Deterministic Formulization of Bandwidth Efficiency for Multicast Systems

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Abstract

End-System multicasting (ESM) is a promising application-layer scheme that has been recently proposed for implementing multicast routing in the application layer as a practical alternative to the IP multicasting. Moreover, ESM is an efficient application layer solution where all the multicast functionality is shifted to the end users. However, the limitation in bandwidth and the fact that the message needs to be forwarded from host-to-host using unicast connection, and consequently incrementing the end-to-end delay of the transmission process, contribute to the price to pay for this new approach. Therefore, supporting high-speed real-time applications such as live streaming multimedia, videoconferencing, distributed simulations, and multiparty games require a sound understanding of these multicasting schemes such as IP multicast and ESM and the factors that might affect the end-user requirements. In this paper, we present both the analytical and the mathematical models for formalizing the bandwidth efficiency of both IP and ESM multicast system. Specifically, our proposed formalization of the bandwidth efficiency is based on the end-to-end delays proposed by [11] for both IP and ESM multicast systems. For the sake of the experimental verifications of the proposed models, several numerical and simulation results are presented in this paper. Finally, the proposed formulization can be used to design and implement a more robust and efficient multicast systems for the future networks.

Keywords – Unicast, multiple unicast, IP multicast, End-System multicasting, and overlay networks.
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Abstract— End-System multicasting (ESM) is a promising application-layer scheme that has been recently proposed for implementing multicast routing in the application layer as a practical alternative to the IP multicast. Moreover, ESM is an efficient application layer solution where all the multicast functionality is shifted to the end users. However, the limitation in bandwidth and the fact that the message needs to be forwarded from host-to-host using unicast connection, and consequently incrementing the end-to-end delay of the transmission process, contribute to the price to pay for this new approach. Therefore, supporting high-speed real-time applications such as live streaming multimedia, videoconferencing, distributed simulations, and multiparty games require a sound understanding of these multicasting schemes such as IP multicast and ESM and the factors that might affect the end-user requirements. In this paper, we present both the analytical and the mathematical models for formalizing the bandwidth efficiency of both IP and ESM multicast system. Specifically, our proposed formulization of the bandwidth efficiency is based on the end-to-end delays proposed by [11] for both IP and ESM multicast systems. For the sake of the experimental verifications of the proposed models, several numerical and simulation results are presented in this paper. Finally, the proposed formulization can be used to design and implement a more robust and efficient multicast systems for the future networks.

I. INTRODUCTION

There is an emerging class of Internet and Intranet multicast applications that are designed to facilitate the simultaneous delivery of information from a single or multiple senders to multiple receivers. Different approaches of multicasting have been suggested to improve the overall performance of networks especially the Internet. These approaches are: multiple unicast, IP multicast, and end-system multicast (ESM). All of these methods have some advantages and disadvantages but the last two approaches (IP multicast, and ESM) mentioned above have had more research effort in terms of performance evaluation of networks. ESM uses an overlay structure, which is established on top of the traditional unicast services. In this way, every pair of edges (source-destination) is a unicast connection. The overlay has its meaning from the fact that the same link can have multiple unicast connections for multiple pair of edges. Although, ESM seems to have many advantages (no further changes to the network are required, user has more control of the application layer, no need of special multicast router capability, etc), there is a penalty to pay. In the overlay structure, hosts are able to multicast information and consequently use the same link to redirect packets increasing the end-to-end delay of the entire transmission process. Another problem is the number of receivers that a potential “multicast” host can support. End users have a limited bandwidth and suffer the last mile problem.

While these different multicasting approaches can displace some of the costs of face-to-face communications, their true potential business benefit lies in improving the accessibility and timeliness of information, vastly increasing its value to both the organization and individual employees. Although research on multicast dates back to the early days of the Internet, it has yet to produce a multicast service that is ubiquitously and economically available. In spite of the performance advantages, commercial deployment of multicast has not yet been fully realized. One of factors that prevent the wide-range deployment of multicast is the difficulty in providing reliable multicast transport.

II. THEORETICAL ANALYSIS

In this section, we will theoretically analyze the problems of different level of multicasting, which hinder their performance with respect to the bandwidth utilization and latency.

A. IP Multicast

The IP-multicast capable version of the network shown in Fig. 1 consists of network with native multicast support. IP multicast capable routers are consider along the path. The traditional process includes the construction of a source-rooted tree together with the members of the multicast group. Since only one copy of the message is required, we can say that a minimum bandwidth effort is being used for the transmission of the message to all group members connected in the network. The problem for IP multicast is that there is no commercial support for multicast routers. Investors still think
that there is not enough multicast application demand and that multicast traffic could take their routers down due to congestion problems. The IP-multicast transmission takes the same bandwidth on source host's network as a single copy, regardless of how many clients are members of the destination host group in the Internet.

Besides the advantages of IP multicast, there are also certain drawbacks of this approach. One of them is the deployment problem of IP multicast, which imposes dependency on routers. The main disadvantage of IP multicast is the need of commercial routers supporting multicast protocol. Existing IP multicast proposals [1] [2] embed an assumption of universal deployment, as all routers are assumed to be multicast capable. The lack of ubiquitous multicast support limits the deployment of multicast applications, which in turn reduces the incentive for network operators to enable multicast. Therefore, from the above discussion one can expect that we need another multicast alternative in which network routers have not to do all of the work; instead each of the host will equally contribute in the overall multicast process of the messages.

B. End-system Multicast (ESM)

Because of the limitations in IP multicast, researchers have explored an alternative architecture named ESM, which built a system on top of the unicast services with multicast functionalities. ESM is a very promising application layer solution where all the multicast functionality is shifted to the end users as shown in Fig. 2. However, doing multicasting at end-hosts incurs in some performance penalties. The structure of the ESM is an overlay in a sense that each of the paths between the end systems corresponds to a unicast path. Here the membership and replication functionality is performed by the end receivers, which connect together over unicast channels to form a multicast tree, rooted at one data source. The end receivers could play the role of parent or children nodes. The parent nodes perform the membership and

Fig. 1. Example of IP Multicast with four sources and routers along with 10 destination systems

Fig. 2. Example of ESM, solid and dotted lines represent two ways and one way packet transmission, respectively.
replication process. The children nodes are receivers who are getting data directly from the parent nodes. There is one central control server and one central data server residing in the same root source. Any receiver can play the role of parent to forward data to its children. Each client has two connections: a control connection and a data connection.

III. PROPOSED FORMULIZATION OF BANDWIDTH EFFICIENCY FOR MULTICAST SYSTEMS

This section presents the formulization of the bandwidth efficiency for all multicasting schemes. For the ease of simplicity, we divide our proposed formulization for each type of multicasting approach such as unicast, multiple unicast, IP multicast, and ESM.

A. Model and Assumptions

Let G is an irregular graph that represents a network with a set of N vertices and M edges such as: \( G = \{N, M\} \). Let L is a direct communication link between a single pair of source (s) and destination (d) where both source and destination belong to N such as: \( \{s, d\} \in N \). In addition, each packet transmitted between source (s) and destination (d) must traverse one or more communication links in order to reach the final destination.

Let the value of \( D(L) \) denotes packet-delay that is associated with each direct communication link. Therefore, each transmitted packet will typically experience a delay of \( D(L) \) on a particular link. The delay includes transmission, processing, and propagation delays such as: Link Delay = \( D(L) = Transmission \) Delay + Propagation Delay + Processing Delay where \( L \in M \). In connection less communication such as IP network, there might be multiple routes exist between a pair of source and destination. As a result, each packet might follow a different route in order to reach the final destination where each route requires traversing of one or more communication links (L). A single route between a pair of source and destination can be defined as: \( R\{s, d\} \) where \( \{s, d\} \in N \).

In order to approximate the bandwidth for Unicast, multicast, and ESM, we use the classical definition of computing the transmission time. Based on this definition, the transmission time \( (T_T) \) can be defined as a product of the packet size \( (P_r) \) which we transmit between a pair of source (s) and destination (d) and the inverse of the bandwidth \( (B_w) \). Mathematically, this can be expressed as follows:

\[
T_T = \left( \frac{1}{B_w} \right) \left( \frac{1}{P_r} \right)
\]

Let the value of \( D(s \rightarrow d) \) denotes the total packet-delay which is associated with each direct communication link. Therefore, each transmitted packet will typically experience a delay of \( D(s \rightarrow d) \) on a particular link (L) between a pair of source (s) and destination (d). This delay is the sum of the transmission time, the queuing and the propagation delays such as: \( D(s \rightarrow d) = \) Transmission Time \( (T_T) + \) Propagation Delay \( (\tau) + \) Queuing Delay \( (D_Q) \). Also, it can be expressed as:

\[
D(s \rightarrow d) = T_T + \tau + D_Q
\]

Changing the above expression for the transmission delay, we got

\[
T_T = (D(s \rightarrow d)) - (\tau) - (D_Q)
\]

Recall our classical definition for the transmission time, equation (1) can be rewritten as:

\[
\left( \frac{1}{B_w} \right) P_r = (D(s \rightarrow d)) - (\tau) - (D_Q)
\]

Solving equation (2) for approximating the bandwidth, we got

\[
B_w = \frac{P_r}{(D(s \rightarrow d)) - (\tau) - (D_Q)}
\]

It should be noted that the propagation delay \( (\prod L) \) is a ration between the distance for a communication link \( (L) \) and the speed of light \( (SoL) \). This allows us to further extend equation (3).

\[
B_w = \frac{P_r}{(D(s \rightarrow d)) - \left( \frac{L}{SoL} \right) - D_Q}
\]

Simplifying the above equation (4), we got

\[
B_w = \frac{(P_r)(SoL)}{\left(SoL \right) \left(D(s \rightarrow d) - L_D - (SoL)(D_Q)\right)}
\]

For the sake of simplicity, we can ignore the queuing delay. Equation (5) can now be written as:

\[
B_w = \frac{(P_r)(SoL)}{\left(\left(\frac{SoL}{L_D} \right) \left(D(s \rightarrow d) - L_D\right)\right)}
\]

B. Bandwidth Efficiency Formulization for a Unicast System

In unicast, a packet is sent from one point (source) to another point (destination). As mentioned earlier, when packet transmit from one source (s) to a specified destination (d), there exist multiple routes where each route can have multiple links. This implies that the packet-delay for unicast is entirely dependent on the number of links a packet needs to traverse in order to reach the final destination system. Based on the above argument, one can define the packet delay such as:

\[
D(R) = D(L_1) + D(L_2) + \ldots \ldots + D(L_n)
\]

where \( n \) is the maximum number of links that need to be traversed on route \( R \) between s and d.
We generalize the delay for one particular route \((R)\) that exists between source \((s)\) and destination \((d)\) as:

\[
D(R) = \sum_{i=1}^{n} D(L_i)
\]

where

\[
\sum_{i=1}^{n} (L_i) = L_1 + L_2 + \ldots + L_n \in M.
\]

This expression is further extended as:

\[
\text{Delay} = D_{s-d} = \sum_{L \in R_{s-d}} D(L)
\]

where \(L \in R_{s-d}\) represents the value of the total delay associated with the route \(R\) between source \(s\) and destination \(d\).

For a unicast system, taking the above expressions into account, the available estimated bandwidth \((B_w)\) for a communication link \((L)\) that exists between a pair of source \((s)\) and destination \((d)\) can be approximated in the following equation:

\[
B_w = \frac{(P_s)(SoL)}{(SoL) \left( \sum_{L \in R_{s-d}} D(L) \right) - L_D}
\]

(7)

The \(D(L)\) in (7) represents the link delay where as the \(L \in R_{s-d}\) represents the value of the total delay associated with the route \(R\) between source \(s\) and destination \(d\).

The above equation (7) represents the approximated bandwidth which can be used by the transmitted packet for each individual communication link between the source and destination. It should be noted that the above equation is not representing the bandwidth approximation for one particular route between the source and destination. Instead, it represents the bandwidth approximation for \(n\) number of links that need to be traversed on route \(R\) between source \((s)\) and destination \((d)\).

Based on the above derivation, one can also simply derive a mathematical expression for an average-bandwidth, denoted by \(AB_w\). The average bandwidth represents the available bandwidth that each transmitted packet may utilize if it traverses one of the available routes. Equation (6) can be modified for the average delay between a pair of source \((s)\) and destination \((d)\), denoted by as follows:

\[
AB_w = \frac{(P_s)(SoL)}{(SoL) \left( \sum_{L \in R_{s-d}} D(L) \right) - L_D}
\]

(8)

The mathematical expression for an estimated \(AB_w\) can be derived as follows:

\[
AB_w = \frac{(P_s)(SoL)}{(SoL) \left( \sum_{i=1}^{y} D(R_i) \right) - L_D}
\]

(9)

Where \(D(R) = \sum_{i=1}^{n} D(L_i)\)

\[
\sum_{i=1}^{n} (L_i) = L_1 + L_2 + \ldots + L_n \in M
\]

and \(y\) represents the maximum number of possible routes between source \(s\) and destination \(d\).

In addition to the average bandwidth, one can also choose the optimal route with respect to the minimum bandwidth that each packet may require when traverses from one particular source \((s)\) to a destination \((d)\). In order to derive an expression for the optimal bandwidth, we may need to modify equation (6) for the optimal delay. This is due to the fact that we assume that for each link that offers minimum bandwidth must have an optimal delay. This leads us to the following modification of (6), such as:

\[
OB_w = \frac{(P_s)(SoL)}{(SoL) \left( \text{Min} \left\{ \left[ \frac{\sum_{i=1}^{y} D(R_i)}{y} \right] \right\} \right) - L_D}
\]

(10)

where \(OB_w\) represents the optimal delay with respect to the minimum delay that each packet may experience when traverses from one particular source to a destination. Based on (10), we can derive an expression for the optimal bandwidth, denoted by \(OB_w\), between a pair of source \((s)\) and destination \((d)\) such as:

\[
OB_w = \frac{(P_s)(SoL)}{(SoL) \left( \text{Min} \left\{ \left[ \frac{\sum_{i=1}^{y} D(R_i)}{y} \right] \right\} \right) - L_D}
\]

(11)

where \(D(R) = D(L_1) + D(L_2) + \ldots + D(L_n)\) and \(n\) is the maximum number of links that need to be traversed on route \(R\) between \(s\) and \(d\).

C. Bandwidth Efficiency Formulization for a Multiple Unicast

In addition to unicast systems, we can derive the similar mathematical expressions for the multiple unicast system where a single source \((s)\) can transfer a packet simultaneously to multiple destinations. In other words, in a multiple unicast system, there exist a unicast route between a source \((s)\) and one of the destinations. This hypothesis leads us to the following argument: multiple routes can be established between the source \((s)\) and each destination \((d_1, d_2, \ldots, d_y)\) where \(y\) represents the maximum number of unicast routes established in multiple unicast. Based on this hypothesis, we can modify (6) that account the total delay such as:

\[
B_w = \frac{(P_s)(SoL)}{(SoL) \left( \sum_{i=1}^{y} D(R_i) \right) - L_D}
\]

(12)
The following mathematical expression can be used to estimate the total bandwidth that the entire packet transmission utilizes in a multiple unicast system:

$$B_{W_{s\rightarrow M}} = \frac{(P_s)(SoL)}{(SoL)\left\{\sum_{i=1}^{y} D(R_i)\right\} - L_{D}}$$

where

$$D(R_i) = \sum_{i=1}^{n} D(L_i)$$

Although, in multiple unicast system, a single packet can be transmitted from one source to multiple destinations, the transmitted packet may follow a different route in order to reach the appropriate destination. In particular, a bandwidth is always associated with links rather than the complete routes between the source and destinations.

As a result, each transmitted packet may use a different amount of bandwidth with respect to the number of links that the packet needs to traverse on the chosen unicast route. This implies that, in order to estimate an average bandwidth that each packeter might utilize, one should consider the number of maximum links a unicast route has. This leads us to the following mathematical expression for an average available bandwidth for the multiple unicast system:

$$AB_{W_{s\rightarrow d}} = \frac{(P_s)(SoL)}{(SoL)\left\{AD_{s\rightarrow d}\right\} - L_{D}}$$

Further, solving (14) for average bandwidth approximation results (15) such as:

$$AB_{W_{s\rightarrow d}} = \frac{(P_s)(SoL)}{(SoL)\left\{\sum_{i=1}^{y} D(R_i)\right\} - L_{D}}$$

where

$$D(R_i) = \sum_{i=1}^{n} D(L_i)$$

where $y$ represents the maximum number of unicast routes between a source $s$ and multiple destinations and $n$ represents the maximum number of links that a unicast route has.

### D. Bandwidth Efficiency Formulation for a IP Multicast

In IP multicast system, a single source $(s)$ sends a packet to a group that consists of multiple destination systems. In addition, a packet is sent only once by the source system where as the intermediate routers along the route perform replications with respect to the number of destinations a group has. Let $M_G$ denotes a multicast group that consists of one or more destination systems whereas $Z$ represents the size of the group such as $Z = |M_G|$. In an IP multicast system, in order to efficiently transmit a packet from a specific multicast source to all multicast destinations, all multicast groups $(M_G)$ can be typically organized in a spanning tree $(T)$. For the ease of mathematical expression, we only consider a spanning tree rooted at the multicast source $(s)$ consisting of one of the multicast groups $(M_G)$ that has a size of $Z$.  

Based on the above discussion, we describe the spanning tree such as: $T = (N_T, M_G)$ rooted as multicast source $(s)$ where the numbers of destinations in one multicast group $(M_G)$ belong to the total number of nodes present in the network such as: $M_G = \sum_{i=1}^{n} D(L_i)$ where $M$ represents the total edges that the network has. The terms $N_T$ and $M_T$ represent the vertices and the edges of the spanning tree $(T)$, respectively. It should be noted that we consider a spanning tree $(T)$ that includes only the multicast destinations of a multicast group $(M_G)$ with the exception of intermediate routers. In other words, we assume that $N_T$ of the spanning tree $(T)$ only consists of one or more destination nodes. The reason for this assumption is to simplify the process of estimating the total available bandwidth involves with the packet transmission in an IP multicast system.

Based on the above proposed model, we can give the following hypothesis: The total available bandwidth $(TB_{W})$ utilizes by multicast packets when transmitted from a root node $(s)$ to a multicast group $(M_G)$ can be defined as a rational of packet size and the available bandwidth on each link of a spanning tree $(T)$ from the root nodes $(s)$ to all destinations $(d \in M_G)$ with the bandwidth available to one or more intermediate routers of each link.

The above hypothesis leads us to the following expression for total available bandwidth $(TB_W)$ utilizes by multicast packets transmitted from root node $(s)$ to a destination node $(d)$:

$$TB_{W_{s\rightarrow d}} = \frac{(P_s)(SoL)}{(SoL)\left\{D_{s\rightarrow d}(M_G)\right\} - L_{D}}$$

Computing the total delay for the IP multicast and using its resulting values in (16), we provide an expression for the total available bandwidth such as:

$$TB_{W_{s\rightarrow M}} = \frac{(P_s)(SoL)}{\left\{\sum_{i=1}^{Z} D(L_i) + \sum_{i=1}^{n} D(L_i)\right\} - L_{D}}$$

where $Z$ in the denominator of (17) represents the number of destination systems in one multicast group of a spanning tree $(T)$ where $n$ represents the total number of links a route has.

The first term of the denominator of (17) $(\sum_{i=1}^{Z} D(L_i))$ yields the total delay associated with the number of links within a spanning tree when a packet is transmitted from a root node (source) to all the leaf and non-leaf nodes (i.e., the multiple receivers with in $T$ excluding the source node $(s)$). In other words, according to spanning tree, if a message is transmitted from a source node $(s)$ to all the destination nodes, then the packet must traverse $Z$ (i.e., the
number of destinations in one multicast group) number of links which consequently experience a delay on each link.

The second main term of the denominator ($\sum_{i=1}^{n} D(L_i)$) in (17) provides a total delay that a packet may experience when transmitted along a certain route (i.e., from a source node ($s$) to multicast group ($M_G$) via one or more routers along the route). The last term of the denominator cannot be considered as a design parameter and therefore its value completely depends on the type of network.

The above equation can be further generalized for one of the specific destinations ($d$) within a multicast group such as $d \in M_G$, if we assume that we have a route within a spanning tree ($T$) from multicast source ($s$) to a specific destination ($d$) such as $R_T$ ($s$, $d$), then the multicast packets transmitted from a source node may utilize a total available bandwidth such as:

$$TB_{W\{s\rightarrow(d\in M_G)\}} = \frac{P_s}{D_{\{s\rightarrow(d\in M_G)\}} - \left\langle \frac{L_D}{(SoL)} \right\rangle}$$

(18)

Determine the total delay for the multicast packets that transmit from a source node to one of destinations within a group and using the resulting expression in (18), we got,

$$TB_{W\{s\rightarrow(d\in M_G)\}} = \frac{(P_s)(SoL)}{-L_D} \left\langle \sum_{L_{n,Z}\in R_T(s,d)} D(L_{n,Z}) \right\rangle$$

(19)

where $L_{n,Z}$ in (19) represents the total number of links (i.e., $Z \in R_T$) that a packet needs to traverse in order to reach the specific destination $d$ along a path of $R_T$ within the tree $T$ as well as the number of links from source $s$ to a multicast group $M_G$. Since only one copy of the message is required in IP multicast, we can say that a minimum bandwidth effort is being used for the transmission of the message to all group members connected in the network. This minimum bandwidth is achieved due to the fact that a minimal transmission time is required for the IP multicast which in turns reduces the overall end-to-end delay as can be seen in the denominator of (19).

E. Bandwidth Efficiency Formulation for ESM

An ESM group can have at most $N$ end-system nodes where we focus on one of the end-system nodes ($s$) that multicast information to the other participating nodes of a multicast end-system group. From the source host point of view, this ESM multicast group can be considered a group of destination systems. For the sake of mathematical model, let $ESM_G$ denotes an ESM group that consists of one or more end-system-destinations where as $X$ represents the size of the group such as $X = 1 \ ESM_G$.

In an overlay network, all participating end-system nodes are fully connected to each other via the unicast links. Based on the derived expression of unicast in the previous sections, these unicast links that provide connection between end-system nodes can not exceed to $M$ such as: $\{m_1, m_2, \ldots, m_r\} \in M$ where one of the edges ($m$) provides a unicast connection between the two end-system nodes such as: $\{m \in M\} \stackrel{unicast-link}{\rightarrow} \{n_1, n_2\} \subset \{s, N\}$.

The structure of the ESM is an overlay network in a sense that each of the paths between the end-systems corresponds to a unicast path. This implies that an overlay network consisting of a set of $N$ end-system nodes connecting though $M$ number of edges where one of the end-system is designated as a source host ($s$) can be expressed as: $G = \{s, N, M\}$.

This also shows that an ESM is built on top of the unicast services using a multicast overlay network that can be organized in a spanning tree such as $T = (N, M_T)$ rooted as an ESM source ($s$) where the numbers of destinations in one multicast group ($ESM_G$) belong to the total number of nodes present in the network such as: $ESM_G \in M$. Here, the membership and replication functionality is performed by the ESM receivers, which connect together over unicast channels to form a multicast tree ($T$), rooted at one data source ($s$). The end receivers (i.e., the number of end-systems in $ESM_G$) in a multicast tree could be a parent or a child node depending on the location of the node. In a multicast spanning tree ($T$), all the non-leaf nodes can be both parent and child at the same time where as all the leaf nodes are considered to be the child nodes. In other words, the parent nodes perform the membership and replication process where as the children nodes are receivers that receives multicast packet from the parent nodes.

Based on the above argument, one can say that a multicast packet originated from the root ($s$) of a spanning tree ($T$) need to traverse typically two links; source to non-leaf node ($P_s$, $C_n$) and a non-leaf node to a leaf node ($C_n$). Lets $R_T$ ($s$, non-leaf node) represents a route between a source node ($s$) and non-leaf nodes that could be parent or child nodes such as: $R_T = \{P_s \vee C_n \in ESM_G\}$ where $P_s$, $C_n \in \{s \cup N\}$.

Similarly, $R_T$ ($P_s$, $C_n$) represents a route from a parent node to a child node such as: $R_T = \{P_s, C_n \in ESM_G\}$. The above arguments lead us to the following expression for computing the total bandwidth available for transmitting a multicast packet from a source node to one or more parent nodes (i.e., the bandwidth associated with the first link of transmission):

$$TB_{W\{s\rightarrow(\text{multicast-unicast})(P_s \vee C_n \in \{s \cup N\})\}} = \frac{P_s}{D_{\{s\rightarrow(\text{multicast-unicast})(P_s \vee C_n \in \{s \cup N\})\}} - \left\langle \frac{L_D}{SoL} \right\rangle}$$

(20)

Determining the mathematical expression for the total delay involve in transmitting a multicast packet from a source node to one or more parent nodes and using the resulting expression in (20) yields (21) for approximating the total bandwidth such as:
\[ TB_{\text{\( W \) \( \rightarrow \) \( \text{multiple-unicast} \) \( \rightarrow \) \( C_n \)}} = \frac{P_s}{\sum_{i=1}^{n} D\left(R_{i\rightarrow P,C_n}\right) - \left(L_D/SoL\right)} \quad (21) \]

where \( D\left(R_{i\rightarrow P,C_n}\right) \) represents the maximum bandwidth available for a multicast packet which is transmitted from a source node \((s)\) to a child node \((C_n)\) as follows:

\[ TB_{\text{\( W \) \( \rightarrow \) \( \text{multiple-unicast} \) \( \rightarrow \) \( C_n \)}} = \frac{P_s}{\sum_{i=1}^{n} D\left(R_{i\rightarrow P,C_n}\right) - \left(L_D/SoL\right)} \quad (22) \]

IV. CONCLUSIONS

In this paper, we presented both analytical and mathematical models for all the multicast approaches currently available for multimedia applications. We first presented a mathematical model for multiple unicast systems in which the source host has to send a single copy of data to every single receiver. Our proposed formulization suggested that this approach wastes a lot of significant network bandwidth. Secondly, we presented a mathematical model for the IP multicasting approach which is a more efficient concept where the data source only sends one copy of data which is replicated as necessary when propagating over the network towards the receivers. Our proposed formulization shows that the IP multicast demonstrates some good bandwidth efficiency characteristics than the other multicast approaches. Finally, we presented a complete formulization of bandwidth efficiency for ESM systems, which is an alternate to router-dependent multicast service that allows end-systems participating in a multicast group to replicate and forward packets to other receivers. Our proposed formulization of bandwidth efficiency suggests that the ESM is a feasible, especially for sparse, medium size group.

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