A Probabilistic model of \(k\)-coverage in Minimum Cost Wireless Sensor Networks

S. Ali Raza Zaidi  
University of Leeds  
Leeds, LS29JT, U.K.  
elsarz@leeds.ac.uk

Maryam Hafeez  
SEECs, NUST  
Rawalpindi, Pakistan  
47maryam@niit.edu.pk

D.C. McLernon  
University of Leeds  
Leeds, LS29JT, U.K.  
d.c.mclernon@leeds.ac.uk

M. Ghogho  
University of Leeds  
Leeds, LS29JT, U.K.  
m.ghogho@leeds.ac.uk

ABSTRACT
One of the fundamental problems in the wireless sensor networks is the coverage problem. The coverage problem fundamentally address the quality of service (surveillance or monitoring) provided in the desired area. In past, several studies have proposed different formulation and solutions to this problem. Nevertheless none of them has addressed minimum cost solution to the coverage problem. In this paper we consider minimum cost wireless sensor network which provides full coverage to any arbitrary geometric profile under deterministic/random deployment. We then formulate probabilistic coverage model which provides \(k\)-coverage probability for minimum cost wireless sensor networks.\(^1\)

1. INTRODUCTION
Recent advances in semiconductor industry have expanded new horizons for the development of the state of the art wireless sensor networks [1]. These wireless sensor networks and low power personal area networks (LoWPANs) have emerged as a definitive platform for environmental monitoring, surveillance, health care, entertainment and defense related applications. Certainly this emerging sensor network paradigm has posed several challenges in terms of deployment, monitoring, operations and maintenance [2]
Coverage problem is one such fundamental problem in wireless sensor networks [3]. The coverage problem fundamentally addresses the quality of service i.e. surveillance or monitoring provided in desired area. In past several studies [3]-[6] have presented different formulations and solutions for coverage problem. However most of these studies do not provide any useful pre-deployment information. One such useful pre-deployment information is the minimum number of sensors required to cover the desired area. The amount of sensors required to achieve the desired coverage, indeed provides an estimate of the cost associated with wireless sensor network. In this paper we consider minimum cost wireless sensor network (Section 2) which provide full coverage of any arbitrary geometric profile under deterministic or random deployment. We then formulate probabilistic coverage model which provides \(k\)-coverage probability for minimum cost wireless sensor networks under random deployment (Section 3)

2. MINIMUM COST WIRELESS SENSOR NETWORKS
Essentially cost is a subjective notion, thus it can have different meanings in different scenarios. However in most general terms the cost associated with wireless sensor networks generally depends on the number of sensor required to provide full coverage in a specified geometric profile. In [7] we demonstrated that minimum number of sensor nodes required for any arbitrary shape region to provide full coverage under deterministic setup can be computed as:

\[
\begin{align*}
\ell \min &= \frac{(m + k) \sum_{i=1}^{n} (x_{y_i+1} - x_i y_i) + (x_{y_i} - x_{y_i+1})}{2mr^2 \{ \pi - 2(\theta - \sin \theta) \}} \\
\end{align*}
\]

Note that this mathematical relation is only valid under certain assumptions, they are:
- In order to provide redundancy \(k\) additional sensors for every \(m\) sensor are deployed (This is especially useful when \(k - coverage\) is required. Usually \(k \geq 2\) is required for fault tolerance, while \(k \geq 3\) is required for triangulation based tracking)
- Area of an arbitrary shape region can be computed using Green’s theorem as:

\[
A_{\text{arbitrary}} = \frac{1}{2} \sum_{i=1}^{n} (x_{y_i+1} - x_i y_i) + (x_{y_i} - x_{y_i+1})
\]

Note here author has fitted an arbitrary shape curve with a parametric equation of line. More accurate expression can be obtained using quadratic fits.
- Each sensor \(s_i\) and \(s_{i+1}\) has an overlap resultantly each sensor has a coverage area of:

\[
A_{\text{sensor}} = \pi r^2 \{ \pi - 2(\theta - \sin \theta) \}
\]

where \(r\) = sensing radius
\(\theta\) = angle subtended by chord bisecting overlap

Under all these assumption equation (1) provides minimum number of sensors required for full coverage in a deterministic setup. Nevertheless deterministic deployment is not always possible. For example when sensors have to be deployed for military surveillance from air borne or they have to be deployed in forest for monitoring fire. In such a case the random deployment is the only option available.
In [7] we presented a novel probabilistic framework for determination of the minimum number of sensor nodes required to provide the full coverage in an arbitrary shape geometric profile under random deployment. We demonstrated that for random deployment the amount of sensors required is at least seven times of what required for covering the same region under deterministic conditions.

Mathematically: \( n_{\text{min}}^\text{ran} \approx 7 \times n_{\text{min}}^\text{det} \).

3. PROBLEM STATEMENT

The probabilistic framework presented in [7] for randomly deployed wireless sensor network does not provide the \( k \)-coverage probability.

Definition 1: Any point \((x_m, y_m)\) is said to be \( k \)-covered if it can be covered by \( k \)-sensor nodes.

In this paper we present a probabilistic model for randomly deployed wireless sensor network which provides relationship between the number of sensor nodes and \( k \)-coverage probability.

4. \( K \)-COVERAGE MODEL

Random deployment of sensors nodes is considered as a Poisson process [8]. In such a case inter-sensor distance is exponentially distributed. For a 2-D case this distance can easily be mapped to the notion of area. Hence inter-sensor area becomes exponentially distributed. Consequently probability of full coverage in an arbitrary shape region becomes:

\[
P_{\text{cov}}^{2D} = 1 - e^{-4\frac{\lambda}{\pi}}. \quad (2)
\]

The term \( 4r^2 \) comes from the fact that in order to ensure 1-coverage the distance between two neighboring sensors should be less than \( 2r \) on both \( x \) and \( y \) axis. Hence the total area between sensor should be less than \( 4r^2 \).

In order to modify equation (2) for, \( k \)-covered wireless sensor network, let us consider an arbitrary shape region with the area \( D \). Considering random deployment of sensors as a Poisson process, we know the average number of sensors deployed in the area \( D \):

\[
\lambda = n_{\text{min}}^\text{ran} \quad (3)
\]

Since area \( D \) has \( n_{\text{min}}^\text{ran} \) arrivals. The sensing region of sensor \( s_i \) which has an area \( A_{\text{sensor}} = \pi r^2 \) has \( n_{\text{min}}^\text{ran} \pi r^2 / D \) arrivals. Let us consider \( \rho = n_{\text{min}}^\text{ran} / D \) is the density of sensors in region \( D \). Using Poisson distribution:

\[
\Pr\{n = k\} = \frac{\lambda^k e^{-\lambda}}{k!}. \quad (4)
\]

The probability that \( k \) sensors will be present in the sensing radius of sensor \( s_i \) is then given as:

\[
\Pr\{n = k\} = \frac{(\rho \pi r^2)^k e^{-\rho \pi r^2}}{k!}. \quad (4)
\]

Hence combining equation (2) and (4)

\[
\Pr\{\text{Desired area is covered and has } k \text{ or more neighbors}\} = P_{\text{cov}}^{2D} \cap P_k = \sum_{k=0}^{\infty} \frac{(\rho \pi r^2)^k e^{-\rho \pi r^2}}{k!} \times \left(1 - e^{-4\frac{\lambda}{\pi}}\right). \quad (5)
\]

In order to provide a better insight we simulate equation (5) with \( n_{\text{min}}^\text{ran} \) as a multiple of \( n_{\text{min}}^\text{det} \) for a 2-D setup. \( L = W = 100 \ m \) room with \( r = 5 \ m \). Simulation results show that probability of \( k \)-coverage increases with increase in number of sensors. Hence using \( n_{\text{min}}^\text{ran} \approx 7 \times n_{\text{min}}^\text{det} \) does not only provide 100% coverage but sufficient probability of 2 and 3-coverage as well.

Figure 1: Probability of \( k \)-coverage

5. CONCLUSION

In this paper we developed a novel probabilistic model for \( k \)-coverage in wireless sensor networks. We demonstrated that the random deployment need about seven times more sensors than deterministic deployment to provide 100% coverage. However using such large amount of sensors also provides sufficient probability of 2 and 3-coverage.

6. REFERENCES