaller share of the extra profit than the risk-neutral one.

**Example 1.** Consider a risk-averse retailer with the exponential utility function \(U(\Delta \pi) = 1 - \exp(-\Delta \pi)\) \((\alpha > 0)\), and a risk-neutral manufacturer with the utility function \(U_m(\Delta \pi_m) = \Delta \pi_m\). It is clear that \(U(\Delta \pi)\) is strictly concave, increasing, twice differential, and \(U(0) = 0\). By Proposition 7, we know that Nash’s bargaining model predicts that the risk-averse retailer will obtain a smaller share of the extra profit than the risk-neutral manufacturer, and the option contract that the retailer and manufacturer both agree to implement is given by

\[
(\bar{\theta}, \bar{\gamma}) = \left( \bar{\lambda}(c - v), (1 - \bar{\lambda})p + \lambda pv \right),
\]

where \(\lambda\), from the proof of Proposition 7, is the unique solution of (24):

\[
\exp(2\lambda\pi_c) + i\pi_c^2 \exp(i\pi_m) - [\pi_c(\pi_c - \pi_{\min}) + 1] \exp(i\pi_m) = 0.
\]

6.2. The case of Eliashberg’s model involving negotiating power

It is worth pointing out that Nash’s bargaining model does not take individual members’ negotiating powers into account while predicting the outcome, which is a severe deficiency of the model because the selection of a contract clearly depends on supply chain members’ negotiating powers. In order to overcome this deficiency, an alternative way is to apply the approach introduced by Eliashberg (1986). Eliashberg’s model predicts an option contract that maximizes the group utility function reflecting the joint preferences of the supply chain members. By Eliashberg’s model, we can incorporate supply chain members’ negotiating powers into the ultimate implementation outcome, with the use of aggregation weights that measure the relative negotiating powers of the supply chain members. We consider an example for illustration in the following.

**Example 2.** Consider a supply chain consisting of a risk-averse retailer with the exponential utility function \(U(\Delta \pi) = -\exp(-\Delta \pi)\) and a manufacturer also with the exponential utility function \(U_m(\Delta \pi_m) = -\exp(-\beta \Delta \pi_m)\), where \(\alpha, \beta > 0\). By the Pratt–Arrow risk aversion functions, it is easy to know that a larger \(\alpha\) or \(\beta\) indicates a more risk-averse member. We suppose the retailer’s relative power is measured by \(r_1\) and the manufacturer’s by \(r_2\). Without loss of generality, we assume \(r_1 + r_2 = 1\). Then, Eliashberg’s model predicts an option contract in the core \(N\) with \(\lambda\) that solves the following programming problem:

\[
P_4: \quad \max_{\lambda} U(\Delta \pi, \Delta \pi_m) = -r_1 \exp(-\Delta \pi) - r_2 \exp(-\beta \Delta \pi_m)
\]

s.t. \(\lambda \in [\lambda_{\min}, \lambda_{\max}]\),

where \(U(\Delta \pi, \Delta \pi_m)\) is the supply chain system utility function over \((\Delta \pi, \Delta \pi_m)\) under the linear aggregation rule. Denote the optimal solution for problem \(P_4\) as \(\bar{\lambda}\). Similar to the proof of Proposition 7, we can show the strict concavity of \(U(\Delta \pi, \Delta \pi_m)\) with respect to \(\lambda\), by which we obtain the following results (we omit the details for the sake of conciseness):

(i) If \(\bar{\lambda} < \frac{\pi_{\max}}{\pi_{\max} + \pi_{\min}}\), then the optimal solution \(\lambda = \lambda_{\max}\) the corresponding option contract is given by \((\bar{\theta}, \bar{\gamma}) = (\lambda_{\max}(c - v),\)

\[
(1 - \lambda_{\max})p + \lambda_{\max}pv, \quad \text{and the allocation of the extra profit} \quad \Delta \pi \text{between the retailer and the manufacturer is given by} \quad (\Delta \pi, \Delta \pi_m) = (\Delta \pi, 0).
\]

(ii) If \(\lambda > \frac{\pi_{\max}}{\pi_{\max} + \pi_{\min}}\), then \(\lambda = \pi_{\min}\), the corresponding option contract is given by \((\bar{\theta}, \bar{\gamma}) = (\pi_{\min}(c - v), (1 - \pi_{\min})p + \pi_{\min}pv)\), and the allocation of the extra profit \(\Delta \pi\) is given by \((\Delta \pi, \Delta \pi_m) = (\Delta \pi, \pi_{\min})\).

(iii) If \(\lambda \in \left(\frac{\pi_{\max}}{\pi_{\max} + \pi_{\min}}, \pi_{\min}\right)\), then

\[
\lambda = \frac{\alpha}{\alpha + \beta} \lambda_{\min} + \frac{\beta}{\alpha + \beta} \lambda_{\max} - \ln \frac{\pi_{\min}}{\pi_{\max}} \ln \frac{\pi_{\min}}{\pi_{\max}} + \pi_{\min}\pi_{\max}/(\alpha + \beta).
\]

the corresponding option contract is given by \((\bar{\theta}, \bar{\gamma}) = (\lambda^\prime, c - v)\), \((1 - \lambda^\prime)p + \lambda^\prime pv\), where \(\lambda^\prime\) is given by (26), and the allocation of the extra profit \(\Delta \pi\) is given by

\[
\Delta \pi_m(\lambda^\prime) = \frac{\beta}{\alpha + \beta} \Delta \pi - \ln \frac{\pi_{\min}}{\pi_{\max}} \ln \frac{\pi_{\min}}{\pi_{\max}} + \pi_{\min}\pi_{\max}/(\alpha + \beta).
\]

Hence, based on Eliashberg’s model, for such a supply chain, the retailer will obtain a share \(\bar{\lambda}_{\min}\) from the extra profit \(\Delta \pi\) and the manufacturer will obtain a share \(\bar{\lambda}_{\max}\). A compensation fee between the retailer and the manufacturer is \(\bar{\lambda}_{\max}\), which represents a fee paid by the retailer to the manufacturer if \(\bar{\lambda}_{\max} > \frac{\pi_{\max}}{\pi_{\max} + \pi_{\min}}\); otherwise, from the manufacturer to the retailer. Clearly, we see from (27) that the proportions shared by the retailer and the manufacturer do not depend on their relative power measurements \(r_1\) and \(r_2\) but only depend on their risk aversion measurements \(\alpha\) and \(\beta\). The more risk-averse a member is, the less share it will obtain from the extra profit \(\Delta \pi\). As for the compensation fee, we observe that when \(\frac{\pi_{\max}}{\pi_{\max} + \pi_{\min}} < \frac{\alpha}{\alpha + \beta}\) an increase in \(r_2\) or a decrease in \(r_1\) means an increasing compensation fee from the retailer to the manufacturer. Particularly, when \(r_2\) increases or \(r_1\) decreases to the point where \(\frac{\pi_{\max}}{\pi_{\max} + \pi_{\min}} < \frac{\alpha}{\alpha + \beta}\), the manufacturer will receive an extra profit from the retailer that is just equal to \(\frac{\alpha}{\alpha + \beta}\) \(\Delta \pi\). In other words, with an increase in the relative power of the manufacturer with respect to the retailer, it will receive a higher compensation fee from the retailer. When the manufacturer’s relative power is high enough with respect to the retailer (e.g., \(\frac{\alpha}{\alpha + \beta} > \frac{\pi_{\max}}{\pi_{\max} + \pi_{\min}}\)), the result of coordination will be that the manufacturer captures all the extra profit, whereas the retailer receives nothing from coordination. A similar analysis is applicable to the case where \(\frac{\alpha}{\alpha + \beta} > \frac{\pi_{\max}}{\pi_{\max} + \pi_{\min}}\). Besides, it is worth noting that when the retailer and the manufacturer are equally risk-averse, i.e., \(\alpha = \beta\), they will split the extra profit in equal proportions, and their relative power measurements \(r_1\) and \(r_2\) will be the only factors that decide whether a member receives a positive or negative compensation fee from the other one.

7. Conclusions

In view of the current industry environment in which cooperative relations are increasingly becoming prevalent in supply chains, we took a cooperation approach in this paper to address the coordination issues for manufacturer–retailer supply chains using option contracts. Given that the wholesale price mechanism is the prevalent form used in manufacturer–retailer supply chains in practice, we took the profit level under the wholesale price mechanism as the benchmark against which we compared our developed option contracts. Our study demonstrates that, compared with the wholesale price mechanism, any option contract in the core can coordinate the manufacturer–retailer supply chain with Pareto-improvement. In addition, we also explored analytically the issues concerning implementation of the coordinating option contract form, taking into account supply chain members’ risk preferences and negotiating powers. Our study demonstrates that...
under the option mechanism: (i) an individual’s risk preference plays a significant role in the coordination outcome. Specifically, when the retailer and manufacturer are both risk-averse, the more risk-averse a member is, the smaller share of the extra profit it will obtain. When the retailer and the manufacturer are equally risk-averse or both are risk-neutral, they will split the extra profit in equal proportions. (ii) In addition to risk preferences, an individual’s negotiating power also has significant effects on the ultimate outcome. The higher a member’s relative negotiating power is, the higher the compensation fee it will receive from the other side, and vice versa. Our findings provide a relatively comprehensive insight on how option contracts can be used to coordinate a manufacturer–retailer supply chain.

It is worth noting that for any option contract in the core N, both the manufacturer and the retailer operate under voluntary compliance, which means that the manufacturer’s optimal production quantity coincides with the retailer’s optimal reserved quantity, given the option contract terms (which can be seen from Proposition 4). Clearly, to a large extent, this property improves the robustness of the option contracts developed in this paper. Besides, it should also be noted that the option contracts in the core N are “distribution-free”, which implies that they can be utilized to coordinate the supply chain without knowing the demand distribution of the retailer. This property renders their implementation easier in practice. In addition, as a remark, we should point out that in our model we neglected the penalty cost for demand that is not satisfied. In fact, we have checked that such an additional consideration has no effect on the results developed in this paper, other than complicating the notation. In addition, we wish to point out that the buy-back contract and the option contract differ fundamentally in their impacts on the behaviours of supply chain members and on the cash flow of the supply chain. For example, in contrast to the buy-back contract under which the retailer often places the order after the commencement of the production season, the option contract prompts the retailer to order before the production season begins. Hence, under the option contract, the retailer can take advantage of order quantity flexibility to better accommodate changes in demand. On the other hand, the manufacturer enjoys the retailer’s early commitment and can have better capacity and materials planning. However, this is not the case for buy-back contracts. As a natural extension of our work, future studies should consider coordination issues in supply chains with bidirectional options by which the retailer can adjust freely the initial order quantity both upwards and downwards. Attempts to study this issue have been made by Milner and Rosenblatt (2002) and Wang and Tsao (2006), who considered the issue from the buyer’s perspective.

Acknowledgements

The authors thank an editor and three anonymous referees for their constructive comments and suggestions on earlier versions of this paper. This work was supported in part by the Logistics Research Centre of The Hong Kong Polytechnic University.

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version, at doi:10.1016/j.ejor.2010.05.017.

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