Balancing IT with the Human Touch: Optimal Investment in IT-Based Customer Service

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To cut costs, companies have chosen to deliver a variety of service offerings online. However, the digital systems providing such services (e-service) have always been complemented with or supported by human-based service (h-service). Whereas h-service has total costs that increase with the demand for services, e-service mainly requires a fixed investment upfront, which can be amortized over the totality of customers served. Considering the different nature of the costs of h-service and e-service and the heterogeneity of customer preferences for services, we derive the optimal mix of h-service and e-service for a service-providing company vis-à-vis its competitor. Our theoretical analysis finds the subgame-perfect Nash equilibria that determines the optimal positions in a duopoly setting. We further study the competitive dynamics of the system to examine how firms stay on the equilibrium paths. Using simulation, we investigate the effects of starting positions, small adjustments in h-service and/or e-service, and monotonic expansions of e-service on the final positioning and profits of the firms. Our results demonstrate that when firms follow a local best-reply strategy, they may end up in a position of low profitability, and when only monotonic expansions of e-service are allowed, both firms may end up overinvesting in e-service.

Key words: e-service quality; customer service; price competition; service differentiation; competitive strategy; competitive dynamics

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1. Introduction
Digital systems enabled by technology advancement are revolutionizing the way business is conducted and reshaping how companies interact with their customers. Over the last few years, companies have realized the potential savings from more efficient processes by delivering services digitally, and how their choices on the use of digital systems could influence the firm’s competitiveness (Chi et al. 2008, p. i). For example, online brokers (such as E*Trade, TD Ameritrade) were among the first to use digital systems to deliver online the traditionally human-intensive trading services (such as order taking over the phone or in person). Traditional brick-and-mortar businesses that sell physical goods have also made use of the Web to provide cost-saving services and improve their bottom lines. Tracking courier packages, booking tickets, scheduling car maintenance, applying for mortgage loans are just a few examples of services that one can now expect to do using digital systems, as opposed to the human intervention needed a decade ago. The spectrum of digital services offered seems to expand more and more to include, for example, legal services (e.g., LegalZoom, TheAttorneyStore), accounting and tax services (e.g., H&R Block, TurboTax), and other services where it was previously thought that human intervention was necessary. Indeed, the use of digital systems has dramatically impacted a firm’s competitive actions, which have been found to influence the firm’s performance and competitiveness (Chi et al. 2007, 2008).

On the other hand, the services delivered and how they are delivered differ from firm to firm. That is, the competitive actions taken by the firms to deliver their services can be quite different. For example, traditionally, legal services such as incorporating a business and estate planning are done by interacting with an attorney who would draft the appropriate documents. Nowadays, a number of law firms provide a comprehensive collection of online technology resources and legal knowledge to automate certain services. Clients can obtain these
services through the firm’s website. The two main players in this space are http://www.legalzoom.com and http://www.theattorneystore.com; both provide online preparation of legal documents as well as support for the preparation process. One firm offers additional services by having an attorney check the final documents, even file the documents with the appropriate authorities.

Traditional firms have also been investing heavily in IT-based customer services (e-service). An IDC survey found that e-service (such as online order taking and order tracking, payment, and after-sales support) provision absorbed about 50% of the total investment in new IT at a typical company (Tsikriktsis et al. 2004). Anecdotal evidence confirms this trend. For example, Verizon Business operates three call centers to handle service requests from their business customers. Recently its IT department is charged with the task of implementing a web-based self-service system. The system is supposed to handle routine type of service requests such as adding an additional phone line, the mailing of a duplicate bill, etc. Verizon Business’s goal is to deliver more customer services through IT and reduce the amount of customer services that need to be handled by service representatives.

As much as the trend seems to favor service provision using digital systems, not all services can be expected to be performed without human intervention. Some services may be too complex, rather rare, or may need to be complemented by human interaction. In addition, some customers feel more comfortable with the assistance of a human operator to guide them through the completion of their process. Prior research has shown that customers value the responsiveness a human being can provide, the customized service to their needs, and the flexibility and spontaneous delight during a service encounter (Bitner et al. 1990, Meuter et al. 2000). These aspects of the service encounter are often times lost when customers perform IT-based self-service. While some customers may prefer to use an IT-based system, as customers are heterogeneous, others might prefer to have more of a human touch. That is, they may prefer to interact with human service representatives when a service need arises.

In addition to customer preferences, firms also need to consider the costs involved in providing different types of services. Scholars (such as Shapiro and Varian 1999) have argued that the cost structure of e-service delivery has the unique characteristic of large fixed cost but low marginal cost. Human-based services (h-service), on the other hand, have typically been considered as more costly, with a large variable cost component. The different cost structures obviously will have significant implications for a firm’s bottom line. Therefore, firms need to strategically choose the proportion of e-service and h-service that they offer.

Given that a firm has the option of offering services digitally and/or with human assistance, what are the choices available to the firm when determining the appropriate level for each type of service, taking into account consumer preference, as well as the cost of providing each service? How does its action affect its competitiveness in a marketplace where competitors also strategically choose their competitive actions? This paper intends to fill this void by examining how a firm could position itself along the two service dimensions to maximize profit in light of its competitors. Note that we do not model how a firm should shift from one service to another. Rather, we consider these two types of services as two strategic actions that a firm could take, each action involving a different level of offering that the firm must decide on. In addition, we use profit to measure a firm’s competitiveness as suggested by Chi et al. (2008).

We build a game-theoretic model to study the competition between two firms that sell substitutable products. In our model, a product can be a physical product bundled with services, or a bundle of mere services. For example, the product may be a camcorder bought online, where firms differ in services they provide, such as order tracking and after-sales support. A product can also be incorporating a business or acquisition of securities, which require only a bundle of services to be performed. The two firms can provide customer service using both IT-based and/or human-based options. We analyze what the firms’ optimal mix of the e-service and h-service should be. In other words, what mix of the different levels of the two services would yield maximal profit for a firm given the competitor’s strategic choices of price and services? Understanding this optimal mix of service delivery could offer managers important guidelines in the competitive environment.
The results of our game-theoretic model show that there exist multiple possible subgame-perfect Nash equilibria, which depend on a firm’s cost structure, as well as the feasible range of levels along the two service delivery options. We further study the competitive dynamics of the system to examine how firms stay on the equilibrium paths. Using simulation, we investigate the effects of starting positions, small adjustments in h-service and/or e-service, and monotonic expansions of e-service on the final positioning and profits of the firms. Our results demonstrate that when firms follow a local best-reply strategy, they may end up in a position of low profitability, and when only monotonic expansions of e-service are allowed, both firms may end up overinvesting in e-service.

This study makes several important contributions to the literature on digital systems and online competition. First, our theoretic model identifies the existence of an asymmetric equilibrium where no firm’s differentiation dimension strictly dominates another. This asymmetric equilibrium, to the best of our knowledge, has never before been established in the differentiation literature. Second, contrary to previous calls for more e-service (e.g., Rust and Kannan 2003), our analysis clearly illustrates that a firm’s most profitable strategic choice also depends on its initial level of service provisions, as well as the cost structure of providing each type of service. In fact, the cost structures are the main driver of a firm’s competitive positioning. Finally, this paper offers some practical insights on the integration of digital systems and firm’s competitiveness. We show that before a firm can undertake major initiatives to build or improve its digital systems, the firm needs to carefully analyze the feasible competitive space in the service dimensions as well as the dynamic interactions vis-à-vis its competitors.

The rest of this paper is organized as follows. After reviewing the research background in §2, we present in §3 our theoretical modeling framework. The short-term price equilibrium is then analyzed in §4, followed by the longer-term equilibrium analyses in §5. Section 6 examines the competitive dynamics of the firms and shows how their initial service offerings and the speed of their adjustments in service offerings could affect their equilibrium positions. Section 7 discusses the implications of the research and concludes the paper with future research directions.

2. Research Background

Service is an important measure of success in competitive markets. Firms traditionally rely on h-service for relationship building with their customers. In the quest to reduce operating expenses, firms are removing live representatives from customer service encounters, and automating service provisions through the use of IT-based digital systems. The number, type, and function of e-services have been increasing over the last few years. Services such as order tracking, product inquiry, and bill payment are now routinely delivered through e-service solutions. These e-services provide a number of benefits for consumers, such as access 24 hours a day and seven days a week, speed of service, customized information, and more customer control (Drinjak et al. 2001). Additionally, it is also becoming commonplace for consumers to be penalized for not using self-service technologies such as kiosks, automated teller machines, and Internet-based service channels (Featherman et al. 2006). For example, banks typically charge a fee for various in-person banking services; airlines charge a fee if a customer books an airline ticket through the airline’s reservation agents, instead of using the airline’s online booking system.

From a theoretical standpoint, prior research suggests that services delivered through digital systems are an important innovation deserving further research (Bitner et al. 2000, Meuter et al. 2000, Parasuraman and Grewal 2000, Ruyter et al. 2001, Stafford 2003, Rahman 2004, Carter and Bélanger 2005). One of the dominant underlying premises in e-service research is that this effort will assist business organizations in enhancing their capabilities to meet their customers’ needs in the most effective manner. However, in today’s fast-moving business environment, with constantly changing markets and a profusion of service offerings, companies often find it hard to discern what customers really want and are willing to pay for. Making things even more challenging is the quantification of the relative importance of the e-service component, compared with h-service, related to customer choices and their willingness to pay (Verma et al. 2008). Previous research in the banking industry has suggested that not all customers accept e-service as an innovative change. Some customers, after adopting self-service technologies,
have abandoned them and reverted back to obtaining services from a human staff member in a bank (Prendergast and Marr 1994).

Bateson (1985) is among the few researchers who have examined consumers’ preference for different service delivery channels. He found that there are distinctive consumer preferences toward both h-service and e-service delivery. As researchers have noted, e-business success is determined less by business models than by delivering top-notch repeatable services that result in satisfied customers (Marshall 2001). Instead of blindly investing in e-service solutions, companies urgently need to understand how consumers value the two types of services, and to weigh the costs involved in moving services online against the costs incurred by h-service delivery.

Economic choice theory assumes that individuals’ choice behavior is generated by maximization of preferences or utility. Louviere (1988) defines utility as “judgments, impressions, or evaluations that decision makers form of products or services, taking all the determinant attribute information into account.” The idea of utility maximization and its relation to human choice behavior is not new. McFadden (1986) quotes from a 1912 economics text by Taussig: “An object can have no value unless it has utility. No one will give anything for an article unless it yields him satisfaction.” Drucker (1974) claims that “What business thinks it produces is not of first importance. … What the customer thinks he or she is buying, what he or she considers value, is decisive. … And what the customer buys and considers value is never a product. It is always utility—that is, what a product does for him or her.”

In this paper, we present a model assuming that firms compete on price as well as the positioning of their offerings in terms of h-service and e-service. A consumer chooses a firm that maximizes his utility based on how he values the service delivery options net the product price. We model the decision choice of two competitors using a three-stage sequential game in which firms first decide their desired e-service level, then h-service level, and finally their product price. We view such decisions as sequential because price changes can be made relatively easily, whereas improving h-service offerings involve more operational adjustments and employee training that may take a little longer. In addition, developing an e-service system or changing the technological setup to accommodate e-service offerings typically takes much longer.

The modeling setup of our work is related to the vertical differentiation literature in economics and marketing. Some of the classical work include Gabszewicz and Thisse (1979), Shaked and Sutton (1982), and Tirole (1989), who show that, in a one-dimensional setting, competing firms will choose to locate at the extreme ends of the quality spectrum to reduce price competition. Moorthy (1988) extends the basic model by incorporating a variable production cost and demonstrates that in equilibrium firms choose products that are differentiated. Other researchers have subsequently studied this game under different settings. Economides (1989) and Neven and Thissse (1990) both model a two-dimensional differentiation situation where firms choose first product (consisting of one horizontal differentiation and one vertical) and subsequently price. Both models show that differentiation will still only occur in one dimension; however, under different conditions, it can occur in the horizontal or vertical dimension.

The theoretical model we develop incorporates two vertical dimensions of differentiation: e-service and h-service. Bontems and Requillart (2001), Garella and Lambertini (1999), and Vandenbosch and Weinberg (1995) have all modeled a similar setting before. There are, however, some significant differences. First, whereas the previous work demonstrated the existence of maximum differentiation along both dimensions, with one firm being superior in both dimensions, our model identifies the existence of an asymmetric equilibrium where no firm’s differentiation dimension strictly dominates another. Second, our model is more general and more realistic than the above models in that it allows nonzero costs of differentiation. In fact, the nonzero differentiation cost causes the equilibrium results to be dramatically different from the previous literature—we have shown that the asymmetric equilibrium only occurs when differentiation costs are not negligible.

3. Model
Two firms, indexed 1 and 2, sell substitutable products. Each firm chooses a level of service delivered
through their digital system (e-service), h-service (for simplicity of expression, we henceforth use the term h-service), and product price. We model the firm's decision choice using a three-stage sequential game. In the first stage, firms have to choose their desired e-service offerings. In the second stage, firms choose the h-service level they are going to provide. In the third stage, firms choose their prices. The firms then compete to maximize profits. When both firms offer the same product bundle, i.e., they set the same service levels and price, the market structure is symmetric. Each firm will capture half of the total demand and Bertrand competition results. This paper will focus on the differentiated market, which arises when firms choose different price margins (i.e., the markup above the marginal product cost), and different levels of e-service as well as h-service.

Let \( p_j, H_j, \) and \( E_j \) represent the choice of firm \( j \)'s \((j = 1, 2)\) price margin,\(^1\) h-service, and e-service level accordingly. We assume that a firm incurs a fixed cost for setting up the digital system to deliver e-service (Hitt and Frei 2002), and this cost is independent of the units of product sold. In addition, the firm incurs a variable cost for h-service, which is an increasing function of quantity sold (Desiraju and Moorthy 1997). The general profit for firm \( j \) can therefore be expressed as

\[
\Pi_j(p_j, H_j, E_j, D_j) = [p_j - f(H_j)]D_j - g(E_j),
\]

where \( f(H_j) \) is the average cost of providing h-service at level \( H_j \) for one unit of good sold \( (f(H_j) \text{ is assumed to be a strictly increasing function}) \), \( D_j \) is the demand for firm \( j \), and \( g(E_j) \) is the cost of setting up the IT infrastructure to deliver e-service at level \( E_j \). Since products from different firms are substitutes, the demand for one firm’s product is determined not only by its own price and service offerings, but also by its competitor’s price and service offerings, hence

\[
D_j(p, H, E) = \frac{\bar{D}_j}{2}(p_j - p) - \frac{\beta}{2}g(E_j),
\]

\footnotetext{1}{If the marginal cost of the product is \( C \), then the price choice of firm \( j \) will be \( p_j + C \).}

\footnotetext{2}{We use the shorthand notation \( p, H, E \) to represent the vectors \( (p_1, p_2), (H_1, H_2), (E_1, E_2) \).}

The cost function of e-service, \( g(E_j) \), is increasing and convex. This assumption is reasonable because delivering e-service requires a substantial investment in IT infrastructure and usually takes a considerable amount of time. The convex function also captures the diminishing returns of IT investment (Harter and Slaughter 2003). For simplicity, it takes on a quadratic form, i.e., for a given e-service level \( E_j \), a firm incurs a fixed cost of \( g(E_j) = \beta E_j^2 \), where \( \beta \) is a constant. On the other hand, h-service costs usually depend on the number of items sold. To make the model analytically tractable, we assume that the cost function \( f(H_j) \) is linear: more specifically \( f(H_j) = \alpha H_j \), where \( \alpha \) is a constant and the same across firms. We can now formulate the decision problem of firm \( j \) as

\[
\max_{p_j, H_j, E_j} \Pi_j(p, H, E) = (p_j - \alpha H_j)D_j(p, H, E) - \beta E_j^2. \tag{1}
\]

### 3.1. Consumer Behavior

There is a fixed pool of consumers who would like to buy a unit of product from one of the two firms. Consumers are heterogeneous with respect to service offerings. We use two parameters to capture consumer’s service sensitivity. The e-service sensitivity parameter \( \gamma \) measures the utility a consumer derives from using e-service, i.e., how much she is willing to pay to buy from a firm with more e-service offerings as compared to one with lower level of e-service. Prior research has indicated that some consumers really value e-service because of the speed of service delivery and perceived control over the transaction (Ba and Johansson 2008, Zeithaml et al. 2002, Meuter et al. 2000). We assume that \( \gamma \) takes on a value from the range \([\bar{\gamma}, \gamma]\), with \( \bar{\gamma} \geq 0 \).

In addition to e-service sensitivity, consumers are also characterized by a sensitivity parameter \( \theta \) toward h-service, which denotes the utility (or willingness to pay) that a consumer derives from h-service provided by a firm. Previous research has argued that some consumers prefer the inter-personal nature of talking to a service representative and the flexibility and spontaneous delight during a service encounter (Bitner et al. 1990, Meuter et al. 2000). We assume that \( \theta \) takes on a value from the range \([\bar{\theta}, \theta]\) and that \( \bar{\theta} \geq 0 \). Consumers are uniformly distributed over \([\bar{\theta}, \theta]\times[\bar{\gamma}, \gamma]\).

Finally, we assume the market is covered, i.e., each consumer purchases the product or service either from firm 1 or firm 2. This means that the purchase decision has already been made and the consumer only faces the decision of which firm to choose.
Because the products provided by the two firms are substitutes, the two firms are differentiated by their product price and two types of service offerings. A typical consumer’s utility can then be expressed (Srinivasan 1982) as \( U(H_j, E_j, p_j) = U_0 + \theta H_j + \gamma E_j - p_j \) for \( j = 1, 2 \), where \( U_0 \) is the utility derived from consuming the product. The consumer will choose the product from a firm that maximizes her utility.

### 3.2. Market Segmentation

In the rest of this paper, we assume that firm 1 is the one with the higher level of e-service, i.e., \( E_1 \geq E_2 \). In what follows, the quantity \( \varphi \equiv (H_1 - H_2)/(E_1 - E_2) \) will play an important role. The magnitude of \( \varphi \) measures the amount of h-service differentiation relative to e-service differentiation of the two firms. The sign of \( \varphi \) indicates whether the high h-service firm also provides high e-service (\( \varphi > 0 \)), or whether the low e-service firm provides the highest level of h-service (\( \varphi < 0 \)).

Based on the proposed utility function, a consumer is indifferent between buying from either firm if and only if \( \gamma E_1 + \theta H_1 - p_1 = \gamma E_2 + \theta H_2 - p_2 \), or \( \gamma = -\varphi \theta + (p_1 - p_2)/(E_1 - E_2) \). The indifference curve is a straight line (see, e.g., Figure 1) in the consumer preference space \([\theta, H] \times [\gamma, \bar{\gamma}]\). In Figure 1, consumers with a preference profile \((\theta, \gamma)\) below the indifference line will choose from firm 2; those above the indifference line will choose from firm 1. Note that the value of \( \varphi \) determines the slope of the indifference curve, and the slope of the indifference curve in the consumer preference space in Figure 1 is actually equal to \(-\varphi\). When \( \varphi > 0 \), the firm with the highest e-service also provides the highest h-service (solid line in Figure 1), when the firm with the highest e-service provides inferior h-service, we get \( \varphi < 0 \) (dashed line in Figure 1). The price difference (weighted by the difference in e-service) only determines the location (or intercept) of the indifference curve.

We say that firms are mainly e-service differentiated when \(|\varphi| < (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)\) and h-service differentiated when \(|\varphi| \geq (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)\). Figure 1 shows the case when firms are mainly differentiated in e-service; when firms are mainly differentiated in h-service, a similar figure would show the indifference curves crossing the horizontal axes. When the firms are mainly differentiated in the h-service dimension, the more vertical the indifference curve, the less the firms are differentiated in e-service. When the indifference curve is completely vertical, there is no differentiation in the e-service dimension and the model will reduce to a model of two-dimensional competition (see Tirole 1989). A similar discussion applies when firms are mainly differentiated in e-service: the flatter the line, the less the firms are differentiated in h-service. When the line is completely flat, there is no differentiation in h-service: the two firms are only differentiated in the e-service dimension.

### 4. Short-Term Price Competition

In this section, we derive the equilibrium prices \( p^* = (p_1^*, p_2^*) \) when the h-service and e-service levels, and hence \( \varphi \), are fixed. Ba et al. (2007) previously analyzed a similar setup, but only on the price dimension—competitive actions were assumed to be exogenous. In this paper, we extend this short-term analysis for the cases in which the indifference curve intersects one horizontal and one vertical axis (see Appendix B). In the following discussions, we distinguish two cases based on whether firms mainly differentiate in h-service (§4.1) or e-service (§4.2). Under each case, there are two situations depending on if \( \varphi > 0 \) or \( \varphi < 0 \); the analysis of the two situations for each case is similar.

#### 4.1. When Firms Mainly Differentiate in H-Service

From Figure 1, we can see that when \( \varphi > 0 \) (i.e., \( H_1 > H_2 \) since we suppose \( E_1 \geq E_2 \)), the demand
for firm 2 is equal to the area of the lower trapezoid, and that for firm 1 is the area of the upper trapezoid (when \( \varphi < 0 \), the lower trapezoid becomes the demand for firm 1 and the upper trapezoid the demand for firm 2). For notational convenience, let \( \theta_1 \) be the intersection of the indifference curve with \( \gamma = \bar{\gamma} \) and \( \theta_2 \) be that with \( \gamma = \gamma \), i.e., \( \theta_1 = (p_1 - p_2)/(H_1 - H_2) - \bar{\gamma}/\varphi \) and \( \theta_2 = (p_1 - p_2)/(H_1 - H_2) - \gamma/\varphi \). Similarly, let \( \gamma_1 \) be the intersection of the indifference curve with \( \theta = \bar{\theta} \) and \( \gamma_2 \) be that with \( \theta = \theta \). Hence \( \gamma_1 = (p_1 - p_2)/(E_1 - E_2) - \theta_\varphi \) and \( \gamma_2 = (p_1 - p_2)/(E_1 - E_2) - \theta_\varphi \).

Because the indifference curve intersects both vertical axes, \( \bar{\theta} \leq \theta_1 \leq \bar{\theta} \) and \( \bar{\theta} \leq \theta_2 \leq \bar{\theta} \). The demand for \( \varphi > 0 \) can then be computed as the area of the trapezoid:

\[
D_2 = \frac{\bar{\gamma} - \gamma}{2}(\theta_1 + \theta_2 - 2\theta) \quad \text{and} \quad D_1 = D - D_2 = \frac{\bar{\gamma} - \gamma}{2}(2\bar{\theta} - \theta_1 - \theta_2),
\]

where \( D \equiv (\bar{\theta} - \theta)(\bar{\gamma} - \gamma) \) is the total demand. Substituting the demand into the profit functions \( \Pi_1(p, H, E) = (p_j - \alpha H_j)D_j \) (the term \( \beta E_j \) is a sunk cost at this point, and hence can be omitted), we get

\[
\Pi_1(p, H, E) = (p_1 - \alpha H_1) \frac{\bar{\gamma} - \gamma}{2}(\theta_1 + \theta_2 - 2\theta),
\]

and

\[
\Pi_2(p, H, E) = (p_2 - \alpha H_2) \frac{\bar{\gamma} - \gamma}{2}(\theta_1 + \theta_2 - 2\theta),
\]

which then yield the following reaction functions:

\[
p_1(p_2, H, E) = \frac{1}{\alpha}[2p_2 + 2\alpha H_1 + 2(H_1 - H_2)\bar{\theta} + (E_1 - E_2)(\bar{\gamma} + \gamma)],
\]

and

\[
p_2(p_1, H, E) = \frac{1}{\alpha}[2p_1 + 2\alpha H_2 - 2(H_1 - H_2)\bar{\theta} - (E_1 - E_2)(\bar{\gamma} + \gamma)].
\]

The short-term price equilibrium can then be obtained by solving the above best reaction functions simultaneously. The existence of the equilibrium depends on the relative values of the h-service cost (\( \alpha \)) and consumer heterogeneity (\( \bar{\theta}, \varphi, \bar{\gamma}, \) and \( \gamma \)). In particular, the cost of providing h-service (\( \alpha \)) is not extreme and needs to be bounded within the range defined below:

\[
\alpha_1 \leq \alpha \leq \alpha_2 \quad \text{for} \ \varphi \geq 0,
\]

where

\[
\alpha_1 = -\bar{\theta} + 2\bar{\theta} + \frac{2\bar{\gamma} - \gamma}{\varphi} \quad \text{and} \quad \alpha_2 = 2\bar{\theta} - \theta + \frac{2\gamma - \bar{\gamma}}{\varphi}.
\]

When \( \varphi < 0 \), the cost bounds for providing h-service (\( \alpha \)) are now

\[
\alpha_3 \leq \alpha \leq \alpha_4,
\]

where

\[
\alpha_3 = -\bar{\theta} + 2\bar{\theta} + \frac{2\gamma - \bar{\gamma}}{\varphi} \quad \text{and} \quad \alpha_4 = 2\bar{\theta} - \theta + \frac{2\gamma - \bar{\gamma}}{\varphi}.
\]

It should be noted that when \( \alpha \) is outside the above range, niche equilibrium will arise; please refer to Appendix B for such an analysis. Proposition 4.1 summarizes the short-term equilibrium prices.

**Proposition 4.1.** (i) When \( \varphi \geq (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta) > 0 \) and \( \alpha_1 \leq \alpha \leq \alpha_2 \), the price equilibrium is given by

\[
p_1^* = \alpha H_1 + \frac{1}{\alpha}(E_1 - E_2)[(\bar{\gamma} + \gamma) + 2\varphi(2\bar{\theta} - \theta - \alpha)],
\]

and

\[
p_2^* = \alpha H_2 + \frac{1}{\alpha}(E_1 - E_2)[(\bar{\gamma} - \gamma) + 2\varphi(2\bar{\theta} - \theta + \alpha)].
\]

(ii) When \( \varphi \leq -(\bar{\gamma} - \gamma)/(\bar{\theta} - \theta) < 0 \) and \( \alpha_3 \leq \alpha \leq \alpha_4 \), the price equilibrium is given by

\[
p_1^* = \alpha H_1 + \frac{1}{\alpha}(E_1 - E_2)[(\bar{\gamma} + \gamma) + 2\varphi(2\bar{\theta} - \theta + \alpha)],
\]

and

\[
p_2^* = \alpha H_2 + \frac{1}{\alpha}(E_1 - E_2)[(\bar{\gamma} - \gamma) + 2\varphi(2\bar{\theta} - \theta - \alpha)].
\]

The proof of the proposition can be found in Appendix A.

We now derive implications and give insights on the results of our price equilibrium. Specifically, how will the model results change when some of the parameters are varied?
Effects of H-Service Cost. It can be seen from Proposition 4.1 that $\frac{\partial p_j^*}{\partial \alpha} > 0$; i.e., the optimal price increases with $\alpha$. This is as expected because as the cost of providing h-service increases, firms need to increase prices to recover the high cost. The per unit net profit margins $(p_j - \alpha H_j)$, however, exhibit mixed trends with $\alpha$. It can be verified that for the high h-service firm, the per unit net profit margin is always decreasing with $\alpha$, whereas for the low h-service firm it is always increasing.

Comparing the per unit net profit margin, we see that the difference between the two firms’ per unit net profit margin is

$$\frac{1}{2} \text{sgn}(\varphi)(E_1 - E_2)[(\bar{\gamma} + \gamma) + \varphi(\bar{\theta} + \theta - 2\alpha)],$$

where $\text{sgn}(x)$ refers to the sign of $x$. One can check that this quantity is positive when $\varphi > 0$ and $\alpha = \alpha_1$ (or $\alpha = \alpha_3$ when $\varphi < 0$), but becomes negative when $\alpha = \alpha_2$ (or $\alpha = \alpha_4$ when $\varphi < 0$). That is, per unit net profit margin for the high h-service firm is higher than that for the low h-service firm as long as the h-service cost is not too high, i.e., as long as

$$\alpha \leq \frac{1}{2} \left( \bar{\theta} + \theta + \frac{\bar{\gamma} + \gamma}{\varphi} \right).$$

Comparing the market share of the two firms, we see that the difference between the market share of the two firms is

$$\frac{1}{3} (\bar{\gamma} - \gamma) \left[ \frac{\bar{\gamma} + \gamma}{\varphi} + (\bar{\theta} + \theta - 2\alpha) \right],$$

which is positive as long as $\alpha$ does not exceed $\frac{1}{2}(\bar{\theta} + \theta + (\bar{\gamma} + \gamma)/\varphi)$. Again, we see that the high h-service firm has a bigger market share when $\alpha = \alpha_1$, but the low h-service firm has a bigger market share when $\alpha = \alpha_2$, assuming $\varphi > 0$.

In summary, when the cost is relatively low to provide h-service, the high h-service firm has higher profit because the per unit profit margin as well as the market share are both higher. However, the advantage of being the high h-service firm decreases when the h-service costs go up.

Effects of Differentiation Along Both Service Dimensions. Let us now investigate how profits, prices, and market shares change under varying degrees of differentiation. It can be seen from the expressions of equilibrium prices and demands that a firm’s optimal net price and market share depend on the difference in h-service levels (i.e., $\Delta H \equiv H_1 - H_2$) and the difference in e-service levels (i.e., $\Delta E \equiv E_1 - E_2$). The net price of the high h-service firm will increase when the h-service difference between the two firms increases. Its market share will, however, decrease even though profit is increasing. This means that the high h-service firm will increasingly concentrate on the less price-sensitive customers who value h-service highly; its price increases but it is losing its most price-sensitive customers to the low h-service firm. With increasing service differentiation, the low h-service firm responds by lowering its price, thereby attracting the high h-service firm’s most price-sensitive customers.

Both profit functions are increasing monotonically with increasing h-service differentiation. In fact, if we allow both firms to change their h-service levels, we would see that in equilibrium maximal h-service differentiation would result (see §5): firm 1 chooses the highest possible h-service level and firm 2 chooses the lowest possible (but under the condition that $U = U_0 + \theta_1 H_j + \gamma E_j - p_j \geq 0$, lest the market is not covered). This is consistent with the previous literature in vertical differentiation (see Moorthy 1988 and Tirole 1989). When the h-service differentiation decreases, the products of the two firms look more similar and the price competition increases, resulting in less profit for both.

The effects of different degrees of differentiation in e-service, unlike those of h-service differentiation, are not monotonic. It can be shown that profits for both firms are increasing for small $\Delta E$, but then top off and start going down. In other words, there is an optimal level of differentiation beyond which both firms will see a decrease in profits (see §5.2). From Equations (4)–(7), we know that $\frac{\partial p_j^*}{\partial (\Delta E)} = -\frac{\partial p_j^*}{\partial (\Delta E)} = \frac{1}{3}(\bar{\gamma} + \gamma)$. So, if firm $j$ increases its e-service level, it will always result in a price increase, ceteris paribus. While this may sound intuitive, it is not the case when h-service levels are increased.

4.2. When Firms Mainly Differentiate in e-Service

When firms are mainly differentiated in e-service, the indifference curve will cut across the e-service dimension twice. Since $\gamma \leq \gamma_1 \leq \bar{\gamma}$ and $\gamma \leq \gamma_2 \leq \bar{\gamma}$, $D_2 = ((\bar{\theta} - \theta)/2)(\gamma_1 + \gamma_2 - 2\gamma)$ and $D_1 = D - D_2$. The
equilibrium prices that maximize profits for the two firms can then be solved as the following proposition.

**Proposition 4.2.** When \( 0 < \varphi \leq (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta) \) and \( \alpha_2 \leq \alpha \leq \alpha_1 \), or \(- (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta) \leq \varphi < 0 \) and \( \alpha_4 \leq \alpha \leq \alpha_3 \), the price equilibrium is given by

\[
p^*_1 = \alpha_1 H_1 + \frac{1}{\delta_1} (E_1 - E_2) [2(2\bar{\gamma} - \gamma) + \varphi(\bar{\theta} + \theta - 2\alpha)],
\]

and

\[
p^*_2 = \alpha_2 H_2 + \frac{1}{\delta_2} (E_1 - E_2) [2(\bar{\gamma} - 2\gamma) + \varphi(2\alpha - \bar{\theta} - \theta)].
\]

**Model Implications.** Most of the results discussed under the scenario of h-service differentiation (see §4.1) hold true for the e-service differentiation with two important exceptions. First, when firms are mainly differentiated in the e-service dimension, profits of both firms decrease as h-service differentiation (i.e., \( \Delta H \)) increases. This implies that equilibrium firms will choose to have minimum differentiation in h-service, i.e., \( H_1 = H_2 \) (see §5.1): two firms will decide to differentiate in the e-service dimension because there is not much room to differentiate in the h-service dimension.

For the rest, it can be checked that the following hold:

- If \( \bar{\gamma} > 2\gamma \), both prices increase when the difference in e-service levels increases;
- An increase in h-service cost (\( \alpha \)) results in higher prices;
- An increase in customer heterogeneity in the e-service dimension increases the market share of the high e-service firm more than that for the low e-service firm; and
- The per unit net profit and the market share are both higher for the high h-service firm as long as \( \alpha \leq \frac{1}{2}(\bar{\theta} + \theta + (\bar{\gamma} + \gamma)/\varphi) \).

## 5. Mid- and Long-Term Equilibrium Analysis

### 5.1. Midterm Equilibria

Besides price choices, firms can also change their h-service levels. Such changes are more difficult to achieve than price adjustments, but easier than changing the IT infrastructure to provide e-service adjustments. Hence we consider h-service decisions as a midterm strategy and the e-service choice as a long-term strategy. Suppose \( H_i \) and \( E_i \) are bounded above and below, i.e., \( H_i \in [H_1, H] \) and \( E_i \in [E_1, E] \). We will also use the following definition.

**Definition.** If \((H - \bar{H})(\bar{\theta} - \theta) \geq (E_i - E_2)(\bar{\gamma} - \gamma)\), we say that h-differentiation is feasible.

Substituting the short-term equilibrium prices obtained in §4 into the profit functions, we can obtain the midterm equilibria in the h-service dimension as summarized in Proposition 5.1 (see Appendix A for detailed derivations).

**Proposition 5.1.** The equilibrium choices depend on the cost and the feasible range of providing h-service, as well as firms’ positions in the e-service dimension. When h-differentiation is not feasible, there is a unique equilibrium; when h-differentiation is feasible, there are two candidate equilibria; the equilibrium that prevails is the one that yields firm 2 the highest profit. More specifically:

(i) If h-differentiation is not feasible, the unique equilibrium exhibits minimal h-service differentiation as follows:

\[
(H_1^*, H_2^*) = (H, \bar{H}) \quad \text{when} \quad \alpha \leq \frac{\bar{\theta} + \theta}{2},
\]

\[
(H_1^*, H_2^*) = (H, H) \quad \text{when} \quad \alpha > \frac{\bar{\theta} + \theta}{2}.
\]

(ii) If h-differentiation is feasible, there are two candidate equilibria: one exhibiting maximal h-differentiation, the other minimal h-differentiation. Firm 1’s strategy is uniquely determined; firm 2 will choose the strategy that yields the highest profits among the following candidate equilibria:

\[
(H_1^*, H_2^*) = (\bar{H}, \bar{H}) \quad \text{or} \quad (\bar{H}, H) \quad \text{when} \quad \alpha \leq \frac{\bar{\theta} + \theta}{2},
\]

\[
(H_1^*, H_2^*) = (\bar{H}, \bar{H}) \quad \text{or} \quad (H, H) \quad \text{when} \quad \alpha > \frac{\bar{\theta} + \theta}{2}.
\]

The interpretation of Proposition 5.1 is as follows. There is a candidate equilibrium where firms choose not to differentiate in providing h-service, anticipating that they could build digital systems in the long run to provide differentiated e-service (see §5.2). The cost of providing h-service will determine where both firms are located in the h-service dimension. If the cost is high (i.e., \( \alpha > (\bar{\theta} + \theta)/2 \)), both firms choose to provide minimum h-service, otherwise both choose to provide maximum h-service. However,
there is an alternative candidate equilibrium when h-differentiation is feasible. It is possible that firms choose to differentiate (maximally) in h-service, but this is feasible only when the h-service range weighed by the consumers’ tastes for h-service (i.e., $[H, \bar{H}]$) is “big enough” relative to the firm’s e-service differentiation and consumers’ tastes in e-service. This means that if firms differ little in providing e-service, the firms could decide to mainly differentiate in the h-service dimension. Proposition 5.1(ii) shows that firm 1’s optimal strategy is uniquely determined by the cost of providing h-service; the final determination of the equilibrium is up to firm 2, which will choose minimal or maximal h-differentiation depending on which one yields the highest profit—under the current fixed values of $E_1$ and $E_2$.

Under maximum h-service differentiation, the high e-service firm (firm 1) chooses to position at high h-service ($H_1^* = \bar{H}$) if the cost of providing such service is low. If the cost of providing h-service is high (i.e., $\alpha > (\bar{\theta} + \bar{\theta})/2$), providing low h-service is more profitable. Therefore, in this case, an asymmetric equilibrium could result: firm 1 may enjoy the highest e-service, but provide the lowest level of h-service, whereas firm 2 will invest less in building the digital system, but focus more on improving the h-service (see §5.2). The existence of such equilibrium has not been shown in the literature until now, and our result shows clearly how this equilibrium is linked to substantial differentiation costs. It is not hard to find practical examples of such equilibrium. For instance, E*Trade provides a digital system where customers can place international orders using the digital system (better e-service but inferior h-service) while customers from Schwab need to seek human assistance to place international trades (i.e., Schwab does not provide digital systems with automated international orders).

5.2. Long-Term Equilibria

Given the possible midterm equilibria presented in the previous section, we can again find the long-term equilibria in the e-service dimension. When firms choose the same h-service in the second stage, our model will reduce to the traditional quality differentiation with fixed e-service cost (see Tirole 1989, p. 296). In this case, the firms will choose different e-service levels, resulting in a dominant e-service differentiation strategy. This equilibrium is described below.

**Proposition 5.2.** When firms decide not to differentiate in h-service, then the following is an equilibrium in the e-service dimension:

$$E_1^* = \frac{(\bar{\theta} - \theta)(2\bar{\gamma} - \gamma)^2}{18\beta}, \quad (14)$$

$$E_2^* = \bar{E}.$$  

When firms choose to provide different h-service in the second stage, things get complicated and the existence of long-term equilibria depends on the specific values of the e-service cost parameter $\beta$. Let us first define the following threshold values, introducing $V \equiv (H - \bar{H})$:

$$\beta^* \equiv \frac{(\bar{\gamma}^2 - \gamma^2)(\bar{\gamma} + \gamma)}{36V}, \quad (16)$$

$$\beta_1 \equiv \frac{(\bar{\gamma}^2 - \gamma^2)[(\bar{\gamma} + \gamma)(\bar{E} - E) + 4V \max[2(\bar{\theta} - \theta - \alpha, \bar{\theta} - 2\theta + \alpha]]}{36V(\bar{E} + E)}, \quad (17)$$

$$\beta_2 \equiv \frac{(\bar{\gamma}^2 - \gamma^2)[-(\bar{\gamma} + \gamma)(\bar{E} - E) + 4V \min[2(\bar{\theta} - \theta - \alpha, \bar{\theta} - 2\theta + \alpha]]}{36V(\bar{E} + E)}. \quad (18)$$

**Proposition 5.3.** When firms differentiate in h-service (i.e., h-differentiation is feasible), the equilibrium values in the e-service dimension are given by

(i) when $\beta \geq \beta^*$

$$E_1^* = \min\{E_1 + \max\{E - E_1, 0\}, \bar{E}\}, \quad (19)$$

$$E_2^* = \min\{E_2 + \max\{E - E_2, 0\}, \bar{E}\}, \quad (20)$$

with $E_1^*$ and $E_2^*$ defined as the following quantities:

$$E_1^* \equiv ((\bar{\gamma}^2 - \gamma^2)(\bar{\theta} - \theta)(12V\beta - (\bar{\gamma}^2 - \gamma^2)(\bar{\gamma} + \gamma)) + 12V\beta \max[\bar{\theta} - \alpha, \alpha - \theta])$$

$$\cdot (12\beta[18V\beta - (\bar{\gamma}^2 - \gamma^2)(\bar{\gamma} + \gamma)])^{-1}, \quad (21)$$

$$E_2^* \equiv ((\bar{\gamma}^2 - \gamma^2)(\bar{\theta} - \theta)(12V\beta - (\bar{\gamma}^2 - \gamma^2)(\bar{\gamma} + \gamma)) + 12V\beta \min[\bar{\theta} - \alpha, \alpha - \theta])$$

$$\cdot (12\beta[18V\beta - (\bar{\gamma}^2 - \gamma^2)(\bar{\gamma} + \gamma)])^{-1}. \quad (22)$$

(ii) when $\beta_1 < \beta < \beta^*$:

$$E_1^* = E_2^* = \bar{E}. \quad (23)$$
(iii) when $\beta_2 \leq \beta \leq \min\{\beta_1, \beta^*\}$:

$$E_1^* = \bar{E} \quad \text{and} \quad E_2^* = \bar{E}.$$  \hspace{1cm} (24)

(iv) and finally when $\beta < \min\{\beta_2, \beta^*\}$:

$$E_1^* = E_2^* = \bar{E}.$$ \hspace{1cm} (25)

If the above values for the e-service levels are such that h-differentiation is feasible, then Proposition 5.1(ii) will dictate whether maximal or minimal h-differentiation will occur. If for the values of $(E_1^*, E_2^*)$, h-differentiation is not feasible, Proposition 5.1(i) determines the optimal values in the h-service dimension. It should be noted that the value of $\beta^*$ determines when the firms’ profit functions turn from convex into concave. If the cost of e-service $\beta$ is more than $\beta^*$, the firms’ profit functions become concave. Hence, when the feasible range of $E_j$ is large, the optimal level of e-service is in the interior with value $E_j^*$. When the value of $E_j$ is outside $[\bar{E}, \hat{E}]$, the optimal level will be on the boundary. It can be checked that Equations (19) and (20) yield either an interior solution when feasible or the upper or lower bound when $E_j^*$ is outside the feasible range.

When the cost of e-service is relatively low ($\beta < \beta^*$), the firms’ profit functions become convex. Then, depending on whether the profit functions are increasing or decreasing in the feasible range of $E_j$, a different equilibrium will result. In case (ii), both profit functions are decreasing in $E_j$, hence both firms choose $E^* = \bar{E}$. In case (iii), the profit function of the high e-service firm is increasing whereas the one for the low e-service is decreasing, hence maximal differentiation in e-service results. Note that this could be paired with maximal differentiation in the h-service dimension as well. In case (iv), the e-service costs are low and both firms’ profit functions are increasing. Hence they will both choose the highest level of e-service. In this case, firms will only differentiate in the h-service dimension, because h-differentiation is always feasible and choosing $H_1 = H_2$ yields Bertrand competition where none of the firms makes positive profits.

It is interesting to note that only in case (iii) we could see maximal differentiation in both h-service and e-service levels. It is also interesting to note that in cases (ii) and (iv), there is no differentiation in the e-service dimension, even though the cost of differentiating in the e-service dimension is lower than that in case (i), where e-service differentiation is present.

Combining the results of this section with those of previous sections, we can summarize the possible subgame-perfect equilibria in the following theorem. The theorem summarizes the results qualitatively and references the earlier propositions that support the equilibrium. Readers should refer to those propositions for the mathematical details on the conditions and the expressions for the profits, prices, and equilibrium levels for h-service and e-service, respectively.

**THEOREM 5.1.** The following are possible subgame-perfect equilibria for e-service provision:

1. There exists an equilibrium where firms do not differentiate in h-service and only differentiate in e-service (Propositions 5.2 and 5.1(i)):
   - (a) when the cost of providing h-service is low, both firms choose the same high level for h-service;
   - (b) when the cost of providing h-service is high, both firms choose the same low level for h-service.

2. There may exist an equilibrium where firms differentiate in h-service but do not differentiate in e-service:
   - (a) when the cost of e-service differentiation is relatively low compared to the cost of providing h-service, both firms will choose the same high level for e-service (Propositions 5.3(iv) and 5.1(iii));
   - (b) when the cost of e-service differentiation is relatively high compared to the cost of providing h-service (but not extremely high, see Case 3 (a) below), both firms will choose the same low level for e-service (Propositions 5.3(ii) and 5.1(i)).

3. There may also exist an equilibrium (depending on whether h-differentiation is feasible for the equilibrium values $(E_1^*, E_2^*)$ and which candidate equilibrium in Proposition 5.1(ii) yields the highest profit for firm 2) with both h-service and e-service differentiation:
   - (a) when the cost of providing e-service is very high, firms will position as follows:
     - one firm dominates the other in both e-service and h-service when the cost of providing h-service is low;
     - one firm chooses high e-service, low h-service; the other low e-service, high h-service when the cost of providing h-service is high.
   - However, because of the high cost of providing e-service, firms will not seek to position at the extremes of the e-service dimension (Propositions 5.3(i) and 5.1(ii)).
(b) when the cost of providing e-service is medium, firms will position as follows:

- one firm dominates the other in both e-service and h-service when the cost of providing h-service is low;
- one firm chooses the highest e-service, lowest h-service; the other lowest e-service, highest h-service when the cost of providing h-service is high.

Because the cost of providing e-service is lower than in case 3(a), firms will now position themselves at the extremes of the e-service spectrum (Propositions 5.3(iii) and 5.1(ii)).

The different equilibrium scenarios of parts 2 and 3 of the above theorem are graphically displayed in Figure 2. Note that Figure 2 does not show the possible equilibrium in part 1 of the theorem (that equilibrium does not involve h-service differentiation and depends only on the cost of providing h-service). There are some important points to note from Figure 2. The cost structure, i.e., the relative costs of investment in IT infrastructure compared to the variable cost of providing h-service, is the main driver of the equilibrium positions. When the cost of building IT infrastructure is relatively low, both firms will attempt to build a more or less identical system with full functionality, and they will differentiate themselves by providing different levels of h-service. However, when the cost of building a system goes up, the differentiation now occurs in both the h-service and e-service dimension. When the cost of providing h-service is low compared to the customers’ willingness to pay for service, a dominant firm will appear: this firm will dominate the other firm by providing a better system as well as a higher level of h-service. Such firm will, of course, cater to the customers who are willing to pay for more e-services and higher levels of h-service, whereas the inferior firm will capture the customers who are more price sensitive with respect to e-service and h-service. However, when the cost of providing h-service is high (relative to customers’ willingness to pay for h-service), being a dominant firm becomes too costly: the result will again be a maximum differentiation in both dimensions, but now the firm with the better h-service will have a system that is not as well built as the other firm’s system; however, the former firm now has chosen to compensate for his inferior system by providing a higher level of h-service.

When the cost of acquiring IT infrastructure to provide high levels of e-service increases even more (compared to the cost of providing h-service), both firms will be content with building a rather identical but rudimentary system and differentiate themselves by the level of h-service they provide. Apart from price competition, firms solely compete in h-service. The interesting part now is that if the cost of providing e-service keeps increasing, there is a threshold beyond which firms suddenly decide to start differentiating in e-service again. Moreover, this threshold for the cost of providing e-service is independent of the cost of providing h-service. However, because of the high cost of differentiating in e-service, there will be only limited differentiation. As before, depending on the cost for providing h-service, there will either be a dominant firm providing both high e-service and high h-service (when the cost for h-service is low), or there will be an asymmetric equilibrium where the firm with the better system offsets this advantage by providing lower h-service and vice versa (in this case, the cost of providing h-service is higher).

6. Competitive Dynamics

The previous sections derived the static Nash equilibria, which yield a subgame-perfect equilibrium for the firms’ positions in e-service and h-service. If both firms have perfect information and can make arbitrary adjustments in h-service and/or e-service, such
equilibrium will result. In this section, we investigate what might happen if the firms do not have perfect knowledge of the effect of their strategic actions on the competition and how it will affect their profit. For example, firms may only have local knowledge of their profit functions: they may only be able to estimate the changes in profits in a restricted neighborhood of the present values for $E$ and $H$. Similarly, it may not be possible to instantaneously adjust the level of h-service and/or e-service and adjustments may only take place in small steps. The purpose of this section is to look at the competitive dynamics, which is important to our understanding of how firms compete and how the interfirm competition affects market evolution (Soberman and Gatignon 2005, Chi et al. 2008). We resort to a computational study and investigate under which conditions the theoretical equilibria will occur and what other final (nonequilibrium) scenario might occur. The focus will be on the strategic positions in the $(E, H)$-space.

The study is set up as follows. Starting with two arbitrary out-of-equilibria positions, firms use a myopic best-reply strategy to compute their next position. That is, holding the other firm’s position constant, a firm computes which increase or decrease of $\Delta H$ in h-service (or $\Delta E$ in e-service) improves its profit most and then moves to that position. One important parameter is the maximum rate of change in $H$ allowed; we change the adjustment bounds in $H$ from $\bar{H} \leq \epsilon$ to the case where instantaneous changes in $H$ are possible ($|\Delta H| \leq \infty$). Because the changes in h-service were assumed to be faster than changes in e-service, a firm can do $\kappa$ adjustment steps in $H$-space, before doing one adjustment in $E$-space. In other words, adjustments in h-service can be made $\kappa$ times faster than e-service adjustments.

Figure 3 shows three different pairs of simulation paths. The first pair in solid lines shows how making only small adjustments in h-service may lead to a final situation that is not an equilibrium. The true equilibrium is one with maximal h-differentiation and an asymmetric positioning (Theorem 5.1, case 3(a), second bullet). However, the combination of the starting position and small adjustment in $H$-space force the firms to choose a positioning without h-differentiation: the position from Theorem 5.1, case 1(b). This position yields inferior profits for firm 2, hence, it cannot be an equilibrium point. When allowing changes in h-service to be big enough, firms can avoid being trapped in this nonequilibrium position: the pair of dashed lines show that with the same starting position of firms 1 and 2, firms end up in the equilibrium point when bigger adjustment steps in $H$ are allowed.

Figure 4 shows the profits according to the different paths. We see that the profit lines associated with the nonequilibrium path (solid line with square markers) are not only lower for firm 2 (the firm that determines whether the theoretical equilibrium will exhibit minimal or maximal $H$-differentiation) but are lower...
for firm 1 as well: the dashed lines of the equilibrium profits are above the solid lines of the nonequilibrium profits. In other words, both firms would be better off if they could reach the equilibrium position. The possibility of getting trapped in a nonequilibrium position is caused by the fact that both profit functions are convex in \( \Delta H \) when the condition of Proposition 5.1(ii) is satisfied, and contains a local minimum. Hence, when the starting position of the firms is located at the \textit{wrong} side of the local minimum, firms will climb up the \textit{wrong} part of the curve and end up at the wrong extreme point. One may hence conclude that—when only small adjustments are allowed—the final position of the firms is merely determined by their starting position in \( H \) (or, equivalently, their \( H \)-differentiation). However, it is actually more complicated than that: not only is the initial \( H \)-differentiation important, but the starting positions in e-service as well. The pair of dotted lines in Figure 3 illustrates this: both firms have the same starting \( h \)-levels but a different position in e-service. The small adjustments in \( h \)-service still allow the firms to reach the equilibrium position. In conclusion, Figure 3 illustrates the dangers of making only small strategy changes: firms may both end up in an undesirable nonequilibrium situation. If an equilibrium is to be reached, firms may have to commit to radical changes in strategy to avoid being trapped in a local optimum.\(^3\)

The strategy paths in Figure 3 show that in many situations, firms will move to \textit{reduce} the level of e-service. Such a move may be hard to interpret in practice: in our model, we assumed that e-service requires a fixed investment, and whereas it may be practical to lower the level of e-service provided by a system (e.g., by disabling some functionalities), the investment cannot be recuperated by such a move. Hence it is difficult to rationalize why firms would have made such a big initial investment and later deliberately lower the e-service level: Why make the investment in the first place? Therefore we next investigate how the outcomes would change if we were to allow e-service levels to go up only. That is, over

![Figure 5 Simulation Paths When e-Service Can Only Increase Over Time](image)

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\(^3\)Parameters used for this simulation are: \( \alpha = 3, \beta = 1, \theta \in [1, 4], \gamma \in [1, 3], E \in [1, 8], H \in [2, 12], |\Delta H| \leq 0.1 \) for scenarios 1 and 3, \( |\Delta H| < \infty \) in scenario 2, \( \kappa = 3 \).
Figure 6 Profits Associated with Paths When Only Increase in e-Service Is Allowed

in low profits. Whereas firm 1 captures the less price-sensitive customers, profits are not that high because of the high cost of e-service.4

Merely changing the starting position of firm 2 leads to a very different conclusion. This scenario is represented by the dotted lines. Firm 2 now starts out with a higher level of e-service while all other parameters remain the same. Because of the lower level of e-service differentiation in the starting position, firms choose to differentiate in the h-dimension initially. This high level of h-differentiation then allows firm 2 to increase its e-service provision to capture the more e-service-sensitive customers. The result is that the levels of e-service chosen (and, hence, the investment in IT) are much higher than the optimal (equilibrium) levels. It is also worth noting that in this scenario, firm 2 actually makes a higher profit (dotted line with solid circles in Figure 6) than its profit when it reaches the equilibrium position (solid line with diamonds in Figure 6). This illustrates the point that if one firm (firm 1 in this case) starts out with a very sophisticated system, the best reply for firm 2 is to also increase its offering in e-service. Both firms overinvest in IT as compared to the equilibrium levels, but the firm playing catch up actually is better off: its profits in this situation are higher than when the equilibrium positions would occur. In conclusion, changing some of the assumptions of the game-theoretic model may lead to very different final positions. If the firms do not want to overinvest in IT, it is best to start out with a more basic system, future expansions may guarantee that the equilibrium position is reached. Starting out with a system that contains expanded functionalities (firm 1 in the third scenario) may lead to lower profits, and the competitor firm can be better off.

As a final note, it is important to see how the costs drive the final equilibrium. Whereas the value of $\beta$ determines the optimal level of e-service, the value of $\alpha$ not only determines the optimal level of h-service, but also dictates whether the final equilibrium will be a dominant or asymmetric position. It is therefore important for the firms to attempt to get an accurate value of the cost of providing h-service as compared to the average customer’s willingness to pay to get a sense of where to position itself in the $E$- and $H$-dimension.

7. Conclusion and Future Research

In this paper, using a game-theoretic model of differentiation along two service dimensions, we demonstrate that, although digital systems can improve a firm’s service efficiency as previous literature has argued, a firm’s profit, as one measure of its competitiveness, actually depends on finding the optimal mix of both e-service and h-service offerings, given how competitors position themselves. Therefore it is critical that firms not invest blindly in digital systems. Managers need to balance the two aspects of service offerings and carefully evaluate their competitive situation before embarking on an action. We offer below insights managers could take away from this research.

7.1. Managerial Implications

Our equilibrium findings have some important managerial implications. First, the cost of h-service provision is an important driver of profitability. Our model shows that the firm providing the highest h-service, regardless of its e-service, may not be the most profitable. Only when the h-service cost is relatively low are its net per unit profit margin and market share, and hence profit, higher than the low h-service firm. In addition, the low h-service firm should not try to join the other firm by upping its h-service provision—when the service differentiation gap closes, the price competition heightens, resulting in lower profits for
both firms. We demonstrate that when the cost of providing h-service is low relative to consumer heterogeneity for h-service, the same product bundled with an improved h-service level may even result in a lower sales price!

If the firms are not very much differentiated in the h-service dimension, but differ a lot in e-service, then it is not worthwhile to attempt to increase the differentiation in h-service. Our analysis suggests that it would be better to have minimum h-service differentiation in that case, and it would be best for both firms to retain a big difference in e-service. The low e-service firm will then get the most price-sensitive customers and the high e-service firm will get the most service-sensitive customers. Again, when the h-service cost is low, the high e-service and high h-service firm has a higher profit. More importantly, it would be self-destructive for the low e-service firm to invest its money in obtaining higher e-service. The resulting increase in price competition would be detrimental to both firms, but the low e-service firm will be hurt more than proportionally. This result suggests that not every firm benefits from investing in e-service. Before firms undertake any major initiative to build or improve their digital systems for better service provision, they should carefully analyze their competitive space and the competitive dynamics between the firms.

Our simulations show that in a dynamic environment the willingness to make substantial quick changes in h-service may be required to avoid getting trapped in a low-profit path. By making small incremental changes, both firms may get trapped in a position that yields low profits. When firms only consider expanding their e-services over time, the initial over-investment (as compared to the equilibrium levels) in IT of one firm may cause the other firm to overinvest in IT as well. This situation would also result in lower profits for both firms and could be avoided by building the e-service system incrementally over time.

Given the trend of providing more e-service in various industries to reduce cost, it is important to understand the implications of such e-service provision decisions, taking into account consumer preference and the costs of providing both h-service and e-services. Our analysis would prescribe that:

- When currently faced with higher h-service differentiation than e-service differentiation, firms should continue to differentiate in the h-service dimension. When no more differentiation is possible, they should switch to e-service differentiation, but only to a certain extent.
- When currently faced with higher e-service differentiation, instead of attempting to differentiate in h-service, firms actually get most out of their investment if they continue to differentiate in the e-service dimension.

7.2. Limitations
This research is, of course, not without limitations. We have assumed in this paper that the distribution of the consumers exhibits no correlation between the two service dimensions. A more general distribution function that allows correlation could possibly change the analytical results. But finding closed-form solutions for the analysis of this case would be extremely difficult.

Another assumption we made in the model is that the market is covered. We derived the conditions on costs and consumer preferences that guarantee that the market is covered. These conditions are automatically satisfied for trapezoid demands (see Appendix A), and they appear quite reasonable and realistic. But it is possible that in some cases, consumers may be priced out of the market if the price charged by each company is too high. Relaxing this assumption would require a completely new analysis.

We also assumed that the e-service provision decision is made in the first stage and h-service the second stage. One can reasonably argue that the two decisions may be made simultaneously. However, our preliminary analysis has shown that the equilibria identified here will continue to hold in such a scenario; whether additional equilibria are also possible is currently unknown.

7.3. Future Research Directions
There are several possible directions for future research. We have considered only the duopoly competition. It would be an interesting exercise to extend the analysis to an oligopolistic setting under h-service and e-service differentiation. Our model is sufficiently general to accommodate other settings where two-dimensional differentiation is carried out as a three-stage game.
This paper is only the first attempt at understanding how the two aspects of service provision affects competition and a firm’s investment decision. One can argue that digital systems are playing an increasingly significant role in business processes and operations. Even h-service provision is starting to rely more on IT and digital systems. Therefore the boundary between h-service and e-service will continue to blur. Modeling the dependence on h-service on IT and understanding how that dependence affects IT investment decisions are important topics of future research.

Finally, in our paper, we treat a firm’s digital system as an insulated system. In reality, a firm’s digital system oftentimes transcends firm boundaries and the activities supported by those systems span through the entire value chain. Chi et al. (2008) have demonstrated how different interorganizational systems (IOS) usage affect firms’ competitive actions, which, in turn, might affect the link between firms’ competitive actions and their competitiveness. So, an obvious extension of our work is to allow the use of IOS for both firms. Incorporating this additional dimension into the decision problem would be a valuable contribution to the competitive dynamics literature.

Appendix A

Proof of Proposition 4.1. We can simplify the first-order conditions $\partial \Pi_j / \partial p_j = 0$ to obtain the following reaction functions:

$$p_1(p_2, H, E) = \frac{1}{2}[2p_2 + 2aH_1 + 2(E_1 - E_2)\bar{\theta} + (E_1 - E_2)(\bar{\gamma} + \gamma)]$$

and

$$p_2(p_1, H, E) = \frac{1}{2}[2p_1 + 2aH_2 - 2(E_1 - E_2)\bar{\theta} - (E_1 - E_2)(\bar{\gamma} + \gamma)].$$

where $\bar{\theta} = \bar{\theta}$ when $\varphi > 0$; $\bar{\theta} = \bar{\theta}$ and $\bar{\theta} = \bar{\theta}$ otherwise. The equilibrium prices can be obtained by solving the linear equations of the reaction functions. The expression to compute the demand assumes that the area is a trapezoid, so it is required that $\bar{e}_1 \leq (H_1 - H_2)\bar{\theta} + (E_1 - E_2)(\bar{\gamma} + \gamma)$, which, after some algebra, reduces to $a_1 \leq a \leq a_2$ if $\varphi > 0$ or $a_3 \leq a \leq a_4$ if $\varphi < 0$. □

Proof of Proposition 4.2. Substituting the demand functions into the objective and simplifying the first-order conditions, we get the reaction functions

$$p_1(p_2, H, E) = \frac{1}{2}[2p_2 + 2aH_1 + 2(E_1 - E_2)\bar{\theta} + (E_1 - E_2)(\bar{\gamma} + \gamma)]$$

and

$$p_2(p_1, H, E) = \frac{1}{2}[2p_1 + 2aH_2 - 2(E_1 - E_2)\bar{\theta} - (E_1 - E_2)(\bar{\gamma} + \gamma)].$$

Equilibrium prices are obtained by solving the above equations simultaneously. Finally $\gamma \leq \gamma \leq \bar{\gamma}$, $i = 1, 2$, imply $a_2 \leq a \leq a_1$ if $\varphi > 0$ or $a_2 \leq a \leq a_3$ if $\varphi < 0$. □

Proof of Proposition 5.1. In the second stage, when $\varphi \geq (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)$, we substitute $p_1^*$ and $p_2^*$ from the short-term equilibrium into the profit functions and get

$$\Pi_1(p^*, H, E) = \frac{(\bar{\gamma} - \gamma)(\bar{\gamma} + \gamma)(E_1 - E_2) - 2(\alpha - 2\bar{\theta} + \bar{\theta})(H_1 - H_2)^2}{36(H_1 - H_2)} - \beta E_1,$$

and

$$\Pi_2(p^*, H, E) = \frac{(\bar{\gamma} - \gamma)(\bar{\gamma} + \gamma)(E_1 - E_2) - (\bar{\gamma} + \gamma)(E_1 - E_2)^2}{36(H_1 - H_2)} - \beta E_2.$$

Both profit functions depend in the service difference $\Delta H \equiv (H_1 - H_2)$ and are convex with respect to $H_1$, because $\partial^2 \Pi_j / \partial H_1^2 = (E_1 - E_2)^2(\bar{\gamma} - \gamma)/(18(H_1 - H_2)^3) > 0$. Since $\alpha > \alpha_1$, we get $\alpha + \bar{\theta} - 2\bar{\theta} > 0$ and $\Delta H \geq (2\bar{\gamma} - \gamma)(E_1 - E_2)/(\alpha + \theta - 2\theta)$. In addition, $\Pi_1$ obtains its minimum at $\Delta H = (\bar{\gamma} + \gamma)(E_1 - E_2)/(2(2\bar{\theta} - \bar{\theta} - \alpha))$, and $\Pi_2$ obtains its minimum at $\Delta H = (\bar{\gamma} + \gamma)(E_1 - E_2)/(2(2\bar{\theta} - \bar{\theta} - \alpha))$.

It can be shown that $(\bar{\gamma} + \gamma)(E_1 - E_2)/(2(\alpha + \bar{\theta} - 2\theta)) < (2\bar{\gamma} - \gamma)(E_1 - E_2)/(\alpha + \theta - 2\theta)$. In other words, $\Pi_2$ strictly increases with $\Delta H$ in the feasible region. When $\theta + \bar{\theta} - 2\alpha > 0$, we can show that $2\bar{\theta} - \bar{\theta} - \alpha > \alpha + \theta - 2\theta > 0$. So $\Pi_2$ also strictly increases with $\Delta H$. Therefore, at equilibrium, both firms would like to choose to have the maximum service differentiation $\Delta H = \bar{H} - H \equiv \bar{V}$; i.e., $H_1^* = \bar{H}$ and $H_2^* = H$.

When $\varphi < -(\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)$, firm 1 chooses low service and firm 2 chooses high service (i.e., $H_1 < H_2$). The profit functions in the second stage become

$$\Pi_1(p^*, H, E) = \frac{(\bar{\gamma} - \gamma)(\bar{\gamma} + \gamma)(E_1 - E_2) + 2(\bar{\theta} - \alpha - \bar{\theta})(H_1 - H_2)^2}{36(H_1 - H_2)} - \beta E_1,$$

and

$$\Pi_2(p^*, H, E) = \frac{(\bar{\gamma} - \gamma)(\bar{\gamma} + \gamma)(E_1 - E_2) + 2(\bar{\theta} - \alpha - \bar{\theta})(H_1 - H_2)^2}{36(H_1 - H_2)} - \beta E_2.$$

Here $\partial^2 \Pi_1 / \partial H_1^2 = -(E_1 - E_2)^2(\bar{\gamma} - \gamma)/(18(H_1 - H_2)^3) > 0$ and $\Pi_1$ is convex with respect to $H_1$. The boundary condition $\alpha < \alpha_1$ implies that $\alpha + \bar{\theta} - 2\theta < 0$ and $\Delta H \leq (2\bar{\gamma} - \gamma)(E_1 - E_2)/(\alpha + \theta - 2\theta)$. In this case, $\Pi_1$ obtains the minimum at $\Delta H = (E_1 - E_2)(\bar{\gamma} + \gamma)/(2(\alpha + \bar{\theta} - 2\theta))$ and $\Pi_2$ obtains the minimum at $\Delta H = (E_1 - E_2)(\bar{\gamma} + \gamma)/(2(\alpha + \bar{\theta} - 2\theta))$. When $2\alpha - \bar{\theta} - \theta > 0$, we can show that both profits increase as $\Delta H$ decreases. Hence, at equilibrium, $H_1^* = \bar{H}$ and $H_2^* = H$. 

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When \(|\varphi| < (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)|\), we obtain the following profit functions by substituting the short-term equilibrium price margins:

\[
\Pi_1(p^*, H, E) = \frac{(\bar{\theta} - \theta)[2(2\bar{\gamma} - \gamma)(E_1 - E_2) + (\bar{\theta} + \theta - 2\alpha)(H_1 - H_2)]^2}{36(E_1 - E_2)} - \beta E_1^2,
\]

and

\[
\Pi_2(p^*, H, E) = \frac{(\bar{\theta} - \theta)[2(\bar{\gamma} - 2\gamma)(E_1 - E_2) + (2\alpha - \bar{\theta} - \theta)(H_1 - H_2)]^2}{36(E_1 - E_2)} - \beta E_2^2.
\]

Because \(\partial \Pi_1 / \partial H_i = ((\bar{\theta} - \theta)(p_i - \alpha H_i))/\left(3(E_1 - E_2)\right)(\bar{\theta} + \theta - 2\alpha)|\), the sign of \(\partial \Pi_1 / \partial H_i\) is the same as that of \((\bar{\theta} + \theta - 2\alpha)|\).

In other words, \(\Pi_1\) strictly decreases with respect to \(H_i\) if \((\bar{\theta} + \theta - 2\alpha)|\geq 0\), or strictly decreases with \(H_i\) otherwise. Hence, at equilibrium \(H_i = \bar{H}\) when \(\alpha < (\bar{\theta} + \theta)/2\), or \(H_i = H\) when \(\alpha > (\bar{\theta} + \theta)/2\).

It is worth noting from the above analysis that when the condition \((\bar{H} - H) > (E_1 - E_2)/((\bar{\gamma} - \gamma)/\bar{\theta})\) is satisfied, firm 2 could choose a strategy \((H_2 = \bar{H}\) or \(H))\) such that the two firms are either maximally or minimally differentiated in h-service. Therefore, in equilibrium, firm 2 will choose the strategy that yields the highest profit.  

**Proof of Proposition 5.2.** If firms choose no differentiation in the service dimension, the profits in the first stage become

\[
\Pi_1(p^*, H^*, E) = \frac{(\bar{\theta} - \theta)(2\bar{\gamma} - \gamma)^2}{9} (E_1 - E_2) - \beta E_1^2,
\]

and

\[
\Pi_2(p^*, H^*, E) = \frac{(\bar{\theta} - \theta)(\bar{\gamma} - 2\gamma)^2}{9} (E_1 - E_2) - \beta E_2^2.
\]

Because \(\Pi_2\) strictly decreases with \(E_2, E_2^* = E\). Solving the first-order condition from \(\Pi_2\), we obtain \(E_1^* = (\bar{\theta} - \theta)(2\bar{\gamma} - \gamma)/18\).  

**Proof of Proposition 5.3(i).** If \(E_1^* - E_2^* \leq ((\bar{\theta} - \theta)/(\bar{\gamma} - \gamma))(\bar{H} - H)|, then firms are mainly service differentiated. When \(\alpha \leq (\bar{\theta} + \theta)/2\), we can substitute the values of \(H_1^*\) and \(H_2^*\) from Proposition 5.1(ii) to obtain the profit functions as

\[
\Pi_1(p^*, H^*, E) = \frac{(\bar{\gamma} - \gamma)(E_1 - E_2)(\bar{\gamma} - \gamma) - 2V(\alpha - 2\bar{\theta} + \theta)}{36V} - \beta E_1^2,
\]

and

\[
\Pi_2(p^*, H^*, E) = \frac{(\bar{\gamma} - \gamma)[2V(\alpha + \bar{\theta} - 2\theta) - (E_1 - E_2)(\bar{\gamma} + \gamma)]}{36V} - \beta E_2^2.
\]

When \(\beta > \beta^* = (\bar{\gamma} - \gamma)/(36V)\), the second-order derivative \(\partial^2 \Pi_i / \partial E_i^2 = (\bar{\gamma} - \gamma)/(\bar{\gamma} - \gamma) - 36V\beta < 0\) for \(i = 1, 2\), hence the profit functions \(\Pi_i\) are concave. This means that, assuming \([E, \bar{E}] = [0, \infty]\), the maximum for \(\Pi_i\) can be found be setting \(\partial \Pi_i / \partial E_i = 0\) and simultaneously solving the equations will yield (noting that we always assume \(E_1 \geq E_2\)):

\[
E_1^* = \frac{(\bar{\gamma} - \gamma)[(\bar{\theta} - \theta)(12V \beta - (\bar{\gamma} - \gamma)(\bar{\gamma} + \gamma) + 12V \beta(\bar{\theta} - \theta)]}{12V[(\bar{\gamma} - \gamma)/(\bar{\gamma} + \gamma)]},
\]

\[
E_2^* = \frac{(\bar{\gamma} - \gamma)[(\bar{\theta} - \theta)(12V \beta - (\bar{\gamma} - \gamma)(\bar{\gamma} + \gamma) + 12V \beta(\alpha - \theta)]}{12V[(\bar{\gamma} - \gamma)/(\bar{\gamma} + \gamma)]}.
\]

When \(\alpha > (\bar{\theta} + \theta)/2\), the roles of \(\Pi_1\) and \(\Pi_2\), \(E_1^*\) and \(E_2^*\) are reversed. Now, in case \([E, \bar{E}] \neq [0, \infty]\), it is possible that the values for \(E_1^*\) and \(E_2^*\) lie outside the range for \(E\), in which case (since the \(\Pi_i\) are concave), the optimum will occur at the boundary. Hence

\[
E_i^* = \begin{cases} E, & \text{if } E_1 < E_1^* \\ E_1, & \text{if } E_1 \leq E < E_1^* \\ E_1^*, & \text{if } E_1 > \bar{E}. \end{cases}
\]

The reader can verify that the expressions in Equations (29)–(30) precisely amount to the above conditions.  

**Proof of Proposition 5.3(ii)–(iv).** When \(\beta < \beta^*\), the second-order derivatives \(\partial^2 \Pi_i / \partial E_i^2 > 0\) and the profit functions are strictly convex. Hence the maximum profit occurs at either \(E\) or \(\bar{E}\). Let us look at the profit difference \(\Delta \Pi_i = \Pi_i(E) - \Pi_i(\bar{E})\). When \(\alpha \leq (\bar{\theta} + \theta)/2\), we have

\[
\Delta \Pi_1 = \frac{(\bar{\gamma} - \gamma)[(\bar{\gamma} + \gamma)(\bar{E} + 2R_2) - 4V(\alpha - 2\bar{\theta} + \gamma)]}{36V} \bar{E} - \Delta E^2
\]

\[
- \beta(\bar{E} - E)^2
\]

\[
- \beta(\bar{E} + E)
\]

\[
\text{and}
\]

\[
\Delta \Pi_2 = \frac{(\bar{\gamma} - \gamma)[4V(\alpha + \bar{\theta} - 2\theta) - (\bar{\gamma} + \gamma)(2R_1 - \bar{E} - E)]}{36V} \bar{E} - \Delta E^2
\]

\[
- \beta(\bar{E} + E)
\]

When \(\alpha > (\bar{\theta} + \theta)/2\), the roles of \(\Delta \Pi_1\) and \(\Delta \Pi_2\) are precisely reversed (i.e., replace subscript 1 with 2 and vice versa in the expressions).  

Depending on the signs of \(\Delta \Pi_i\), we can obtain the following equilibrium solutions:

- \(E_1^* = E_2^* = E\) if \(\Delta \Pi_1 < 0\) and \(\Delta \Pi_2 < 0\), which implies that \(\beta > \beta_1\);
• $E_1' = E$, $E_2 = \bar{E}$ if $\Delta \Pi_1 > 0$ and $\Delta \Pi_2 < 0$, which implies that $\beta_2 < \beta < \beta_1$; and
• $E_1' = E_2 = \bar{E}$ if $\Delta \Pi_1 > 0$ and $\Delta \Pi_2 > 0$, which implies that $\beta < \beta_2$.

Finally, the above solutions require that $\beta < \beta^*$. Interested readers can verify that conditions of $\beta$ in Proposition 5.3(ii)-(iv) take care of this. $\square$

Appendix B

Niche Equilibria

In the main paper, we presented the equilibrium price where the cost of providing service is not extreme, i.e., the value of $\alpha$ was bounded in some range relative to the consumers’ tastes ($\theta$, $\gamma$). When such conditions are not satisfied, one firm becomes a prevalent player and will grab most of the market. The other firm then becomes a niche player, which serves consumers with only extreme preferences. In such cases, the demand for the niche firm is no longer expressed as the area of a trapezoid, but becomes a triangle in one of the corners of the consumers’ preference space as shown in Figure B.1. We now present the niche equilibrium prices below (detailed derivation of the results is available upon request).

Proposition B.1. The equilibrium prices for the niche equilibria are as follows.

**Niche 1.** When $\varphi \geq (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)$ and $\alpha < \alpha_1$, or $0 < \varphi \leq (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)$ and $\alpha < \alpha_2$,

\[ p_1^* = \alpha H_1 + \frac{1}{2} (E_1 - E_2) [\xi_1 + \sqrt{\xi_1^2 + 8D\varphi}], \]  

(B1)

**Figure B.1 Niche Equilibria**

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Firm 1 or 2</td>
<td>Firm 2</td>
</tr>
</tbody>
</table>

and

\[ p_2^* = \alpha H_2 + \frac{1}{2} (E_1 - E_2) [-\xi_1 + \sqrt{\xi_1^2 + 8D\varphi}], \]  

(B2)

where $\xi_1 = \gamma + \varphi(\bar{\theta} - \alpha)$.

**Niche 2.** When $\varphi \leq -(\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)$ and $\alpha < \alpha_3$, or $-(\bar{\gamma} - \gamma)/(\bar{\theta} - \theta) \leq \varphi < 0$ and $\alpha < \alpha_4$,

\[ p_1^* = \alpha H_1 + \frac{1}{2} (E_1 - E_2) [\xi_2 + \sqrt{\xi_2^2 - 8D\varphi}], \]  

(B3)

and

\[ p_2^* = \alpha H_2 + \frac{1}{2} (E_1 - E_2) [-\xi_2 + \sqrt{\xi_2^2 - 8D\varphi}], \]  

(B4)

where $\xi_2 = \gamma + \varphi(\bar{\theta} - \alpha)$.

**Niche 3.** When $\varphi \geq (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)$ and $\alpha > \alpha_2$, or $0 < \varphi \leq (\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)$ and $\alpha > \alpha_1$,

\[ p_1^* = \alpha H_1 + \frac{1}{2} (E_1 - E_2) [\xi_3 + \sqrt{\xi_3^2 + 8D\varphi}], \]  

(B5)

and

\[ p_2^* = \alpha H_2 + \frac{1}{2} (E_1 - E_2) [-\xi_3 + \sqrt{\xi_3^2 + 8D\varphi}], \]  

(B6)

where $\xi_3 = \gamma + \varphi(\bar{\theta} - \alpha)$.

**Niche 4.** When $\varphi \leq -(\bar{\gamma} - \gamma)/(\bar{\theta} - \theta)$ and $\alpha > \alpha_3$, or $-(\bar{\gamma} - \gamma)/(\bar{\theta} - \theta) \leq \varphi < 0$ and $\alpha > \alpha_4$,

\[ p_1^* = \alpha H_1 + \frac{1}{2} (E_1 - E_2) [5\xi_4 + 3\sqrt{\xi_4^2 - 8D\varphi}], \]  

(B7)

and

\[ p_2^* = \alpha H_2 + \frac{1}{2} (E_1 - E_2) [-\xi_4 + \sqrt{\xi_4^2 - 8D\varphi}], \]  

(B8)

where $\xi_4 = \gamma + \varphi(\bar{\theta} - \alpha)$.

It is clear that the market share for the niche firm cannot exceed that for the prevalent firm. Comparing the per unit net profit margin, we can find the following generic expression for the difference in per unit profit between the niche firm and the prevalent firm:

\[
(p_1^* - \alpha H_1) - (p_2^* - \alpha H_2) = \frac{1}{2} (E_1 - E_2) [\text{sgn}(E_n - E_d) \xi_l - \sqrt{\xi_l^2 + 8D\varphi}],
\]

where $n$ is the index for the niche firm and $d$ is the index for the prevalent firm, and $l$ is the index for the niche in question. It can be shown that the above expression is always nonpositive, regardless of the niche or whether the niche firm is a high or low service provider. Hence the total profit for the niche firm is always lower than that for the prevalent firm.

References


