Transmission Strategy of Fountain Code in Cooperative Networks with Multiple Relay Nodes

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Abstract—In this paper, transmission of fountain code over cooperative networks is considered. While it is known that fountain code is optimal for sending data to multiple receivers, extra care is needed when it is used in cooperative networks. On one hand, the presence of relay node can help to improve the overall performance. On the other hand, multiple relay nodes introduce the problem of mutual interference, which can potentially degrade the performance. This work proposes an amplitude modulation scheme to address the above issue. By appropriately scaling the output signal at each node according to their unique identifier, separability of data from several transmitters at the destination can be achieved. A numerical analysis highlighting the advantage of the proposed scheme is then presented.

I. INTRODUCTION

Fountain code [1]-[2] is a promising strategy for transmission over erasure channels. The main idea of fountain code is to continuously transmit encoded packets until sufficient number of packets is received at the destination. As such, fountain code is rateless (the effective code rate is naturally matched to the channel quality), and it has been adopted by several standards such as MBMS (Multimedia Broadcast Multicast Service) and DVB-H (Digital Video Broadcasting for Handheld) [3].

Cooperation [4]-[5] is another interesting concept which allows different terminals to help each other to improve the transmission quality (or to extend the network coverage in cellular environment). It provides an inexpensive way of exploiting spatial diversity in the network by creating multiple paths for the message to reach destination. This advantage, coupled with the superiority of fountain code for large content distribution [6], has attracted research on fountain code transmission over cooperative networks [7]-[11].

Reference [7] and [8] combines distributed space time coding (DSTBC) and fountain code for single carrier and multiple carrier transmission respectively. Although extra diversity gain is achieved, the use of space time coding requires the number of cooperating terminals to be known a priori, which is difficult to achieve in practice. Meanwhile, direct application of fountain code in cooperative network requires modification to the degree distribution of the code, which can be difficult even for the two users case [9]. Without changing the degree distribution, the encoding/decoding process has to cater for online re-coding capability at the relay [10], which will in turn increase the encoding/decoding complexity.

Channel orthogonalisation can provide a simple and efficient alternative for transmitting fountain code over cooperative networks [11]. However, this approach causes a degradation in bandwidth efficiency. This work is motivated to address this issue by proposing a new transmission scheme for fountain codes over cooperative networks, which allows simultaneous transmissions without compromising the bandwidth efficiency. In addition, this paper also provides a mathematical formulation for analysing system performance in terms of the required number of packet transmissions for successful decoding.

The remaining part of this paper is organised as follows. System model used in the analysis and problem definition is given in Section II. The proposed method and numerical analysis of its performance compared to other conventional schemes are explained in Section III and IV respectively. Finally, Section V ends this paper with concluding remarks.

II. SYSTEM MODEL

The cooperative network considered in this paper is a cellular downlink network (as illustrated in Figure 1), whereby one base station (BS) communicates with several users (UTs), potentially with the help of one or more relay nodes (RNs).

The channel for any given two nodes is modelled as erasure channel. For simplicity, the erasure probabilities from BS to any of the UTs are assumed identical, and it is denoted as $P_e^D$. Similarly, the erasure probabilities from BS to any of the RNs as well as from any RNs to any of the UTs are denoted as $P_e^R$ and $P_e^{RD}$ respectively, and they are in general smaller than $P_e^D$ due to physical proximity.
Without the help of RN, transmission from BS to all UTs can be performed using conventional fountain code [6]. When there exist some helper RNs, the overall performance can be improved. We consider the following two methods for transmitting fountain coded packets in cooperative network.

A. Method 1

With this method, any one of the RNs which are able to decode the source message will send an acknowledgment to the source. Once BS received the acknowledgment message, it will stop transmitting, and the successful RN will take over the transmission. The advantage of this approach is its simplicity. Since only one node (either BS or successful RN) is transmitting at any one time, there is no issue with mutual interference. This method also outperforms the direct transmission case due to the smaller erasure probability from BS to RN and from RN to UT, than that of the BS to UT link.

B. Method 2

This method is different from the earlier one in that both BS and all RNs which are able to decode the source message will simultaneously transmit to the UTs. The random code generator matrix used at different transmitter is independent from each other, therefore the information received at the UTs will accumulate, resulting in shorter decoding time. However, since wireless medium is shared among all transmitters, this method suffers from mutual interference, which can potentially degrade the overall performance if not handled properly. Previous work on this issue can be found in [11], whereby unique spreading code is assigned to each transmitter. RAKE receiver is then used at the receiver to peel the individual message from different transmitters. Although this method ensures orthogonality, the spreading code effectively reduces the bandwidth efficiency. In other words, to maintain the same bandwidth utilisation using this method, the symbol duration must be increased by the same factor, which reduces the effective throughput. In the following section, an amplitude modulation scheme will be proposed which supports multiple access over erasure channels without compromising the throughput and bandwidth utilisation.

III. Amplitude Modulation Scheme for Multiple Simultaneous Transmissions

Whenever multiple simultaneous transmissions are involved, the wireless medium has to be shared among all transmitters, which necessitates the use of resource allocation and medium access control (MAC) protocol. The present method provides an alternative solution for transmissions over multiple access erasure channels. By modulating the binary symbols to different amplitudes at different transmitters, it is shown that multiple transmissions can be supported simultaneously without compromising the performance. Therefore, the feedback overhead and inefficient resource utilisation due to resource allocation and MAC protocol respectively, can be avoided.

Binary multiple access erasure channel was first introduced by van der Meulen [12], where multiple senders transmit binary symbols to a common receiver (as illustrated in Figure 2). The output at the receiver is equal to the sum of individual channel output, each of which is a binary erasure channel (BEC) with erasure probability \( P_e^{(k)} \). Here, \( k \in \{1, \ldots, K\} \) is the transmitter index, and \( K \) is the total number of transmitters. The received signal at the destination node can then be expressed as:

\[
y = \sum_{k=1}^{K} \epsilon^{(k)}(x_k)
\]

where \( x_k \) is the modulated binary symbol from transmitter \( k \), and \( \epsilon^{(k)}(\cdot) \) is an erasure function defined as:

\[
\epsilon^{(k)}(x_k) = \begin{cases} x_k & \text{with probability } (1 - P_e^{(k)}) \\ 0 & \text{with probability } P_e^{(k)} \end{cases}
\]

The mapping from binary digit \( b_k \) into symbol \( x_k \) can be arbitrary. Here, symbol \( +A \) and \( -A \) is used to represent bit 1 and 0 respectively; while the choice of amplitude \( A \) is a design parameter.

The central problem of multiple access communication in erasure channel is to determine message vector \( b = [b_1 b_2 \ldots b_K] \in \{0, 1, *\}^K \), by observing \( y \). Note that \( * \) has been used to represent the erasure symbol. When the same amplitude \( A \) is used at all transmitters, it is impossible for the receiver to uniquely decode message vector \( b \). To overcome this issue, we propose to assign unique scaling factors at different transmitters. The objective is to ensure that all \( 3^K \) possible \( b \) produces a unique \( y \) for any \( K < \infty \).

The encoding process is as follows. Each transmitter is associated with an identifier (ID), starting from 1 to \( K \) in no specific order. For simplicity, the node index \( k \) is used as identifier. The scale factor for node \( k \) is then set to \( 3^{-(k-1)} \), such that \( x_k \) is either \( +A \cdot 3^{-(k-1)} \) or \( -A \cdot 3^{-(k-1)} \). Each node only needs to know its identifier, and scales their amplitude accordingly. It can be shown that this ensures a unique \( y \) value for all possible \( 3^K \) message bits and erasure patterns. An example for 2 transmitters case is given in Table I.

![Fig. 2. Erasure Multiple Access Channel.](image-url)

**TABLE I**

<table>
<thead>
<tr>
<th>User 1 data ( b_1 ) (Scale = ( +A ))</th>
<th>User 2 data ( b_2 ) (Scale = ( +A/3 ))</th>
<th>RN Symbol ( y ) (( y = x_1 + x_2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>( 2/3 A )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( 3/3 A )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( 0 )</td>
</tr>
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<td></td>
<td></td>
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</tbody>
</table>
As the mapping from $y$ to the desired information is one to one correspondence, the decoding process can be performed using lookup table. However, this method may not be practical for large $K$. An alternative decoding procedure is described as follows. Starting with $k = 1$, equation (1) is rewritten as:

$$y = e^{(1)}(x_1) + \sum_{k=2}^{K} e^{(k)}(x_k)$$

(3)

Since $e^{(1)}(x_1)$ can be $-A$, 0, or $+A$, a hard decision threshold would be $+A/2$ and $-A/2$. The estimate for $b_1$ is then expressed as:

$$b_1 = \begin{cases} 
1 & y \geq +A/2 \\
0 & -A/2 \leq y < +A/2 \\
-1 & y < -A/2 
\end{cases}$$

(4)

Moreover, due to scaling factor, $e^{(k)}(x_k)$ can be $-A 3^{-(k-1)}$, 0, or $+A 3^{-(k-1)}$. As such, the second term of (3) can be bounded as:

$$-A \sum_{k=2}^{K} 3^{-(k-1)} \leq \sum_{k=2}^{K} e^{(k)}(x_k) \leq A \sum_{k=2}^{K} 3^{-(k-1)}$$

$$-A/2 \leq \sum_{k=2}^{K} e^{(k)}(x_k) \leq A/2$$

(5)

with $\preceq$ notation denotes asymptotic inequality as $K \to \infty$. Therefore, as long as $K < \infty$, the hard decision for data from transmitter 1 in equation (4) will have the same performance as the single transmitter BEC.

Using this fact, the decoding can be iterated for the other $k > 1$ successively. Knowing $b_1$, it is possible to strip its contribution from $y$ by subtracting it with the non-scaled modulated symbol $x_1$ (a 0 value is used for erasure symbol *). Then, multiplying the result with a factor of 3, the same hard decision in (4) can be applied for $b_2$. This process continues until the message vector $b$ is recovered. To better illustrate this idea, an equivalent received signal model and decoder structure is depicted in Figure 3.

This scheme is readily applicable to the cooperative transmission using Method 2. Without loss of generality, transmitter ID $k = 1$ can be assigned to BS, and the remaining $K - 1$ ID numbers are assigned to each individual relay node in no specific order.

A. Simplified Decoder

Simplification to the receiver design is possible as shown in Figure 4, whereby only a single hard decision device is used.

The operation is as follows. In every symbol duration, the baseband signal is sampled. At this point, the decision for $b_1$ is available at the hard decision output. Meanwhile, the modulo device, adder, subtractor, and multiplier strip the contribution of $x_1$ from the present symbol, and feed it into a buffer. Next, the switch is connected to the buffer output, which is then latched $K - 1$ times, each producing a decision for $b_k$ and stripping the contribution of $x_k$ symbol. This process is done within the current symbol duration, thereby ensuring that all transmitters’ message are decoded before the next symbol arrives. Once message vector $b$ is decoded, the switch is connected back to the initial position to get the next symbol, and the process continues.

IV. NUMERICAL ANALYSIS

In this section, performance of fountain code transmission in cooperative network is analysed. As a baseline comparison, the performance of direct transmission (in the absence of relay node) is analysed. Using the probability density function of the required number of transmissions for successful decoding as the performance metric, the advantage of cooperative transmission (with both Method 1 and Method 2) is demonstrated.

For simplicity, we limit the numerical analysis to the case of single relay node. The results obtained can also be extended to the case of multiple relay nodes by following the same approach. Before analysing the performance of cooperative transmission, it is necessary to study the behaviour of the underlying fountain code.

The $M$ message packets can be recovered at the destination after receiving $N \geq M$ encoded packets if and only if the $M \times N$ generator matrix randomly generated by the code is full rank. It can be shown (as given in [13, Appendix A]) that the number of full rank $M \times N$ binary matrices is equal to $2^{MN} \prod_{i=0}^{M-1}(1 - 2^{-i-N})$. When random linear fountain code is used, all $2^{MN}$ possible binary generator matrices are equiprobable. Therefore, the probability of successful decoding after receiving $N$ encoded packets can be expressed as:

$$F(N) = \begin{cases} 
0 & N < M \\
\prod_{i=0}^{M-1} (1 - 2^{-i-N}) & N \geq M 
\end{cases}$$

(6)

The above equation can be interpreted as the cumulative distribution function (CDF) of successful decoding after $N$
Fig. 5. Probability Distribution of the Required Number of Transmissions for Successful Decoding

packets is received. The respective probability density function (PDF) can be expressed as:

\[ f(N) = F(N) - F(N - 1) \]

\[ = \begin{cases} 0 & N < M \\ \prod_{i=0}^{M-1} \left(1 - 2^{i-N}\right) & N = M(7) \\ \prod_{i=0}^{M-1} \left(1 - 2^{i-N+1}\right) & N > M \end{cases} \]

Figure 5 shows the above distributions for \( M = 100 \). When LT code is used, the number of ones at each column of the generator matrix follows a certain degree distribution such as Robust Soliton Distribution [14]. In this case, different binary generator matrices are no longer equiprobable, and the derivation of the density function \( f(N) \) is more complicated. One possible approach to such problem is to use Monte Carlo for generating the PDF. Throughout this work, equation (7) is used for the analysis. Note, however, that the results obtained are also applicable to other choice of degree distributions.

A. Direct Transmission

First, consider point to point transmission over erasure channel with erasure probability \( P_e^D \). As some of the packets are not received at the destination, after \( N \) transmissions, the number of packets received unerased is random. Given that the destination is able to decode after \( N \) transmissions, it is apparent that the \( N^{th} \) packet must not be erased, otherwise it belongs to the event of successful decoding prior to \( N \) transmissions. This happens with probability \( (1 - P_e^D) \). Meanwhile, out of the remaining \( N - 1 \) packets, there can be \( i \) packets that are received unerased; where \( i \) ranges from 0 to \( N - 1 \). Here, \( i \) is binomially distributed, hence the probability that \( i \) packets out of \( N - 1 \) are unerased is \( C_{N-1}^i (1 - P_e^D)^i (P_e^D)^{N-1-i} \). Adding it to the \( N^{th} \) packet which is unerased, there are total of \( i + 1 \) packets available at the destination, and the respective probability of successful decoding is \( f(i + 1) \). Putting it all together, the PDF of the required number of transmission for this channel can be expressed as:

\[
f_{ER}(N, P_e^D) = \left( \frac{1 - P_e^D}{C_1} \right) \times \sum_{i=M-1}^{N-1} f(i + 1) C_{i}^{N-1} (1 - P_e^D)^i (P_e^D)^{N-1-i}
\]

where \( C_1 \) is the normalisation factor required to make \( \sum_N f_{ER}(N, P_e^D) = 1 \). Using the fact that users are independent from each other, and denoting total number of users as \( U \), the PDF of the required number of transmissions until all receivers are able to decode can be expressed as:

\[
f_{Dir}(N) = \sum_{i=1}^{U} \left[ f_{ER}(N, P_e^D) \right]^i \sum_{j=M}^{N-1} f_{ER}(j, P_e^D) \] \[ U-i \]

The above PDF represents the distribution of the required number of transmissions from BS to multiple UTs when there is no RN available. The curves of this PDF is plotted in Figure 6(a) for \( U \in \{10, 20, 30, 40, 50\} \) with erasure \( P_e^D = 0.3 \) and message packets \( M = 100 \).

B. Cooperative Transmission

When there is an available relay node, the overall performance can be improved due to better channel quality from BS to RN and from RN to UT. Here, erasure probability \( P_e^R = P_e^{RD} = 0.1 \) is assumed.

1) Method 1: Using Method 1, the moment RN is able to decode the message, BS stops transmitting and RN takes over the transmissions. Here, there are two possible transmission scenarios. First scenario is when destination is able to decode before the relay, which happens with probability \( \sum_{j \geq N} f_{ER}(j, P_e^R) \). In this case, the probability of successful decoding at the destination is simply \( f_{ER}(N, P_e^D) \). The second scenario is when relay node is able to decode prior to the destination at \( j < N \), which happens with probability \( f_{ER}(j, P_e^R) \). In this case, the packets received at the destination contains the contributions from both source and relay node. Define two auxiliary functions:

\[
g_{y,P_e}(x) = C_x^y (1 - P_e)^x P_e^{y-x} \quad x \in \{0 : y\} \quad (10)
\]

\[
h_{y,P_e}(x) = C_x^{y-1} (1 - P_e)^x P_e^{y-x} \quad x \in \{1 : y\} \quad (11)
\]

When the relay is able to decode after exactly \( j \) transmissions, the probability that \( s \) packets are received unerased during the first phase is \( g_{j,P_e^D}(s) \). For the remaining \( N - j \) transmissions, the probability that \( t \) packets are received unerased is \( h_{N-j,P_e^D}(t) \). Considering that there are \( U \) users, the PDF of the required number of transmissions can be calculated by combining the two phases for all users and considering all possible values of \( j \) as follows:

\[
f_{RL}(N) = \sum_{j \geq N} f_{ER}(j, P_e^R) f_{Dir}(N)
\]

\[ + \sum_{j=M}^{N-1} f_{ER}(j, P_e^R) \sum_{i=1}^{U} C_i^j \left[ A(j, N) \right]^i \left[ B(j, N) \right]^{U-i} \quad (12)
\]

where:

\[
A(j, N) = \sum_{s=0}^{j} \sum_{t=1}^{N-j} g_{j,P_e^D}(s) h_{N-j,P_e^D}(t) f(s+t)(13)
\]

\[
B(j, N) = \sum_{k=M}^{j} f_{ER}(k, P_e^D) + \sum_{k=j+1}^{N-1} A(j, k) \quad (14)
\]
The PDF calculated using equation (12) for different number of users is shown in Figure 6(b). Compared to direct transmission, the PDF is more concentrated (achieves smaller variance). Meanwhile, the average value achieved is smaller only for large number of users case.

2) Method 2: This scheme is similar to Method 1, with the exception that at the second phase, both BS and RN are transmitting to the UTs simultaneously. Using the proposed amplitude modulation scheme described in Section III, this can be done without introducing any interference. The PDF of the required number of transmissions under this scheme can be derived using the same approach as that in Method 1, and its expression is also identical to equation (12). The difference, however, is in the calculation of the auxiliary functions $A(j, N)$, which needs to be replaced with:

$$A'(j, N) = \sum_{s=0}^{j} \sum_{q=1}^{N-j} g_{j, P^D} (s) h''_{N-j}(q) f(s + q) \quad (15)$$

$$h''_{N}(q) = \sum_{k,t\in\{0...N\}}^{N} g_{N, P^D} (k) g_{N, R^{RD}}(t) \quad (16)$$

$$h''_{N}(q) = \frac{1}{C_2} \left[ h''_{N}(q) - (h''_{N-1}(q) P^e R^{RD}) \right] \quad (17)$$

where $C_2$ is the normalising constant. Detailed derivation of the above formula is skipped due to limited space. The PDF of the required number of transmissions under Method 2 for different number of users is depicted in Figure 6(c). It is apparent that this scheme is superior compared to the other two, as it achieves smaller mean and variance values.

V. CONCLUSIONS

In this paper, a novel scheme to transmit fountain codes over cooperative networks is proposed. The scheme combines amplitude modulation technique and multiple access transmission over erasure channel to prevent mutual interference, while keeping the low complexity property of the code. Numerical analysis of the scheme is then given, whereby the distribution of the required number of transmissions for successful decoding is derived. It is shown that the proposed method is able to bring performance improvement by achieving smaller mean and variance on the required number of transmissions.

REFERENCES